Lesson 1: Interpreting negative numbers

Goals

- Interpret directed numbers in the contexts of temperature and height above sea level.
- Order rational numbers, and justify (orally) the comparisons.
- Plot points on a vertical or horizontal number line to represent rational numbers.

Learning Targets

- I can compare rational numbers.
- I can use rational numbers to describe temperature and height above sea level.

Lesson Narrative

In this lesson, students review what they learned about negative numbers earlier in KS3, including placing them on the number line, comparing and ordering them, and interpreting them in the contexts of temperature and height above sea level. The context of temperature helps build students' intuition about directed numbers because most students know what it means for a temperature to be negative and are familiar with representing temperatures on a number line (a thermometer). The context of height above sea level may be less familiar to students, but it provides a concrete (as well as cultural) example of one of the most fundamental uses of directed numbers: representing positions along a line relative to a reference point (sea level in this case). The number line is the primary representation for directed numbers in this unit, and the structure of the number line is used to make sense of the rules of directed number arithmetic in later lessons.

Building On

- Apply and extend previous understandings of numbers to the system of rational numbers.

Addressing

- Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

- Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Building Towards

- Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Collect and Display
- Co-Craft Questions
- Discussion Supports
- Take Turns

Required Materials

Pre-printed slips, cut from copies of the blackline master

Rational numbers Card Sort Set 1

-23	-10	-9	-7
-6	-5	-3	-2
-1	0	1	2
3	5	8	10
11	15	22	23

Rational numbers Card Sort Set 2

$\frac{1}{4}$	$\frac{9}{8}$	2.5	$\frac{8}{3}$
$-\frac{1}{4}$	$-\frac{9}{8}$	-2.5	$-\frac{8}{3}$
$5\frac{5}{6}$	7.7	$\frac{31}{3}$	$22\frac{3}{8}$
$-5\frac{5}{6}$	-7.7	$-\frac{31}{3}$	$-22\frac{3}{8}$

Required Preparation

Print and cut up slips from the Rational Numbers Card Sort blackline master. Prepare 1 copy for every 3 students. Students will need copies of both sets 1 and 2. Keep the slips from set 1 (Integers) separate from set 2 (Rational numbers that are not integers) for each group. Consider using different colours of paper so sets 1 and 2 are easier to separate.

Student Learning Goals

Let's review what we know about directed numbers.

1.1 Using the Thermometer

Warm Up: 5 minutes

The purpose of this warm-up is to remind students about negative numbers. The context of a weather thermometer works like a vertical number line. Students do not need to understand comparative temperatures in Celsius and Fahrenheit. The activity is written with temperatures in Celsius; however, the activity would work the same if the thermometer was labelled in Fahrenheit. These two different systems for measuring temperature is an opportunity to remind students that what counts as zero is arbitrary and was chosen by someone as some point. If desired, explain to students that 0° Celsius is the freezing point of fresh water and 0° Fahrenheit is based on the freezing point of salt water.

Launch

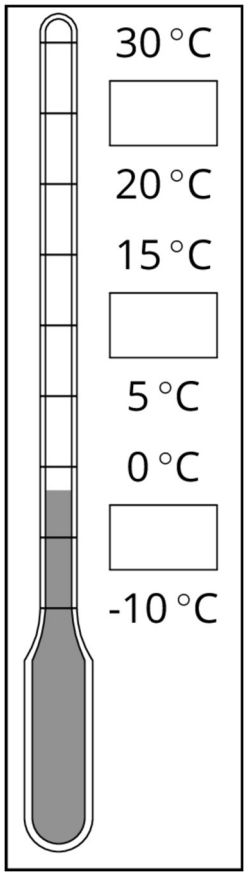
Display the thermometer image for all to see. Explain that degrees Celsius is a way of measuring temperature, like degrees Fahrenheit—but it has a different zero point. Students may already know that 0° Celsius is based on the freezing point of water and 0° Fahrenheit on the freezing point of brine, but these were chosen by people; there's no reason they had to be this way. Give students 1 minute of quiet think time to examine the picture before they start writing.

Anticipated Misconceptions

Some students may think the missing number between 0 and -10 needs to have a magnitude larger than -10, such as -15, because on the positive side of the number line, numbers increase in magnitude as you go up.

Student Task Statement

Here is a weather thermometer. Three of the numbers have been left off.



1. What numbers go in the boxes?
2. What temperature does the thermometer show?

Student Response

1. 25, 10, -5
2. About -2°C

Activity Synthesis

Ask students to share their responses for the first question and explain their reasoning. After each response, ask students to indicate if they agree or disagree. If all students are in

agreement, record and display the missing temperatures for all to see. If they disagree, have students explain their reasoning until they reach an agreement.

Ask students to share their responses to the second question. Because the thermometer is labelled in 5 degree increments, we have to estimate the temperature between 0° and −5°. Ask students to explain their reasoning and record and display possible responses for all to see. Highlight student responses that include the following ideas:

- The location of negative numbers below 0.
- The distance between numbers on the vertical number line.

1.2 Fractions of a Degree

5 minutes

In this activity, students return to the context of a thermometer to examine rational numbers that are not integers. Students compare and interpret the directed numbers to make sense of them in the context, including comparing a temperature that is not pictured to the temperatures that are pictured.

Instructional Routines

- Co-Craft Questions

Launch

Remind students of the warm-up problem about a weather thermometer. Instruct them to estimate when necessary.

Writing, Speaking: Co-craft Questions. To help students make sense of the drawings in this problem and to increase their awareness of the language used when comparing directed numbers, show students just the images of the four thermometers. Ask pairs of students to write their own mathematical questions about the situation. Listen for how students use the idea of numbers being above or below zero. Ask pairs to share their questions with the whole class. Highlight specific questions that are related to comparing numbers above or below zero. This will help students develop meta-awareness of the language used when comparing directed numbers.
Design Principle(s): Maximise meta-awareness; Support sense-making<

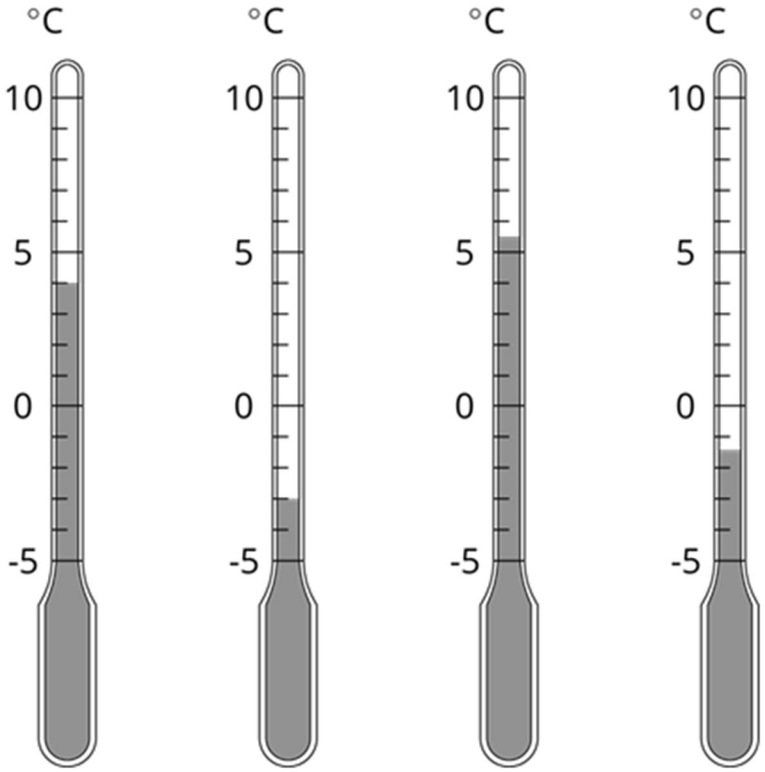
Anticipated Misconceptions

Some students may struggle to estimate the temperature on the last thermometer, since it is between two markings. Ask them to tell what the temperature would be for the lines directly above and directly below the thermometer's level. Then ask what temperature would be halfway in between those two numbers.

Some students may struggle with comparing -4°C to the temperatures shown on the thermometers. Prompt students to point out where -4°C would be on the thermometer that is showing -3°C.

Student Task Statement

1. What temperature is shown on each thermometer?
2. Which thermometer shows the highest temperature?
3. Which thermometer shows the lowest temperature?
4. Suppose the temperature outside is -4°C. Is that colder or warmer than the coldest temperature shown? How do you know?



Student Response

1. 4°C, -3°C, 5.5°C, -1.5°C
2. The third thermometer
3. The second thermometer
4. It is colder because $-4 < -3$.

Activity Synthesis

Ask one or more students to share their response for the temperature for each thermometer.

When discussing the last question, first have students explain their reasoning until they come to an agreement that -4°C is colder than -3°C. Then, if not brought up in students'

explanations, introduce the notation $-4 < -3$ and remind students that this is read, "Negative 4 is less than negative 3." Explain that -4 is farther away from zero than -3 is, and point to the location of -4 on a thermometer to show that is it below -3. On the negative side of the number line, that means -4 is less than -3. Familiarity with less than notation will be useful for describing their reasoning in the next activity.

1.3 Seagulls Soar, Sharks Swim

10 minutes

The purpose of this activity is for students to continue interpreting directed numbers in context and to begin to compare their relative location. A vertical number line shows the heights above sea level or depths below sea level of various animals. The number line is labelled in 5 metre increments, so students have to interpolate the height or depth for some of the animals. Next, they are given the height or depth of other animals that are not pictured and asked to compare these to the animals shown.

As students work, monitor for whether they are expressing relative distances in words, for example "3 metres below," or if they are expressing the same idea with notation, as in -3 metres. Both are acceptable; these ideas are connected in the discussion that follows. Also monitor for students who notice that there are two possible answers for the last question.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Collect and Display

Launch

Display the image for all to see. Tell students to measure the height or depth of each animal's eyes, to the nearest metre. Remind students that we choose sea level to be our zero level, in the same way that we chose a zero level for temperature.

Representation: Internalise Comprehension. Represent the same information with additional structure. If students are unsure where to begin, suggest that they extend a straight horizontal line at each depth to determine the height or depth of each animal.
Supports accessibility for: Visual-spatial processing; Conceptual processing Conversing, Representing: Collect and Display. Use this routine to capture the language students use during discussion with a partner. Collect the language of opposites: "above" or "below." For example, "The albatross is 3 metres above the penguin or the penguin is 3 metres below the albatross." Then, identify students who use negative numbers to describe these differences to share their reasoning during the whole-class discussion. Ensure students connect this language to, "The difference in height is +3 (to represent above) or -2.5 (to represent below)."
Design Principle(s): Maximise meta-awareness

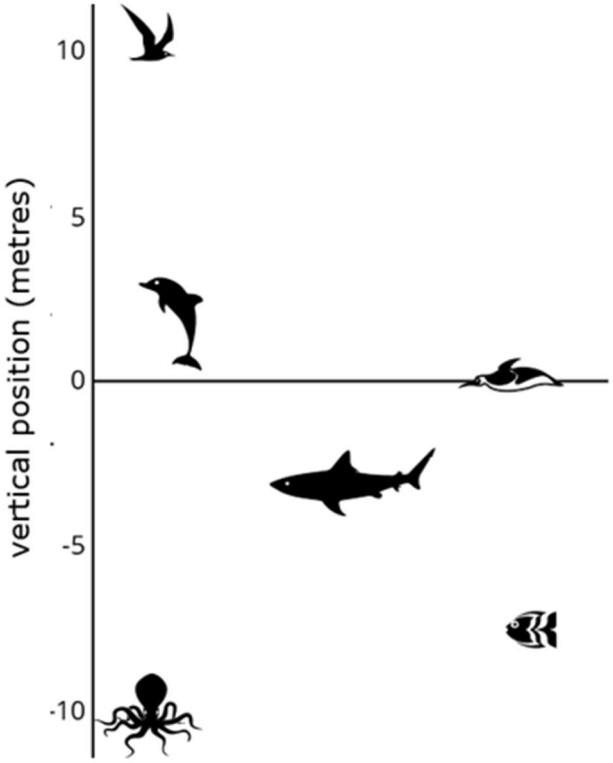
Anticipated Misconceptions

If students measure to the top or bottom of the animal, remind them that we are using the eyes of the animal to measure their height or depth.

Some students may struggle to visualise where the albatross, seagull, and clownfish are on the graph. Consider having them draw or place a marker where the new animal is located while comparing it to the other animals in the picture.

Student Task Statement

Here is a picture of some sea animals. The number line on the left shows the vertical position of each animal above or below sea level, in metres.



1. How far above or below sea level is each animal? Measure to their eye level.
2. A mobula ray is 3 metres above the surface of the ocean. How does its vertical position compare to the height or depth of:

   The jumping dolphin?

   The flying seagull?

   The octopus?
3. An albatross is 5 metres above the surface of the ocean. How does its vertical position compare to the height or depth of:

The jumping dolphin?

The flying seagull?

The octopus?

4. A clownfish is 2 metres below the surface of the ocean. How does its vertical position compare to the height or depth of:

   The jumping dolphin?

   The flying seagull?

   The octopus?

5. The vertical distance of a new dolphin from the dolphin in the picture is 3 metres. What is its distance from the surface of the ocean?

Student Response

1. Seagull is at 10 m. Dolphin is at 3 m. Octopus is at -10 m. Shark is at -3 m. Fish is at -7 m. Penguin is at 0 m.

2. The mobula ray is

   a. 0 m above the dolphin

   b. 7 m below the seagull

   c. 13 m above the octopus

3. The albatross is

   a. 2 m above the dolphin

   b. 5 m below the seagull

   c. 15 m above the octopus

4. The clownfish is

   a. 5 m below the dolphin

   b. 12 m below the seagull

   c. 8 m above the octopus

5. Either 0 m or 6 m, depending on whether the new dolphin is 3 m above or below the dolphin in the picture.

Are You Ready for More?

The north pole is in the middle of the ocean. A person at sea level at the north pole would be 3 949 miles from the centre of Earth. The sea floor below the north pole is at a height above sea level of approximately -2.7 miles. The height above sea level of the south pole is about 1.7 miles. How far is a person standing on the south pole from a submarine at the sea floor below the north pole?

Student Response

About 7 897 miles.

Activity Synthesis

The main point for students to get out of this activity is that we can represent distance above and below sea level using directed numbers. The depths of the shark, fish, and octopus can be expressed as approximately -3 m, -6 m, and -7.5 m respectively, because they are below sea level.

Directed numbers can also be used to represent the relative vertical position of different pairs of animals. Have selected students share their responses and reasoning for how the heights of the albatross, seabird, and clownfish compare to the dolphin, seagull, and octopus. Record and display their verbal descriptions using directed numbers. For example, if a student says the albatross is 7 metres below the seagull, write "-7".

Finally, ask whether students noticed the ambiguity in the last question (about the height of the new dolphin). Ask such a student to explain why there are two possible answers to the last question.

1.4 Card Sort: Rational Numbers

Optional: 15 minutes

This activity reviews ordering integers first, and then rational numbers second. Many of the numbers also have their additive inverse in the set, which can help students use the structure of the number line to order the numbers.

The previous activities in this lesson used vertical number lines to help students make sense of negative numbers being below 0. It is important that students also feel comfortable working with horizontal number lines. As students work on ordering these slips, it is likely they will automatically make the transition to using a horizontal orientation. Watch for any groups that continue to use a vertical orientation and prompt them to consider whether they have really ordered their numbers from least to greatest.

Monitor for students who specifically compare the magnitudes of numbers and translate that into the correct number order (such as $2.5 > 2$ so $-2.5 < -2$) are using the structure of the number line; ask them to share their reasoning in the whole-class discussion.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports
- Take Turns

Launch

Arrange students in groups of 3. Distribute the first set of cards (integers) to each group. Instruct the students to put the cards in order from least to greatest. When a group has finished ordering the first set, give them the second set (rational numbers that are not integers) and have them add these in the correct locations.

Conversing: Discussion Supports. To support students as they explain their reasoning for how they placed the cards, provide sentence frames such as: "First, I ___ because ___.", "I noticed ___ so I....", and "I know ___ is greater than/less than ___ because....".
Design Principle(s): Support sense-making; Cultivate conversation

Anticipated Misconceptions

Some students may struggle with ordering the negative numbers. For example, they may put -2.5 to the right of -2 since they are used to seeing 2.5 to the right of 2. Help students visualise a number line and figure out which number should be farther away from 0.

Student Task Statement

1. Your teacher will give your group a set of cards. Order the cards from least to greatest.
2. Pause here so your teacher can review your work. Then, your teacher will give you a second set of cards.
3. Add the new set of cards to the first set so that all of the cards are ordered from least to greatest.

Student Response

1. -23, -10, -9, -7, -6, -4, -3, -2, -1, 0, 1, 2, 3, 5, 8, 10, 11, 15, 22, 23
2. No answer needed.
3. -23, $-22\frac{3}{8}$, $-\frac{31}{3}$, -10, -9, -7.7, -7, -6, $-5\frac{5}{6}$, -4, -3, $-\frac{8}{3}$, -2.5, -2, $-\frac{9}{8}$, -1, $-\frac{1}{4}$, 0, $\frac{1}{4}$, 1, $\frac{9}{8}$, 2, 2.5, $\frac{8}{3}$, 3, 5, $5\frac{5}{6}$, 7.7, 8, 10, $\frac{31}{3}$, 11, 15, 22, $22\frac{3}{8}$, 23

Activity Synthesis

Select students to share their strategies when sorting. Highlight strategies that used the magnitudes of a number and its additive inverse.

Discuss:

- Which numbers were the hardest to order? Why?
- How did you decide where to put the fractions?
- How is, for example, $-\frac{9}{8}$ related to $\frac{9}{8}$?

Introduce the convention that number lines are usually drawn horizontally, with the negative numbers to the left of 0. If any groups put their slips in order vertically, considering having them reposition their slips to match the orientation of a horizontal number line. Make sure students understand the meaning of the term "opposite" and absolute value notation.

Lesson Synthesis

Main learning points:

- Negative numbers can be used to represent quantities below a chosen zero point.
- Negative numbers can be ordered to the left side of zero on a horizontal number line.
- Absolute value, or magnitude, describes how far away from zero a value is.

Discussion questions:

- Which number is greater, -7 or -12?
- Which number has the greater magnitude, 7 or -12?
- How can we order negative numbers?

1.5 Directed Numbers

Cool Down: 5 minutes

For upcoming work in this unit, it is vital that students can correctly place positive and negative rational numbers on a number line, and that they can compare positive and negative rational numbers. If any students do poorly on this cool-down, they will have plenty of practice with placing positive and negative numbers on a number line in the next several lessons, but they may need more support in doing so.

Student Task Statement

Here is a set of directed numbers: 7, -3, $\frac{1}{2}$, -0.8, 0.8, $-\frac{1}{10}$, -2

1. Order the numbers from least to greatest.
2. If these numbers represent temperatures in degrees Celsius, which is the coldest?
3. If these numbers represent heights above sea level in metres, which is the farthest away from sea level?

Student Response

1. -3, -2, -0.8, $-\frac{1}{10}$, $\frac{1}{2}$, 0.8, 7

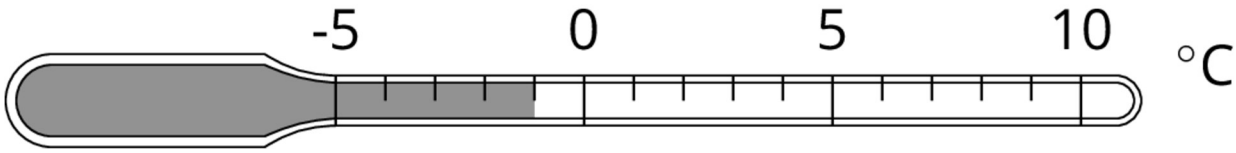
2. -3

3. 7

Student Lesson Summary

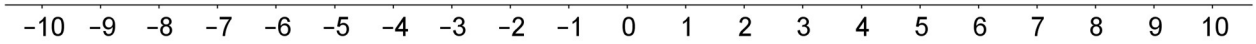
We can use **positive numbers** and **negative numbers** to represent temperature and height above sea level.

When numbers represent temperatures, positive numbers indicate temperatures that are warmer than zero and negative numbers indicate temperatures that are colder than zero. This thermometer shows a temperature of -1 degree Celsius, which we write -1°C.



When numbers represent height above sea level, positive numbers indicate positions above sea level and negative numbers indicate positions below sea level.

We can see the order of directed numbers on a number line.



A number is always less than numbers to its right. So -7 $<$ -3.

We use **absolute value** to describe how far a number is from 0. The numbers 15 and -15 are both 15 units from 0, so $|15| = 15$ and $|-15| = 15$. We call 15 and -15 *opposites.* They are on opposite sides of 0 on the number line, but the same distance from 0.

Glossary

- absolute value
- negative number
- positive number

Lesson 1 Practice Problems

1. **Problem 1 Statement**

   It was -5°C in Copenhagen and -12°C in Oslo. Which city was colder?

   Solution

   It was colder in Oslo because -12 is less than -5.

2. **Problem 2 Statement**

   a. A fish is 12 metres below the surface of the ocean. What is its height above sea level?

   b. A sea bird is 28 metres above the surface of the ocean. What is its height above sea level?

   c. If the bird is directly above the fish, how far apart are they?

   Solution

   a. -12 m

   b. 28 m

   c. 40 m

3. **Problem 3 Statement**

   Compare using >, =, or <.

   a. 3 ____ -3

   b. 12 ____ 24

   c. -12 ____ -24

   d. 5 ____ -(-5)

   e. 7.2 ____ 7

   f. -7.2 ____ -7

   g. -1.5 ____ $\frac{-3}{2}$

   h. $\frac{-4}{5}$ ____ $\frac{-5}{4}$

   i. $\frac{-3}{5}$ ____ $\frac{-6}{10}$

j. $\frac{-2}{3}$ ——— $\frac{1}{3}$

Solution

a. >

b. <

c. >

d. =

e. >

f. <

g. =

h. >

i. =

j. <

4. **Problem 4 Statement**

Han wants to buy a £30 ticket to a game, but the pre-order tickets are sold out. He knows there will be more tickets sold the day of the game, with a markup of 200%. How much should Han expect to pay for the ticket if he buys it the day of the game?

Solution

£90. A 100% increase of a £30 ticket is an additional £30, therefore a 200% increase of a £30 ticket would be an additional £60.

5. **Problem 5 Statement**

A type of green paint is made by mixing 2 cups of yellow with 3.5 cups of blue.

a. Find a mixture that will make the same shade of green but a smaller amount.

b. Find a mixture that will make the same shade of green but a larger amount.

c. Find a mixture that will make a different shade of green that is bluer.

d. Find a mixture that will make a different shade of green that is more yellow.

Solution

Answers vary. Sample response:

a. 1 cup of yellow and 1.75 cups of blue

b. 4 cups of yellow and 7 cups of blue

c. 2 cups of yellow and 4 cups of blue.

d. 2 cups of yellow and 2 cups of blue.