Mathematics Education Innovation

Lesson 4: Add & Subtract decimals with many non-zero digits

Goals

- Add or subtract decimals with multiple non-zero digits, and explain (orally) the solution method.
- Interpret a description (in written language) of a real-world situation involving decimals, and write an addition or subtraction problem to represent it.
- Recognise and explain (orally) that column addition and subtraction are efficient strategies for adding and subtracting decimals, especially decimals with multiple non-zero digits.

Learning Targets

- I can solve problems that involve addition and subtraction of decimals.

Lesson Narrative

This lesson strengthens students' ability to add and subtract decimals, enabling them to work toward fluency. Students encounter longer decimals (beyond thousandths), find missing addends, and work with decimals in the context of situations. They decide which operation (addition or subtraction) to perform and which strategy to use when finding sums and differences. They also reinforce the idea that we can express a decimal in different but equivalent ways, and that writing additional zeros after the last non-zero digit in a decimal does not change its value. Students use this understanding to practise subtracting numbers with more decimal places from those with fewer decimal places (e.g., $1.9 - 0.4563$).

To solve these problems, students must lean heavily on their understanding of base-ten numbers. Given problems such as $7 - ? = 3.4567$ and $0.404 + ? = 1$, they need to think carefully about the meaning of each place value, the meaning of addition and subtraction, and potential paths toward the solution. Along the way, they also begin to see patterns in the calculations, which enables them to become increasingly fluent in finding sums and differences.

Addressing

- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Three Reads
- Discussion Supports
- Notice and Wonder

- Think Pair Share

Required Preparation

Some students might find it helpful to use graph paper to help them align the digits for column addition and subtraction. Consider having graph paper accessible for the Decimals All Around activity.

Student Learning Goals

Let's practise adding and subtracting decimals.

4.1 The Cost of a Photo Print

Warm Up: 5 minutes

This warm-up prompts students to review the placement of the digits when subtracting decimal numbers using an efficient algorithm. The first question gives students a chance to think about how different placements of the 5 (the first number) affects the subtraction and the difference. It also gives insight to how students interpret the 5 and its value. For examples:

- Would they place the decimal point directly after the 5?
- When placing 5 above 7 (in the tenths place), do they see it as 5 ones or as 5 tenths?
- Do they wonder if the 5 is missing a decimal point in the second and third case?

In the second and third questions, a context is introduced for the same subtraction. Notice how students interpret the problem now; the equivalence of 5 and 5.00 should become much more apparent in a money context. Identify a couple of students who have correct responses so they could share later.

Instructional Routines

- Notice and Wonder

Launch

Give students up to 1 minute to think about the first question and then ask them to share what they noticed and wondered. Then, give students 1 minute of quiet work time for the remaining questions and follow with a whole-class discussion.

Student Task Statement

1. Here are three ways to write a subtraction calculation. What do you notice? What do you wonder?

$$\begin{array}{r@{}l} 5 \\ -\ 0.17 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ -\ 0.17 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ -\ 0.17 \\ \hline \end{array}$$

2. Clare bought a photo for 17 pence and paid with a £5 bill. Look at the previous question. Which way of writing the numbers could Clare use to find the change she should receive? Be prepared to explain how you know.

3. Find the amount of change that Clare should receive. Show your reasoning, and be prepared to explain how you calculate the difference of 0.17 and 5.

Student Response

1. Answers vary. Students may notice that no decimal point is shown in the 5 and that 5 is lined up with a different decimal place relative to the 0.17 each time. Students may wonder if the decimal point is missing in the 5 or if it is supposed to be lined up with the decimal point in 0.17.

2. The first setup (in which 5 and 0 line up vertically) is most conducive to correct subtraction. Sample reasoning: The 5 means £5.00, and it helps line up the pounds (the ones) and the pence (the tenths and hundredths) when subtracting.

3. £4.83

Activity Synthesis

Ask a few students to share which way they think Clare could write the calculation (in the second question) in order to find the amount of change. In this particular case, the first setup is the most conducive to correct computation, but there is not one correct answer. Students could find the answer with any one of the setups as long as they understand that the 5 represents £5.00. If any students choose the second or third setup because they can mentally subtract the values without lining them up by place values, invite them to share their reasoning. Ask if they would use the same strategy for dealing with longer decimals (e.g., $5.23 - 0.4879$) and, if not, what approach might be more conducive to correct calculation in those cases.

Encourage more students to be involved in the conversation by asking questions such as:

- "Do you agree or disagree? Why?"
- "Can anyone explain ___'s reasoning in their own words?"
- "What is important for us to think about when subtracting this way?"
- "How could we have solved this problem mentally?"

4.2 Decimals All Around

15 minutes

This activity has two parts and serves two purposes. The first two problems aim to help students see the limits of using base-ten diagrams to add and subtract numbers and to think about choosing more efficient methods. The latter three questions prompt students to reason about addition and subtraction of numbers with more decimal places in the context of situations. Students need to determine the appropriate operations for the given situations, perform the calculations, and then relate their answers back to the contexts.

As students work, check that they remember to line up the decimal points, write zeros when needed, and label their answers with units of measurement (for the last three questions). Identify one student to present each question. For the last question, there are two likely methods for solving (as outlined in the solutions). If both methods arise, they should both be presented.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Three Reads

Launch

Ask students to read the first 2 problems and think about which method of reasoning might work best. Give students 5 minutes of quiet time to work on these problems. Observe which methods students choose and check their work for accuracy.

Afterwards, take a few minutes to discuss their answers and methods. Ask one or more of the following questions before moving onto the second half of the activity:

- "Would base-ten diagrams work well for 318.8 − 94.63? Why or why not?" (No, it is time consuming to draw and keep track of so many pieces. Yes, I can draw 3 large squares for 300, 1 rectangle for 10, 8 small squares for 8, etc., remove pieces in the amount of 94.63, and count what is left.)
- "What challenges did you face in calculating 0.02 − 0.0116 using a diagram?" (I needed to take away 116 ten-thousandths from 2 hundredths, so I had to exchange hundredths with ten-thousandths, which meant drawing or showing many smaller pieces.) "What about using column subtraction?" (The first number is two digits shorter than the second, which meant some extra steps.)
- "When might base-ten diagrams be a good choice for finding a difference (or sum)?" (When the numbers are simple or short so the diagrams do not take too long to draw.)
- "When might an algorithm be a good choice for finding a difference (or sum)?" (For a difference like 6.739258 − 0.8536672, it will be time consuming to draw pictures and label each place value. The algorithm takes some time as well but is more efficient than drawing pictures.)

After this discussion, have students work independently on the remaining problems. During this time, monitor and select individuals to share their work during the whole-class discussion.

Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2–3 minutes of work time.

Check to make sure students have proper decimal point and place value unit alignment prior to calculating the value of each expression. Some students may benefit from access to graph paper to aide in proper alignment of decimals.
Supports accessibility for: Organisation; Visual-spatial processing Reading: Three Reads. Use this routine to support reading comprehension of the problem about Lin's grandmother. After a shared reading of the question, ask students "what is this situation about?" (Lin's grandmother ordered needles; the needles were different lengths). Use this read to clarify any unfamiliar language such as pharmacist or administer. After the second read, students list any quantities that can be counted or measured, without focusing on specific values (length of each type of needle, in inches). After the third read, invite students to discuss possible strategies to solve the problem. If needed, repeat this support with the remaining word problems. This helps students connect the language in the word problem and the reasoning needed to solve the problem.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Students may benefit from using graph paper for calculations; they can place one digit in each box for proper decimal point placement and place value unit alignment. For the last question, students might confuse the amount of zeros in 1 millionth. It might be helpful to write 0.000001 on the board as reference.

Student Task Statement

1. Find the value of each expression. Show your reasoning.
    a. $11.3 - 9.5$
    b. $318.8 - 94.63$
    c. $0.02 - 0.0116$
2. Discuss with a partner:
    - Which method or methods did you use in the previous question? Why?
    - In what ways were your methods effective? Was there an expression for which your methods did not work as well as expected?
3. Lin's grandmother ordered needles that were 0.3125 inches long to administer her medication, but the pharmacist sent her needles that were 0.6875 inches long. How much longer were these needles than the ones she ordered? Show your reasoning.
4. There is 0.162 litre of water in a 1-litre bottle. How much more water should be put in the bottle so it contains exactly 1 litre? Show your reasoning.
5. One micrometre is 1 millionth of a metre. A red blood cell is about 7.5 micrometres in diameter. A coarse grain of sand is about 70 micrometres in diameter. Find the difference between the two diameters in *metres*. Show your reasoning.

Student Response

1.

   a. 1.8

   b. 224.17

   c. 0.0084

2. Answers vary.

3. 0.375 inches. $0.6875 - 0.3125 = 0.375$

4. $1 - 0.162 = 0.838$ 0.838 litres

5. Answers vary. Sample responses:

   - The blood cell is 7.5 micrometres, and the grain of sand is 70 micrometres. The difference is 62.5 micrometres. Since 1 micrometre is 0.000001 metres, this is 0.0000625 metres.

   - The bacteria is 0.0000075 metres, and the grain of sand is 0.00007 metres. The difference is 0.0000625 metres.

Activity Synthesis

Discussions on the first 2 questions are outlined in the Launch section. Focus the discussion here on the last 3 questions, i.e., on how students interpreted the problems, determined which operation to perform, and found the sum or difference (including how they handled any regroupings). Select a few previously identified students to share their responses and reasoning.

If some students chose base-ten diagrams for the calculation, contrast its efficiency with that of numerical calculations. The numbers in these problems have enough decimal places that using base-ten representations (physical blocks, drawings, or digital representations) would be cumbersome, making a numerical calculation appealing.

4.3 Missing Numbers

15 minutes

Students deepen their understanding of regrouping by tackling problems that are more challenging and that prompt them to notice and use structure. Students build on both their work with whole-number differences (such as $1\,000 - 256$) to find differences such as $1 - 0.256$. To add and subtract digits, they may think in terms of grouping and ungrouping base-ten units, but there are also other opportunities to use structure here. Let's take the example $1\,000 - 256$. Since $1\,000 = 999 + 1$, students could calculate $1\,000 - 256$ by first finding $999 - 256 = 743$, and then adding 1 to get 744. They could use the same reasoning to find sums and differences of decimals.

As students work and discuss their responses, notice the different ways students reason about the addition and subtraction. Identify a few students with differing approaches to share later.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 8–10 minutes of quiet work time and 2–3 minutes to discuss their answers with a partner. Follow with a whole-class discussion.

Anticipated Misconceptions

If students use a guess-and-check method without success, encourage them to try working backwards. Consider scaffolding with a simpler problem involving whole numbers, such as $10-? = 3$. We can find the answer by thinking: $3+? = 10$, or $10 - 3 =?$.

Student Task Statement

Write the missing digits in each calculation so that the value of each sum or difference is correct. Be prepared to explain your reasoning.

1.

  0 . 4 0 4
+ ☐ ☐ ☐ ☐
  1

2.

  9 . 8 7 6 5
+ ☐ ☐ ☐ ☐ ☐ ☐
  1 0

3.

  0 . 7
− ☐ ☐ ☐ ☐
  0 . 0 1 2

4.

  7
− ☐ ☐ ☐ ☐ ☐
  3 . 4 5 6 7

5.

  7 0
− ☐ ☐ ☐ ☐ ☐ ☐
  0 . 0 0 8 9

Student Response

1.

  0 . 4 0 4
+ 0 . 5 9 6
  1 . 0 0 0

2.

  9 . 8 7 6 5
+ ☐ 0 . 1 2 3 5
  1 0 . 0 0 0 0

3.

  0 . 7 0 0
− 0 . 6 8 8
  0 . 0 1 2

4.

  7 . 0 0 0 0
− 3 . 5 4 3 3
  3 . 4 5 6 7

5.

  7 0 . 0 0 0 0
− 6 9 . 9 9 1 1
  0 . 0 0 8 9

Are You Ready for More?

In a cryptarithmetic puzzle, the digits 0-9 are represented using the first 10 letters of the alphabet. Use your understanding of decimal addition to determine which digits go with the letters A, B, C, D, E, F, G, H, I, and J. How many possibilities can you find?

```
   I H F . I J
 + J I I . F I
 -------------
   E J I . I E
```

Student Response

- E: 4, F: 0, H: 2, I: 1, J: 3
- E: 9, F: 0, H: 5, I: 2, J: 7

Activity Synthesis

Select previously identified students to share their responses. Ask students how they approached the addition and subtraction problems, whether they work backwards, write additional zeros, or use other strategies to find the missing numbers.

Conclude the discussion by asking students how their work would change in the fourth question ($7-?=3.4567$) if they were to replace 7 with 6.9999. Give students a moment to think about the question, and then ask:

- "How might the number 6.9999 help us find the missing number?" (The 9s make it easier to find each missing digit, so we can find the difference between 6.9999 and 3.4567, and then add 0.0001 to the result because 6.9999 is 0.0001 less than 7.)
- "How would this method work for a problem such as the second question: $9.8765+?=$ 10?" (We can replace 10 with 9.9999, determine the missing number, and then add 0.0001 to that number).

This strategy is effective because it eliminates the "extra zeros" and the need to compose or decompose.

Speaking, Listening, Representing: Discussion Supports. Use this routine to support whole-class discussion. Display each of the five problems for all to see. For each response that is shared, ask students to restate what they heard using precise mathematical language. Annotate the display to illustrate the steps students describe, and label the strategy that was used next to each question (for example, "work backwards," or "write additional zeros"). Invite students to suggest additional details to include on the display that will support their understanding of each approach.
Design Principle(s): Support sense-making

Lesson Synthesis

In this lesson, students practised adding and subtracting numbers with many decimal places, both in and outside of the context of situations. They noticed the benefits of column calculations and used its structure to solve problems.

- Which problems did you solve with addition (or subtraction)? How did you know to use addition (or subtraction)?
- How did you find the sums or differences? Why did you choose that method?
- Did you use the same method for all problems? Why or why not?
- When subtracting a number with fewer decimal places by another number with more decimal places, such as $2.4 - 0.1587$, what strategies might be helpful? (We can think of 2.4 as 2.4000 and use ungrouping to subtract 0.1587 from it. Or we can think of the 2.4 as $2.3999 + 0.0001$, line up and subtract 0.1587 from 2.3999, and then add 0.0001 back to the difference.)

4.4 Taller and Farther

Cool Down: 5 minutes

Student Task Statement

1. Diego is 59.5 inches tall. His brother is 40.125 inches tall. How much taller than his brother is Diego? Show your reasoning.
2. A runner has run 1.192 kilometres of a 10-kilometre race. How much farther does he need to run to finish the race? Show your reasoning.

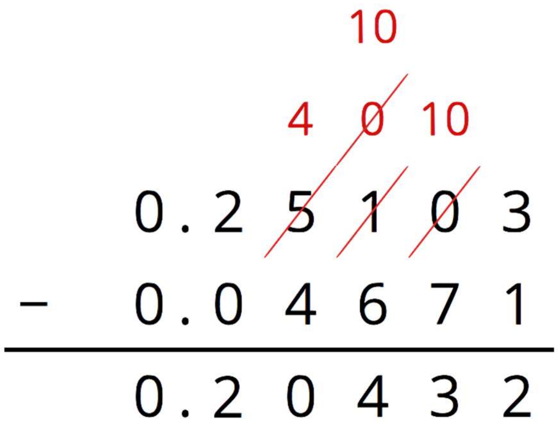
Student Response

1. 19.375 inches because $59.5 - 40.125 = 19.375$
2. 8.808 km because $10 - 1.192 = 8.808$

Student Lesson Summary

Base-ten diagrams work best for representing subtraction of numbers with few non-zero digits, such as $0.16 - 0.09$. For numbers with many non-zero digits, such as $0.25103 - 0.04671$, it would take a long time to draw the base-ten diagram. With column subtraction, we can find this difference efficiently.

Thinking about base-ten diagrams can help us make sense of this calculation.



The thousandth in 0.25103 is ungrouped (or decomposed) to make 10 ten-thousandths so that we can subtract 7 ten-thousandths. Similarly, one of the hundredths in 0.25103 is ungrouped (or decomposed) to make 10 thousandths.

Lesson 4 Practice Problems

1. **Problem 1 Statement**

   For each subtraction problem, circle the correct calculation.

   a. 7.2 − 3.67

   b. 16 − 1.4

a.

$$\begin{array}{r} 7.2 \\ -3.67 \\ \hline 3.05 \end{array} \qquad \begin{array}{r} 07.2 \\ -3.67 \\ \hline 3.05 \end{array} \qquad \begin{array}{r} 7.20 \\ -3.67 \\ \hline 3.53 \end{array}$$

b.

$$\begin{array}{r} 16 \\ -\ 1.4 \\ \hline 0.2 \end{array} \qquad \begin{array}{r} 16.0 \\ -1.40 \\ \hline 0.20 \end{array} \qquad \begin{array}{r} 16.0 \\ -\ 1.4 \\ \hline 14.6 \end{array}$$

Solution

a.

7.2	0 7.2	7.2 0
− 3.6 7	− 3.6 7	− 3.6 7
3.0 5	3.0 5	3.5 3

b.

1 6	1 6.0	1 6.0
− 1.4	− 1.4 0	− 1.4
0.2	0.2 0	1 4.6

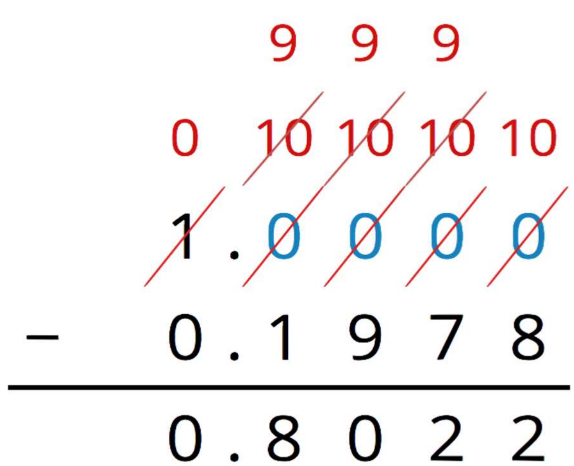
2. **Problem 2 Statement**

Explain how you could find the difference of 1 and 0.1978.

Solution

Answers vary. Sample responses:

- 1 can be ungrouped into 10,000 ten-thousandths. 0.1978 is 1,978 ten-thousandths. To find the difference, we subtract: $10,000 - 1,978 = 8,022$. The difference is 8,022 ten-thousandths or 0.8022.
- 1 can be written as 1.0000. In column subtraction, we can show the 1 being ungrouped into 10 tenths, 1 of those tenths being ungrouped into 10 hundredths, 1 of those hundredths being ungrouped into 10 thousandths, and 1 of the thousandths being ungrouped into 10 ten-thousandths. Subtracting 0.1978 from those digits gives us 0.8022.



3. **Problem 3 Statement**

A bag of chocolates is labelled to contain 0.384 pound of chocolates. The actual weight of the chocolates is 0.3798 pound.

a. Are the chocolates heavier or lighter than the weight stated on the label? Explain how you know.

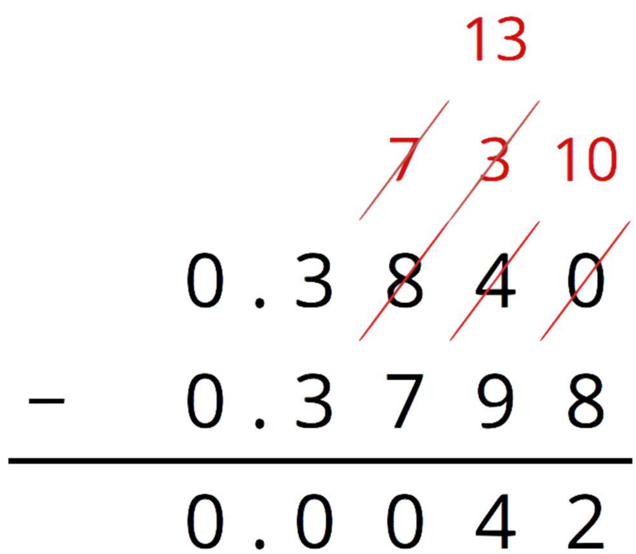
b. How much heavier or lighter are the chocolates than stated on the label? Show your reasoning.

Solution

a. Lighter. Reasoning varies. Sample reasoning: 0.3798 is 3 798 ten-thousandths. 0.384 is 384 thousandths, which is equal to 3 840 ten-thousandths, so 0.384 is greater than 0.3798.

b. 0.0042 ounce lighter. Reasoning varies. Sample reasoning:

- 3 840 ten-thousandths subtracted by 3 798 ten-thousandths is 22 ten-thousandths, because $3\,840 - 3\,798 = 42$.
- 0.3798 is 0.0002 away from 0.3800, and 0.3800 is 0.004 away from 0.384, so 0.3798 is $(0.0002 + 0.004)$ or 0.0042 away from 0.384.
- $0.384 - 0.3798 = 0.0042$.



4. **Problem 4 Statement**

Complete the calculations so that each shows the correct sum.

a.

$$\begin{array}{r} 1.036 \\ +\ \boxed{}\boxed{}\boxed{}\boxed{} \\ \hline 4 \end{array}$$

b.

$$\begin{array}{r} 0.738 \\ +\ \boxed{}\boxed{}\boxed{}\boxed{} \\ \hline 1 \end{array}$$

c.

$$\begin{array}{r} 0.5137 \\ +\ \boxed{}\boxed{}\boxed{}\boxed{}\boxed{} \\ \hline 1 \end{array}$$

Solution

a. 2.964

b. 0.262

c. 0.4863

a.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 1.036 \\ +\ \boxed{2.}\boxed{9}\boxed{6}\boxed{4} \\ \hline 4.000 \end{array}$$

b.

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 0.738 \\ +\ \boxed{0.}\boxed{2}\boxed{6}\boxed{2} \\ \hline 1.000 \end{array}$$

c.

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ 1} \\ 0.5137 \\ +\ \boxed{0.}\boxed{4}\boxed{8}\boxed{6}\boxed{3} \\ \hline 1.0000 \end{array}$$

5. **Problem 5 Statement**

A shipping company is loading cube-shaped crates into a larger cube-shaped container. The smaller cubes have side lengths of $2\frac{1}{2}$ feet, and the larger shipping container has side lengths of 10 feet. How many crates will fit in the large shipping container? Explain your reasoning.

Solution

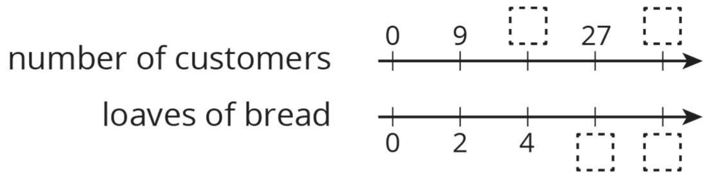
64 crates. Reasoning varies. Sample reasoning:

- Four crates can fit in a length of 10 feet because $4 \times 2\frac{1}{2} = 10$. So the container can fit $4 \times 4 \times 4$ or 64 crates.
- The volume of the larger container is 1000 cubic feet because $10 \times 10 \times 10 = 1\,000$. The volume of a crate is $15\frac{5}{8}$, since $2\frac{1}{2} \times 2\frac{1}{2} \times 2\frac{1}{2} = 15\frac{5}{8}$. Then 64 crates fit inside the container because $1\,000 \div 15\frac{5}{8} = 64$.

6. **Problem 6 Statement**

For every 9 customers, the chef prepares 2 loaves of bread.

a. Here is double number line showing varying numbers of customers and the loaves prepared. Complete the missing information.



b. The same information is shown on a table. Complete the missing information.

customers	loaves
9	2
	4
27	
	14
1	

c. Use either representation to answer these questions.

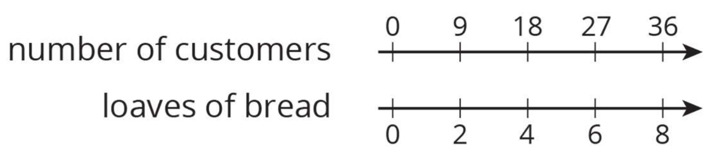
– How many loaves are needed for 63 customers?

– How many customers are there if the chef prepares 20 loaves?

– How much of a loaf is prepared for each customer?

Solution

a. See the double number line.



customers	loaves
9	2
18	4
27	6
63	14
1	$\frac{2}{9}$ or equivalent

- 14 loaves
- 90 customers
- $\frac{2}{9}$ of a loaf

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