Lesson 2: Adjacent angles

Goals

- Comprehend the terms “complementary” and “supplementary” (in spoken and written language) as they describe pairs of angles.
- Explain (orally and in writing) how to find the size of an unknown angle, given adjacent complementary or supplementary angles.
- Generalise (orally) that when a straight angle or a right angle is decomposed, the sizes of the resulting angles add up to 180° or 90°, respectively.

Learning Targets

- I can find the size of unknown angles by reasoning about complementary or supplementary angles.
- I can recognise when adjacent angles are complementary or supplementary.

Lesson Narrative

In this lesson, students are introduced to the terms **complementary**, for describing two angles whose sizes add to 90°, and **supplementary**, for describing two angles whose sizes add to 180°. They practise finding an unknown angle given the size of another angle that is complementary or supplementary.

Many of the angles in this lesson share the same vertex as another angle, so students need to be careful when naming each angle in addition to describing the relationship between pairs of angles.

Building On

- Geometric measurement: understand concepts of angle and measure angles.

Addressing

- Solve real-life and mathematical problems involving the size of angles, area, surface area, and volume.
- Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Instructional Routines

- Clarify, Critique, Correct
- Discussion Supports

Required Preparation

Cut blank paper in half so that each student can have 2 half sheets of paper. It is very important that these cuts are completely straight and exactly perpendicular to the sides being cut for this activity to work.

Prepare to distribute scissors, straightedges, and protractors.

Student Learning Goals

Let's look at some special pairs of angles.

2.1 Estimating the size of Angles

Warm Up: 5 minutes

The purpose of this warm-up is for students to estimate the size of angles in degrees (without a protractor) based on angles that are familiar. In the first two rows, an angle that is close to either a right angle or straight angle is given, and students could use this as a reference angle for the other angles in the row.

Asking students to share a wrong estimate first is a good strategy for launching the activity, because students are more confident in sharing wrong estimates, and it can help them start to consider what would be a more correct estimate.

As student discuss with their partner, monitor for students who use phrases such as:

- "a little more than 90 degrees"
- "almost a straight line"
- "a little less than 360 degrees"

Launch

Arrange students in groups of 2. Do not supply protractors or pattern blocks; let students know that in this activity they are *estimating* the size of each angle in degrees.

Before students begin, ask students to think of an estimate that is *definitely wrong* for angle GHI. Invite a few students to share and explain why it is wrong. Then, ask the same students to come up with an actual estimate.

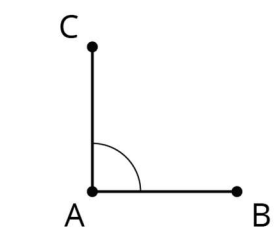
Give students 2 minutes of quiet work time followed by a partner and whole-class discussion.

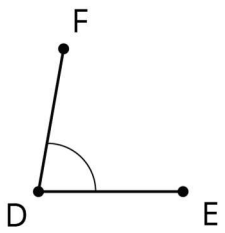
Anticipated Misconceptions

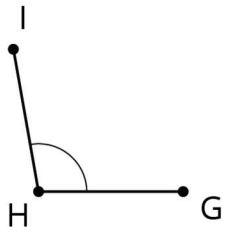
Students will be tempted to figure out the exact angle sizes, encourage students to use estimation to see how close they can get using benchmark angles that they have encountered (90°, 180°, 360°, etc).

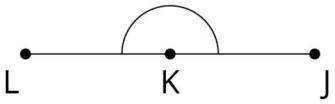
Student Task Statement

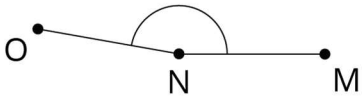
Estimate the size, in degrees, of each indicated angle.

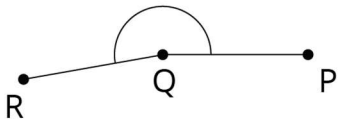


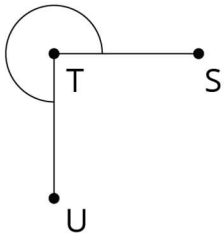


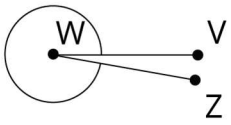












Student Response

- Angle CAB measures about 90 degrees.
- Angle FDE measures about 80 degrees, since it's a little less than a right angle.
- Angle GHI measures about 95 degrees, since it's a little more than a right angle.
- Angle JKL measures about 180 degrees.
- Angle MNO measures about 175 degrees, since it's a little less than a straight angle.
- Angle PQR measures about 185 degrees, since it's a little more than a straight angle.
- Angle STU measures about 270 degrees.
- Angle VWZ measures about 355 degrees since it's almost a full rotation.

Activity Synthesis

Select previously identified students to share an estimate of the size, in degrees, for each angle; record and display their responses for all to see. Poll the class if they agree or disagree after each one.

After each of the angles is discussed, ask students what tools they might use to check their answers themselves. If/when a protractor is mentioned, ask how they could use it to find the size of angles like STU or VWZ when the protractors usually only go to 180. (For example, they could find the size of the angle that is less than 180, and subtract it from 360.)

2.2 Cutting Rectangles

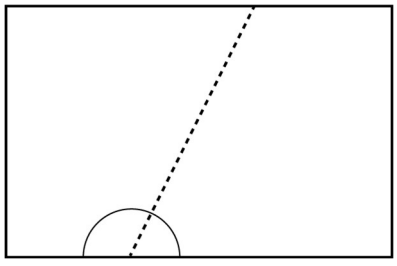
10 minutes

The purpose of this activity is to provide a tangible experience with complementary and supplementary angles, which will be formally defined in the next activity. Students cut sheets of paper in two ways to see the decomposition of straight and right angles. In later activities and lessons, students will continue working with the fact that specific angles can be composed to make straight or right angles as a strategy for finding the size of an unknown angle. In this activity, the language and vocabulary that students use during this task should be allowed to be loose as we will develop it more precisely in the following activities and lessons.

This activity gives students another opportunity to practise using a protractor to measure angles. Especially when students are measuring the angles they cut from the straight angle, it should be readily apparent if they are reading their angle from the wrong side of the protractor. For example, if students think that both of their angles measure 140 degrees, the papers can be positioned on top of each other to show that it is unreasonable to conclude that both angles have the same size.

As students work on the task, monitor for students whose angles sum to exactly 180 degrees (and exactly 90 degrees) and students whose angles sum close to 180 degrees (and close to 90 degrees).

It is recommended **not** to show this image to students or they may try to copy the image rather than making their cut in different ways, but it is included here to clarify the instructions.



Instructional Routines

- Clarify, Critique, Correct

Launch

Distribute two half-sheets of blank paper per student and provide access to straightedges, scissors, and protractors. Emphasise that students should use a straightedge to draw the line they will cut along before they use scissors. Give students 3–5 minutes of quiet work time followed by a whole-class discussion.

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide handouts of an angle representation for students to draw on or highlight and measure.
Supports accessibility for: Visual - Spatial Processing; Conceptual processing Reading, Conversing: Clarify, Correct and Critique. Display a sample student response that illustrates improper placement of the protractor that results in inaccurate measurements or measurements that are greater than 180 degrees when summed. Ask students to work with a partner to identify any errors, and to talk possible reasons for them. Invite students write a corrected response based on their conversation. Improved statements should include reference to the proper placement of protractors and how to determine the sizes of angles with protractors. This helps students evaluate, and improve on, the written mathematical arguments of others.
Design Principle(s): Support sense-making; Maximise meta-awareness

Anticipated Misconceptions

Some students may want to make their first cut perpendicular to the side of the paper that they are cutting from. This could make it harder for them to notice the pattern that their two angles sum to 180 degrees. Encourage them to cut at a different angle.

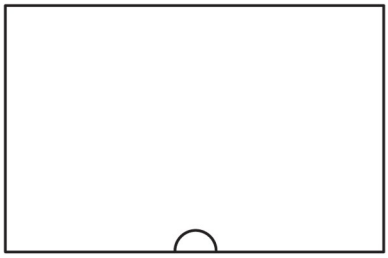
Some students may struggle to position their protractor correctly to measure each cut piece. Prompt them to position the point that represents the centre of the protractor on the vertex of their angle and line up the 0 on their protractor with one side of the angle, so that they can measure to the other side.

Some students may get angle sizes that do not add up to exactly 180 (or 90) degrees. If the sum is close to 180 (or 90) degrees, this should be allowed during the work time and discussed during the activity synthesis. If the sum is not close to 180 (or 90), ask students to show you how they lined up the protractor to measure their angles.

Student Task Statement

Your teacher will give you two small, rectangular papers.

1. On one of the papers, draw a small semi-circle in the middle of one side.



2. Cut a straight line, starting from the centre of the semi-circle, all the way across the paper to make 2 separate pieces. (Your cut does not need to be perpendicular to the side of the paper.)

3. On each of these two pieces, measure the angle that is marked by part of a circle. Label the angle size on the piece.

4. What do you notice about the sizes of these angles?

5. Clare measured 70 degrees on one of her pieces. Predict the size of the angle of her other piece.

6. On the other rectangular paper, draw a small quarter-circle in one of the corners.

7. Repeat the previous steps to cut, measure, and label the two angles marked by part of a circle.

8. What do you notice about the sizes of these angles?

9. Priya measured 53 degrees on one of her pieces. Predict the size of the angle of her other piece.

Student Response

1-3. Answers vary.

4. The two angles should sum to 180°.

5. Clare's other angle should measure 110°, because $70 + 110 = 180$.

6-7. Answers vary.

8. The two angles should sum to 90°.

9. Priya's other angle should measure 37°, because $53 + 37 = 90$.

Activity Synthesis

Select previously identified students to share their angle sizes within the decomposed straight angle. Record each answer displayed for all to see and ask:

- "What do you notice about the sizes of the pairs of angles?" (They all sum to about 180 degrees.)
- "Why do you think this is?" (They started out as a straight angle and were cut apart.)
- "Why do you think some people got measurements that do not sum to exactly 180 degrees?" (measurement error)

Poll the class on the size of Clare's second angle. Invite students to share different strategies they used. It is not important to formalise a process for solving supplementary angles at this point, because that will be addressed more in the next activity.

Select previously identified students to share their angle sizes within the decomposed right angle. Record each answer displayed for all to see and ask similar questions as before to guide students to articulate that these pairs of angles should sum to 90 degrees.

Ask students to think about how they solved for the size of Priya's second angle compared to how they solved for the size of Clare's second angle. "What was the same, and what was different?"

- The process of using one known angle and what they should both add up to was the same.
- The sums were different: 180 for Clare's angles and 90 for Priya's angles.

2.3 Is It a Complement or Supplement?

10 minutes

In this activity, students begin to formalise a process for finding the sizes of angles that are complements and supplements of angles with known sizes. After they have worked on the activity and shared their solutions, they are introduced to the vocabulary terms **complementary** and **supplementary.** Complementary describes angles whose sizes sum to 90 degrees and supplementary describes angles whose sizes sum to 180 degrees.

Monitor for students who use or explain different ways to calculate the size of the unknown angle. For example, when finding the size of angle KOM, some might write $38 + x = 180$ and some might write $180 - 38$.

Instructional Routines

- Discussion Supports

Launch

Remind students that in the previous activity they used straight angles and right angles to help figure out the size of unknown angles. In this activity, they are doing something similar but must figure out the size of the unknown angle without a protractor.

Arrange students in groups of 2. Give students 3–4 minutes of quiet work time, followed by a partner and whole-class discussion.

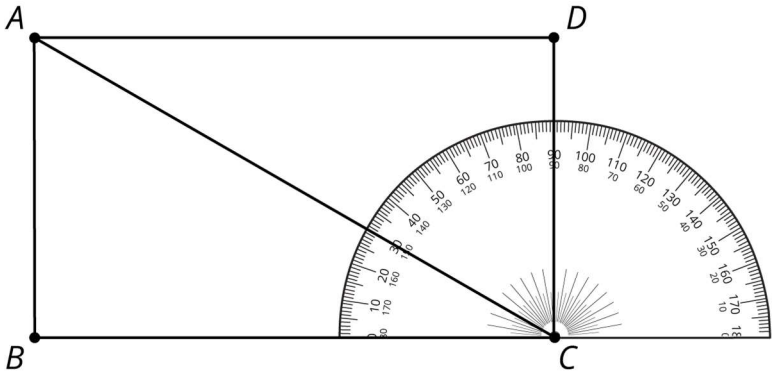
Representation: Develop Language and Symbols. Create and maintain charts to display definitions and examples of complementary angles and supplementary angles.
Supports accessibility for: Conceptual processing; Memory

Anticipated Misconceptions

If students get stuck on the first problem, ask them what measurement do angles BCA and ACD have to add up to. This should get them started noticing the relationships between all the angles involved.

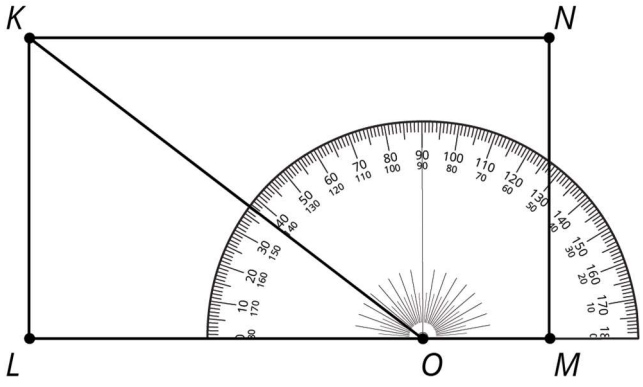
Student Task Statement

1. Use the protractor in the picture to find the size of angles BCA and BCD.



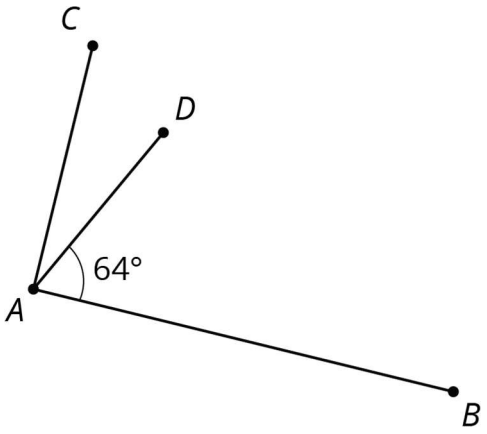
2. Explain how to find the size of angle ACD without repositioning the protractor.

3. Use the protractor in the picture to find the size of angles LOK and LOM.

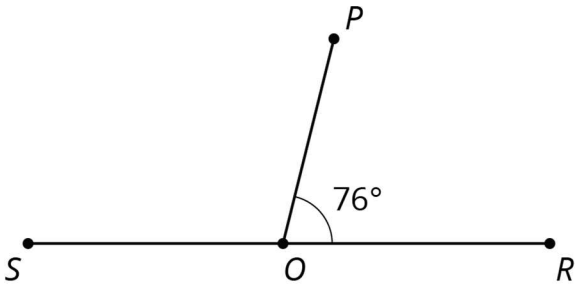


4. Explain how to find the size of angle KOM without repositioning the protractor.

5. Angle BAC is a right angle. Find the size of angle CAD.



6. Point O is on line RS. Find the size of angle SOP.



Student Response

1. Angle BCA is 30 degrees. Angle BCD is 90 degrees.

2. Angle ACD is 60 degrees since it is the angle left when angle BCA is removed from angle BCD and $90 - 30 = 60$.

3. Angle LOK is 37 degrees. Angle LOM is 180 degrees.

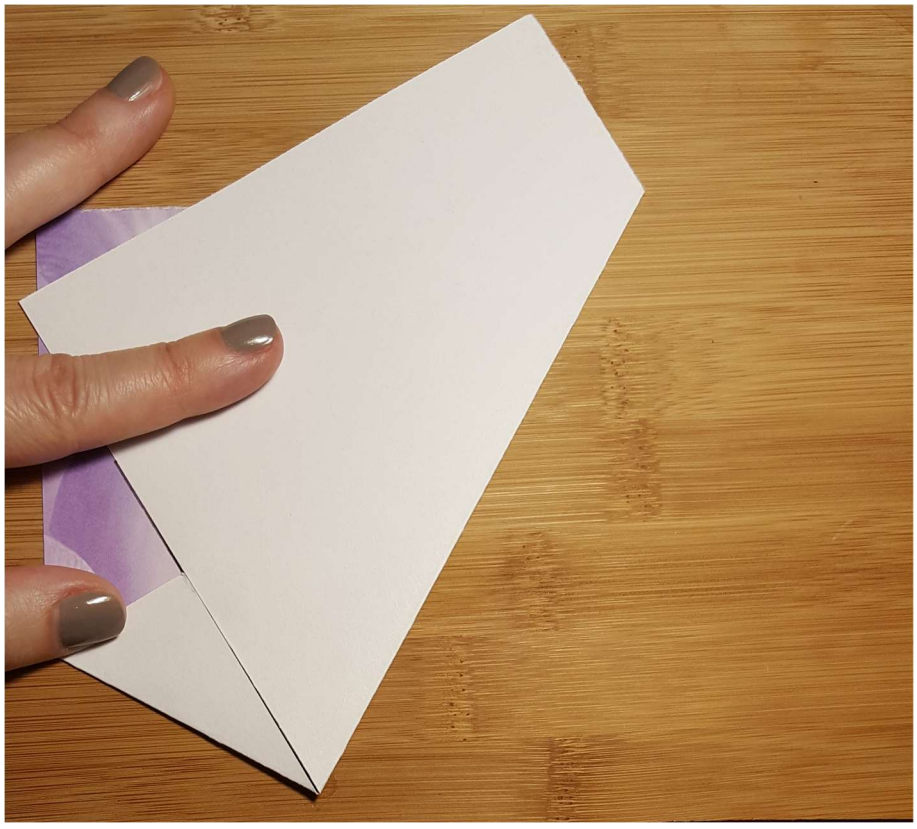
4. Angle KOM is 143 degrees since that is what is left when removing angle LOK from angle LOM and $180 - 37 = 143$.

5. 26° since $90 - 64 = 26$.

6. 104° since $180 - 76 = 104$

Are You Ready for More?

Clare started with a rectangular piece of paper. She folded up one corner, and then folded up the other corner, as shown in the photos.



1. Try this yourself with any rectangular paper. Fold the left corner up at any angle, and then fold the right corner up so that the edges of the paper meet.

2. Clare thought that the angle at the bottom looked like a 90 degree angle. Does yours also look like it is 90 degrees?

3. Can you explain why the bottom angle *always has to be* 90 degrees? Hint: the third photo shows Clare's paper, unfolded. The crease marks have dashed lines, and the line where the two paper edges met have a solid line. Mark these on your own paper as well.

Student Response

Since they were made by folding, there are two sets of angles with equal sizes. Label them $a°$ and $b°$. Since these four angles are adjacent and lie along a line, it must be true that $a + a + b + b = 180$, or $2a + 2b = 180$. Factorising gives $2(a + b) = 2 \times 90$. Therefore, it must be true that $a + b = 90$.

Activity Synthesis

The goal of this discussion is to introduce students to the terms complementary and supplementary for describing relationships between pairs of angles.

First, have students compare answers and strategies for the last two questions with their partners.

Next, display the last two questions for all to see and ask:

- "Which other problem in this activity was similar to the third question? How?" (The first problem, about angle ACD, also involved subtracting from 90.)
- "Which other problem in this activity was similar to the last question? How?" (The second problem, about angle KOM, also involved subtracting from 180.)

Explain to students that the term **complementary** describes a pair of angles whose sizes sum to 90 degrees, and the term **supplementary** describes a pair of angles whose sizes sum to 180 degrees. It is not important at this point to discuss that complementary or supplementary angles do not need to be adjacent, as that will be explored in the next lesson. Ask:

- "Which angles in this activity were supplementary angles?" (angles SOP and POR in the last question, as well as angles LOK and KOM from the second question)
- "Which angles in this activity were complementary angles?" (angles CAD and DAB in the third question, as well as from the first question angles ACD and BCA or angles DAC and BAC, or even from the second question angles OKN and OKL)

Invite students to continue practising using the words *complementary* and *supplementary* throughout the rest of this unit, so they can start to feel more comfortable using them in their vocabulary.

Speaking: Discussion Supports. Provide the following sentence frames to support student use of the words *complementary* and *supplementary*: "Angles ___ and ___ are complementary angles because . . ." and "Angles ___ and ___ are supplementary angles because . . ." Some students may also benefit from choral repetition of these phrases. This will give all students an opportunity to practice these new words in context.
Design Principle(s): Support sense-making; Maximise meta-awareness

Lesson Synthesis

- What does it mean for two angles to be supplementary? (Their sizes sum to 180°.)
- What does it mean for two angles to be complementary? (Their sizes sum to 90°.)

- If you know two angles are supplementary and you know the size of one angle, how can you find the other? (Subtract the known one from 180°.)

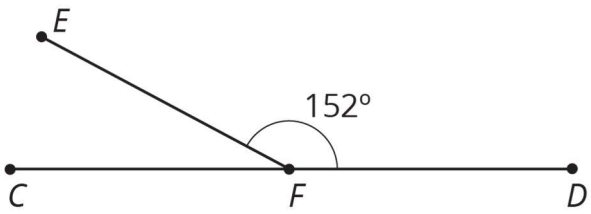
Display diagrams and definitions of new vocabulary somewhere in the classroom so that students can refer back to them during subsequent lessons. As the unit progresses, new terms can be added.

2.4 Finding Measurements

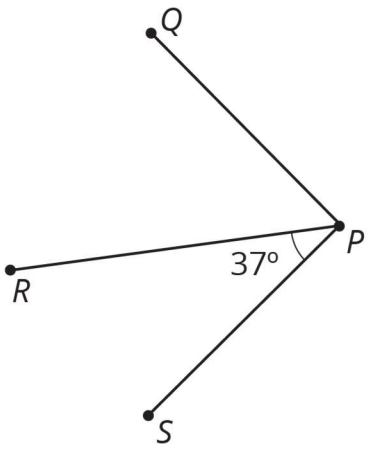
Cool Down: 5 minutes

Student Task Statement

1. Point F is on line CD. Find the size of angle CFE.



2. Angle SPR and angle RPQ are complementary. Find the size of angle RPQ.

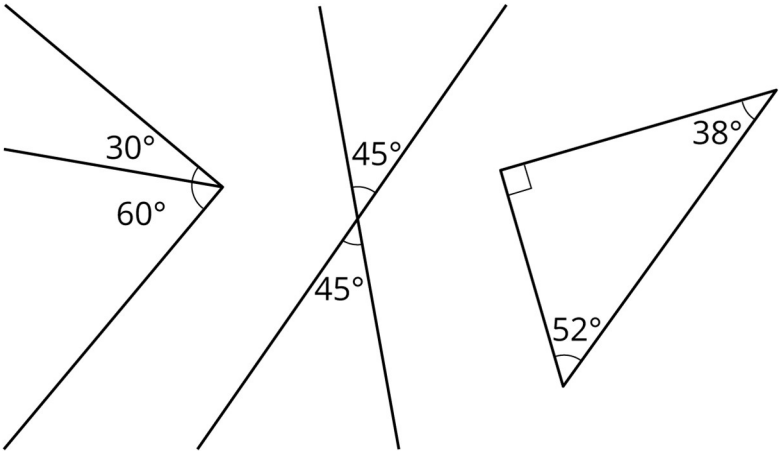


Student Response

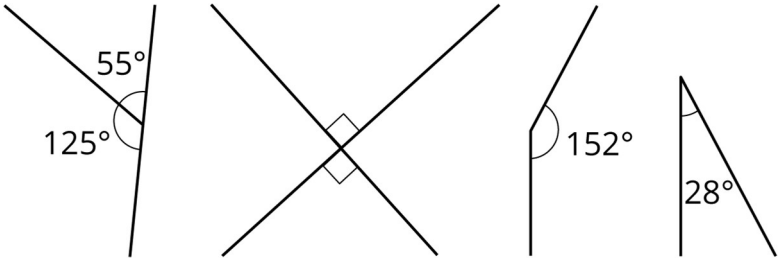
1. Angle CFE is 28 degrees because $180 - 152 = 28$.
2. Angle RPQ is 53 degrees because $90 - 37 = 53$

Student Lesson Summary

If two angles add up to 90°, then we say the angles are **complementary**. Here are three examples of pairs of complementary angles.



If two angles add up to 180°, then we say the angles are **supplementary**. Here are three examples of pairs of supplementary angles.



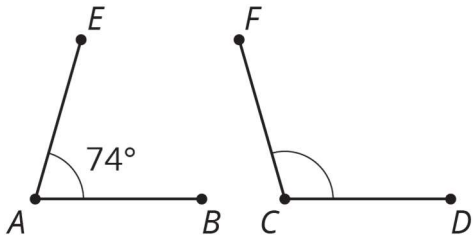
Glossary

- complementary
- supplementary

Lesson 2 Practice Problems

1. **Problem 1 Statement**

   Angles A and C are supplementary. Find the size of angle C.



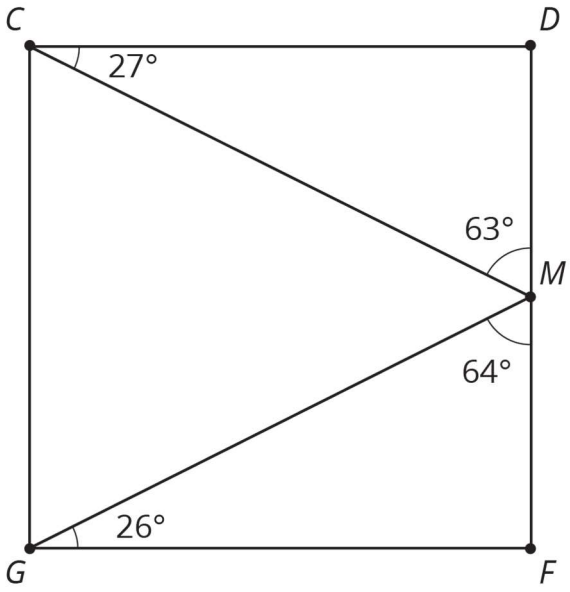
   Solution

   106°

2. **Problem 2 Statement**

   a. List two pairs of angles in square $CDFG$ that are complementary.

   b. Name three angles that sum to 180°.



Solution

   a. Any 2 of these pairs: Angles DCM and MCG, angles MGF and MGC, angles MGF and GMF, or angles DCM and DMC.

   b. Any 1 of these sets: Angles DMC, CMG, and GMF, angles FGM, GMF, and MFG, angles CDM, DMC, and MCD, or angles MCG, CGM, and GMC.

3. **Problem 3 Statement**

Complete the equation with a number that makes the expression on the right side of the equal sign equivalent to the expression on the left side.

$5x - 2.5 + 6x - 3 = \_\_\_\_ (2x - 1)$

Solution

5.5

4. **Problem 4 Statement**

Match each table with the equation that represents the same proportional relationship.

a.

x	y
2	8
3	12
4	16
5	20

b.

x	y
3	4.5
6	9
7	10.5
10	15

c.

x	y
2	$\frac{5}{2}$
4	5
6	$\frac{15}{2}$
12	15

1. $y = 1.5x$
2. $y = 1.25x$
3. $y = 4x$

Solution

- A: 3
- B: 1
- C: 2