Lesson 15: Finding this percentage of that

Goals

- Choose and create diagrams to calculate A% of B, and explain (orally) the solution method.
- Generalise a process for finding A% of B and justify (orally) why this can be abstracted as $\frac{A}{100} \times B$.
- Identify equivalent expressions that could be used to find A% of B and justify (orally) that they are equivalent.

Learning Targets

- I can solve different problems like "What is 40% of 60?" by dividing and multiplying.

Lesson Narrative

Students have practised solving three different types of percentage problems (corresponding to finding A, B, or C respectively when A% of B is C). This lesson focuses on finding "A% of B" as efficiently as possible. While the previous lesson used numbers that students could calculate mentally, the numbers in this lesson are purposefully chosen to be difficult for students to calculate mentally or to represent on a double number line diagram, so as to motivate them to find the simplest way to do the calculation by hand.

The third activity hints at work students will do in Year 8, namely finding a constant of proportionality and writing an equation to represent a proportional relationship.

Building On

- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Addressing

- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percentage.

Instructional Routines

- Stronger and Clearer Each Time
- Compare and Connect

- Discussion Supports
- Number Talk

Student Learning Goals

Let's solve percentage problems like a pro.

15.1 Number Talk: Decimals

Warm Up: 5 minutes

The purpose of this number talk is to help students multiply and divide decimal numbers by 100 in preparation for their work with percentages later in the lesson.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem, and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the task.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Find the value of each expression mentally.

$(0.23) \times 100$

$50 \div 100$

$145 \times \frac{1}{100}$

$7 \div 100$

Student Response

- 23. Possible reasoning: 23 hundredths times 100 is 23.
- 0.5. Possible reasoning: 50 is one half of 100, and one half can be written as 0.5.
- 1.45. Possible reasoning: 145 divided by 100 means the place value of each digit decreases so it becomes 1.45.

- 0.07. The answer is the same as 7 times $\frac{1}{100}$, which is $\frac{7}{100}$.

Activity Synthesis

Select a couple of students to share their answer and strategies for each problem. Record and display their explanations for all to see. After evaluating all four expressions, ask students:

- How is multiplying by $\frac{1}{100}$ related to division?
- What is important to remember about dividing a one digit number by 100?

To involve more students in the conversation, consider asking as the students share their ideas:

- Who can restate ___'s reasoning in a different way?
- Did anyone solve the problem the same way but would explain it differently?
- Did anyone solve the problem in a different way?
- Does anyone want to add on to ____'s strategy?
- Do you agree or disagree? Why?

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

15.2 Audience Size

15 minutes

In this activity, students encounter percentage problems that are inconvenient to solve by drawing double number lines, encouraging them to reason differently and begin noticing that P% of a number is $\frac{P}{100}$ times that number.

The first two questions, which ask students to find 30% and 140% of some values, can be solved using a familiar percentage,10%, as a stepping stone. For example, to solve for 30% of 250, students may find 10% or $\frac{1}{10}$ of 250 and then multiply the result by 3. This intermediate percentage does not work well for the last question, however, prompting them to find a workaround.

Students may resort to using a double number line, but may soon find it impractical. For example, they may decide to divide by 100% into 25 parts to find the value of 4%, which they can then multiply by 11. As time consuming drawing 25 tick marks is, it may

encourage students to look for structure that may open a shorter path to the solution. For example, once they know that they need tick marks every 4% and every 10 people, they can bypass the rest of the tick marks and multiply ($4 \times 11 = 44$ and $10 \times 11 = 110$) to find that 110 people attended literacy night.

Others may see that dividing the given value by 100 to find the value of 1% and then multiplying the result by the targeted percentage works well. Identify students who take this path so they can share later.

Instructional Routines

- Compare and Connect

Launch

Give students quiet think time to complete the activity and then time to share their explanation with a partner.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts. Check in with students after the first 2-3 minutes of work time. If students are finding that double number lines are not practical, invite others to share other strategies they have attempted so far.
Supports accessibility for: Organisation; Attention

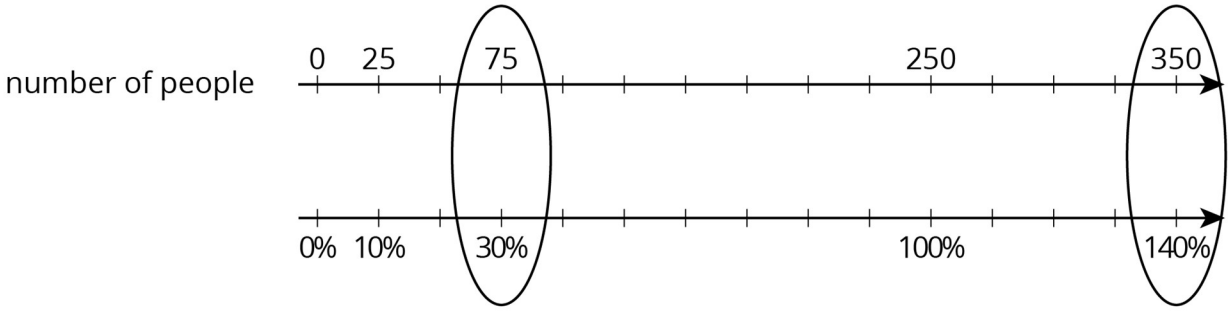
Student Task Statement

A school held several evening activities last month—a music concert, a basketball game, a drama play, and literacy night. The music concert was attended by 250 people. How many people came to each of the other activities?

1. Attendance at a basketball game was 30% of attendance at the concert.
2. Attendance at the drama play was 140% of attendance at the concert.
3. Attendance at literacy night was 44% of attendance at the concert.

Student Response

1. 75 people attended the basketball game.
2. 350 people attended the drama play. Possible strategies:



number of people	percentage
250	100
25	10
75	30
350	140

3. 110 people attended literacy night. Possible strategies:

   - There is 4% for every 10 people, so multiply $10 \times 11 = 110$.
   - First find 1%, then multiply by 44 to find 44%.

number of people	percentage
250	100
2.5	1
110	44

Are You Ready for More?

50% of the people who attended the drama play also attended the music concert. What percentage of the people who attended the music concert also attended the drama play?

Student Response

70%. Half of 350 is 175, and 175 is 70% of 250.

Activity Synthesis

Invite a couple of students to share their work on the first two questions, but focus the whole-class discussion on the last one. Select several students who effectively found 44% of 250 to share, saving the strategy involving 1% for last. If no one used this method, illustrate and explain it. Consider using a table in doing so, as shown below.

people	percentage
250	100
2.5	1
110	44

Guide students to see that this method of finding percentages can be generalised across all such problems, much like finding a unit rate is an effective way to solve any ratio problem.

Representing, Speaking: Compare and Connect. As students share their strategies, make sure you hear from students with different strategies for calculating the number of people who attended literacy night. Encourage students to make comparisons and connections between when they are able to use familiar percentages, and when they cannot. Ask questions such as, "How did you decide which percentages to use?" and "How is using 1%

as a strategy similar to or different from other percentages?" This will foster students' meta-awareness and support constructive conversations as they compare and connect the various ways to find a percentage of a quantity.
Design Principles(s): Cultivate conversation; Maximise meta-awareness

15.3 Everything is On Sale

15 minutes

In general, P% of something is $\frac{P}{100}$ times that thing. The purpose of this activity is to make this explicit. In this activity, students are asked to find 80% of several different values, generalise their process, and express their generalisation in different ways. As they make repeated calculations, students see more explicitly that P% of a number is $\frac{P}{100}$ times that number. Once they arrive at one or more generalisations, students practise articulating why each generalisation always works.

Instructional Routines

- Stronger and Clearer Each Time

Launch

Give students quiet think time to complete the activity and then time to share their explanation with a partner.

Representation: Internalise Comprehension. Activate or supply background knowledge about equivalent fractions. Incorporate explicit opportunities for review and practice if necessary. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing Writing, Speaking, Listening: Stronger and Clearer Each Time. Use this with successive pair shares to give students a structured opportunity to revise and refine their own strategies to determine 80% of any value to find a sale price. Make sure students include a table as well as an expression in their explanations. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., "Can you explain how…", "Will this expression always work? How do you know?", etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own strategies and learn about other ways to find the sale price of an item.
Design Principles(s): Optimise output (for explanation); Maximise meta-awareness

Student Task Statement

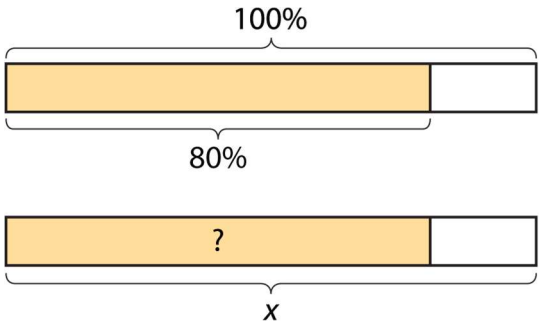
During a sale, every item in a store is 80% of its usual price.

1. If the usual price of a T-shirt is £10, what is its sale price?

2. The usual prices of five items are shown here. Find the sale price of each item.

	item 1	item 2	item 3	item 4	item 5
usual price	£1	£4	£10	£55	£120
sale price					

3. You found 80% of many values. Was there a process you repeated over and over to find the sale prices? If so, describe it.



4. Select **all** of the expressions that could be used to find 80% of x. Be prepared to explain your reasoning.

$\frac{8}{100} \times x$

$\frac{80}{100} \times x$

$\frac{8}{10} \times x$

$\frac{4}{10} \times x$

$\frac{8}{5} \times x$

$\frac{4}{5} \times x$

$80 \times x$

$8 \times x$

$(0.8) \times x$

$(0.08) \times x$

Student Response

1. £8

2. Here is the table:

	item 1	item 2	item 3	item 4	item 5

usual price	£1	£4	£10	£55	£120
sale price	£0.80	£3.20	£8	£44	£96

3. Answers vary. Sample response: Multiply each normal price by 0.8 (or by $\frac{80}{100}$) to find the sale price.

4. These expressions work:

   - $\frac{80}{100} \times x$ (See discussion.)
   - $\frac{8}{10} \times x$ because $\frac{80}{100} = \frac{8}{10}$
   - $\frac{4}{5} \times x$ because $\frac{8}{10} = \frac{4}{5}$
   - $(0.8) \times x$ because $\frac{80}{100} = 0.8$

Activity Synthesis

Make sure students have the correct values in the table for the second question. Then, discuss the expressions, starting with $\frac{80}{100} \times x$.

Guide students to see that if we know the value of 100%, dividing that value by 100 (or equivalently, multiplying it by $\frac{1}{100}$) tells us the corresponding value for 1%. We can then multiply that value by the desired percentage. Consider using a table to organise this argument, as shown below:

percentage	usual price
100	x
1	$\frac{1}{100} \times x$
80	$\frac{80}{100} \times x$

After everyone understands where this expression comes from, ask students to discuss the remaining expressions with a partner. This is a good opportunity for just-in-time review on equivalent fractions, if needed.

Lesson Synthesis

The main idea in this lesson is that to find P% of x, multiply: $\frac{P}{100} \times x$ Ask students to describe a procedure for finding a percentage of a number. If they struggle to describe a general method, ask about some specific examples, like "How could you find 32% of 500?" (You could multiply 500 by $\frac{32}{100}$.)

15.4 Ordering Percentages of Different Numbers

Cool Down: 5 minutes

In addition to assessing students ability to find A% of B, this cool-down allows students to practise arithmetic involving fractions or decimals (using KS2 techniques). This is a good review for upcoming units on fractions and decimals.

Anticipated Misconceptions

Students may put these amounts in order based on just the percentages or just the amount that corresponds to 100%, not realising that both parts of the expression affect the value. Demonstrate calculating 65% of 80 to get them on the right track.

Student Task Statement

Order these three values from least to greatest. Explain or show your reasoning.

65% of 80

82% of 50

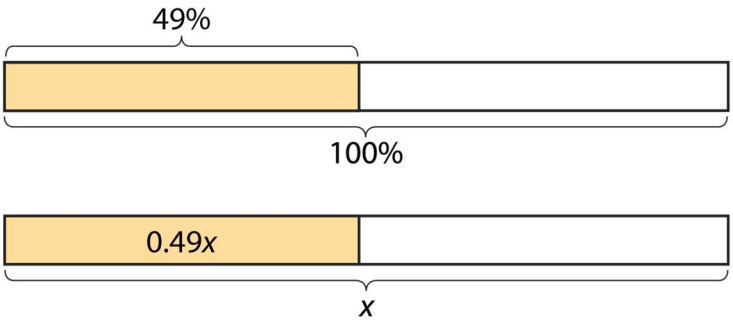
170% of 30

Student Response

The least is 82% of 50, because $(0.82) \times 50 = 41$. Next is 170% of 30, because $(1.7) \times 30 = 51$. The greatest is 65% of 80, $(0.65) \times 80 = 52$.

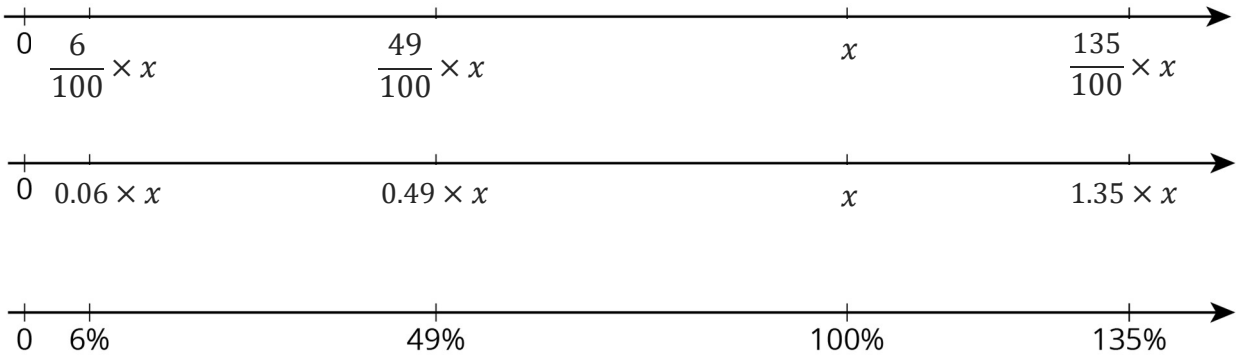
Student Lesson Summary

To find 49% of a number, we can multiply the number by $\frac{49}{100}$ or 0.49.



To find 135% of a number, we can multiply the number by $\frac{135}{100}$ or 1.35.

To find 6% of a number, we can multiply the number by $\frac{6}{100}$ or 0.06.



In general, to find P% of x, we can multiply: $\frac{P}{100} \times x$

Lesson 15 Practice Problems

Problem 1 Statement

a. To find 40% of 75, Priya calculates $\frac{2}{5} \times 75$. Does her calculation give the correct value for 40% of 75? Explain or show how you know.

b. If x represents a number, does $\frac{2}{5} \times x$ always represent 40% of that number? Explain your reasoning.

Solution

a. Yes. 40% is 0.4, and $(0.4) \times 75 = 30$. Using Priya's method: $\frac{2}{5} \times 75 = 30$.

b. Yes. 40% of x is $\frac{40}{100} \times x$. This is the same as $\frac{2}{5} \times x$, since $\frac{40}{100}$ and $\frac{2}{5}$ are equivalent fractions.

Problem 2 Statement

Han spent 75 minutes practising the piano over the weekend. For each question, explain or show your reasoning.

a. Priya practised the violin for 152% as much time as Han practised the piano. How long did she practise?

b. Tyler practised the clarinet for 64% as much time as Han practised the piano. How long did he practise?

Solution

a. 114 minutes. Sample reasoning: 152% of 75 minutes is $\frac{152}{100} \times 75 = 114$.

b. 48 minutes. Sample reasoning: 64% of 75 minutes is $\frac{64}{100} \times 75 = 48$.

Problem 3 Statement

Last Sunday 1 575 people visited the amusement park. 56% of the visitors were adults, 16% were teenagers, and 28% were children ages 12 and under. Find the number of adults, teenagers, and children that visited the park.

Solution

882 adults, 252 teenagers, and 441 children

Problem 4 Statement

Order from greatest to least:

- 55% of 180
- 300% of 26
- 12% of 700

Solution

55% of 180, 12% of 700, 300% of 26.

Problem 5 Statement

Complete each statement.

- 20% of 60 is _____
- 25% of _____ is 6
- _____% of 100 is 14
- 50% of 90 is _____
- 10% of _____ is 7
- 30% of 70 is _____

Solution

a. 12

b. 24

c. 14

d. 45

e. 70

f. 21

Problem 6 Statement

A shopper needs 24 sandwich rolls. The store sells identical rolls in 2 differently sized packages. They sell a six-pack for £5.28 and a four-pack for £3.40. Should the shopper buy 4 six-packs or 6 four-packs? Explain your reasoning.

Solution

6 four-packs is a better deal. The rolls in the six-pack are being sold at a rate of 88 pence each, because $5.28 \div 6 = 0.88$. The rolls in the four-pack are being sold at a rate of 85 pence each, because $3.40 \div 4 = 0.85$. The four-packs are a better deal, because the sandwich rolls have a cheaper unit rate.

Problem 7 Statement

On a field trip, there are 3 chaperones for every 20 students. There are 92 people on the trip. Answer these questions. If you get stuck, consider using a bar model.

a. How many chaperones are there?

b. How many children are there?

Solution

a. 12

b. 80