Lesson 4: Enlargements on a square grid

Goals

- Create an enlargement of a polygon on a square grid given a scale factor and centre of enlargement.

- Identify the image of a shape on a coordinate grid given a scale factor and centre of enlargement.

Learning Targets

- I can apply enlargements to shapes on a square grid.

- If I know the angles and side lengths of a polygon, I know the angle and side lengths of the polygon if I apply an enlargement with a certain scale factor.

Lesson Narrative

In this lesson, students apply enlargements to polygons on a grid, both with and without coordinates. The grid offers a way of measuring distances between points, especially points that lie at the intersection of grid lines. If point Q is three grid squares to the right and two grid squares up from P then the enlargement with centre P of Q with scale factor 4 can be found by counting grid squares: it will be twelve grid squares to the right of P and eight grid squares up from P. The coordinate grid gives a more concise way to describe this enlargement. If the centre P is $(0,0)$ then Q has coordinates $(3,2)$. The image of Q after this enlargement is $(12,8)$.

Students continue to find enlargements of polygons, providing additional evidence that enlargements map line segments to line segments and hence polygons to polygons. The scale factor of the enlargement determines the factor by which the length of those line segments increases or decreases. Using coordinates to describe points in the plane helps students develop language for precisely communicating shapes in the plane and their images under enlargements.

Addressing

- Understand congruence and similarity using physical models, transparencies, or geometry software.

- Describe the effect of enlargements, translations, rotations, and reflections on two-dimensional shapes using coordinates.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- Collect and Display

Required Materials
Geometry toolkits
tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Pre-printed slips, cut from copies of the blackline master

Card Sort: Matching enlargements on a coordinate grid

1.
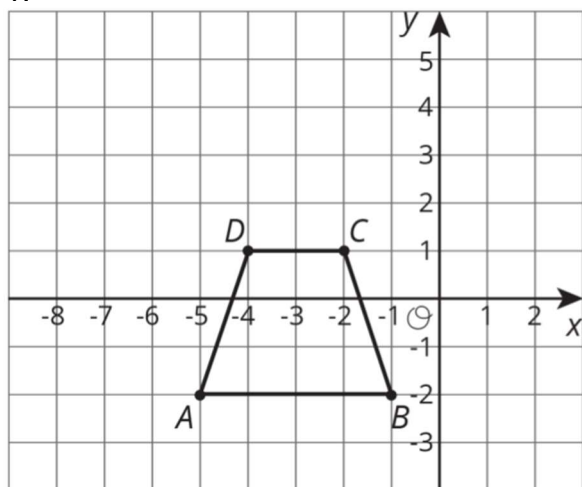
Enlarge the trapezium using centre (-1, -2) and scale factor $\frac{3}{2}$.

Card Sort: Matching enlargements on a coordinate grid
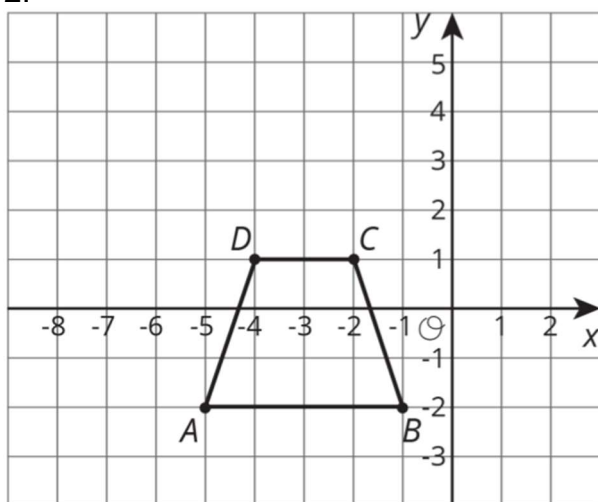
C.

The polygon with vertices at:

$A' = (-7, -2)$

$B' = (-1, -2)$

$C' = (-2\frac{1}{2}, 2\frac{1}{2})$

$D' = (-5\frac{1}{2}, 2\frac{1}{2})$

Card Sort: Matching enlargements on a coordinate grid

2.

Enlarge the trapezium using centre (-3, 4) and scale factor $\frac{1}{2}$.

Card Sort: Matching enlargements on a coordinate grid

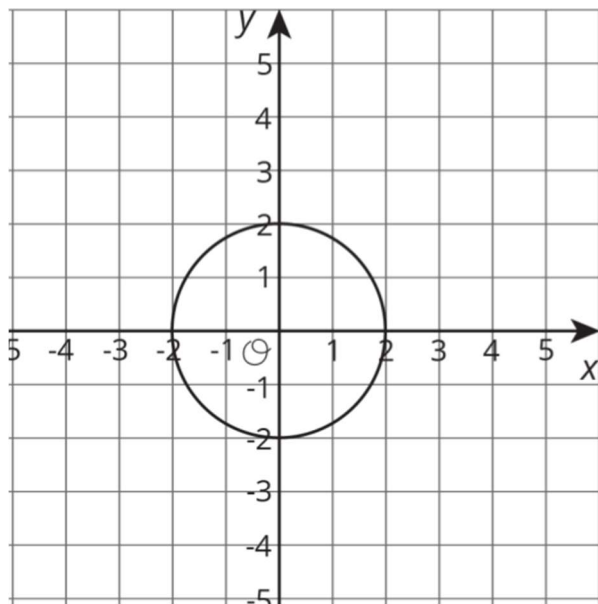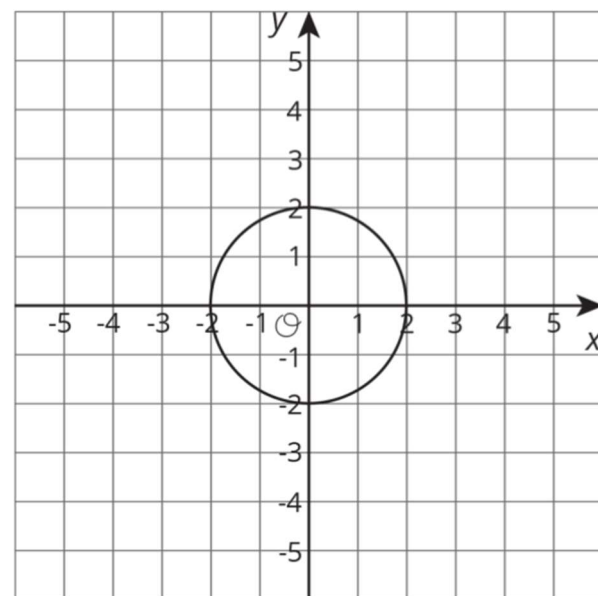**A.**

The polygon with vertices at:

$A' = (-4, 1)$

$B' = (-2, 1)$

$C' = (-2\frac{1}{2}, 2\frac{1}{2})$

$D' = (-3\frac{1}{2}, 2\frac{1}{2})$

3.

Enlarge the circle using centre (0,0) and scale factor 2.

B.

The circle with centre (0,0) and radius 4.

4.

Enlarge the circle using centre $(0,0)$ and scale factor $\frac{1}{2}$.

E.

The circle with centre (0,0) and radius 1.

5.
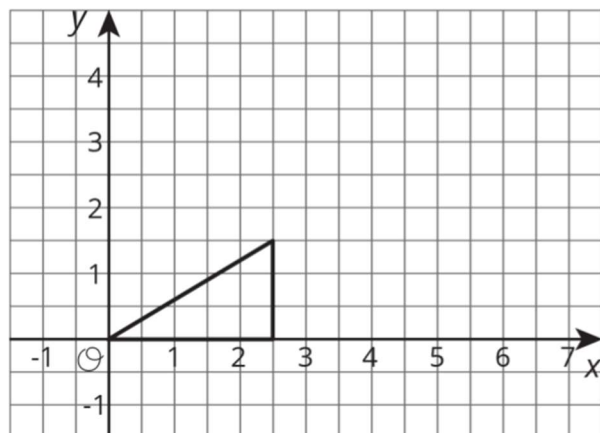
Enlarge the triangle using centre $(0, 0)$ and scale factor 2.

6.
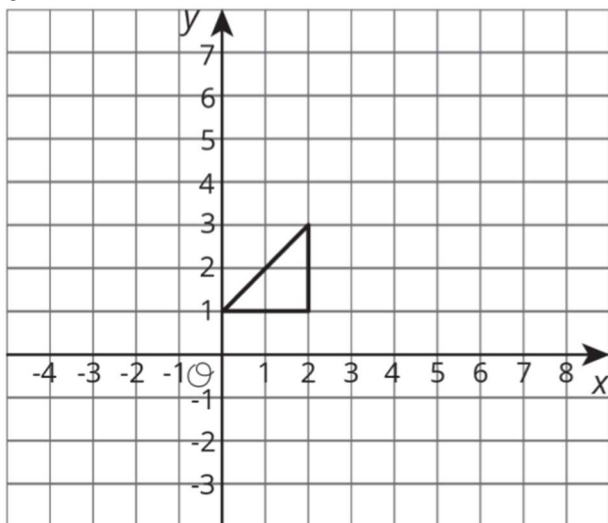
Enlarge the triangle using centre $(-4, -3)$ and scale factor 1.5.

D.

The polygon with vertices at $(2, 3)$, $(5, 3)$, and $(5,6)$.

Required Preparation

Print and cut one copy of the blackline master for each student.

Student Learning Goals

Let's enlarge shapes on a square grid.

4.1 Estimating a Scale Factor

Warm Up: 5 minutes

In this warm-up, students estimate a scale factor based on a picture showing the centre of the enlargement, a point, and its image under the enlargement.

Launch

Tell students they will estimate the scale factor for an enlargement. Clarify that "estimate" doesn't mean "guess." Encourage students to use any tools available to make a precise estimate. Provide access to geometry toolkits.

Student Task Statement

Point C is the enlargement of point B with centre of enlargement A and scale factor s. Estimate s. Be prepared to explain your reasoning.

Student Response

Answers vary. Sample response: about 2.3.

Activity Synthesis

Check with students to find what methods they used to compare distances. Likely methods include using a ruler and division or using an index card and marking off multiples of the distance from A to B.

Ask students:

- "Is the scale factor greater than 1?" (Yes.) "How do you know?" (the point C is further from A than B)

- "Is the scale factor greater than 2?" (Yes.) "How do you know?" (the distance from C to A is more than twice the distance from B to A)

- "Is the scale factor greater than 3?" (No.) "How do you know?" (The distance from C to A is less than 3 times the distance from B to A.)

- "Is the scale factor greater or less than 2.5?" (It is less.) "How do you know?" (The distance from C to A is less than 2.5 times the distance from B to A.)

4.2 Enlargements on a Grid

10 minutes

In previous lessons, students performed enlargements on a circular grid and with no grid. In this activity, they perform enlargements on a square grid. A square grid is particularly helpful if the centre of enlargement and the points being enlarged are grid points. When the extra structure of coordinates is added, as in the next activity, the grid provides an extremely convenient tool for naming points and describing the effects of enlargements using coordinates. As in previous lessons, students will again see that scale factors greater than 1 produce larger copies while scale factors less than 1 produce smaller copies.

Monitor for how students find the enlarged points and the language they use to describe the process. In particular:

- using a ruler or index card to measure distances along the rays emanating from the centre of enlargement

- taking advantage of the grid and counting how many squares to the left or right, up or down

Instructional Routines
- Anticipate, Monitor, Select, Sequence, Connect

Launch

Provide access to geometry toolkits.

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, reference examples from the previous lessons on methods for enlarging points on a circular grid and on no grid to provide an entry point into this activity.
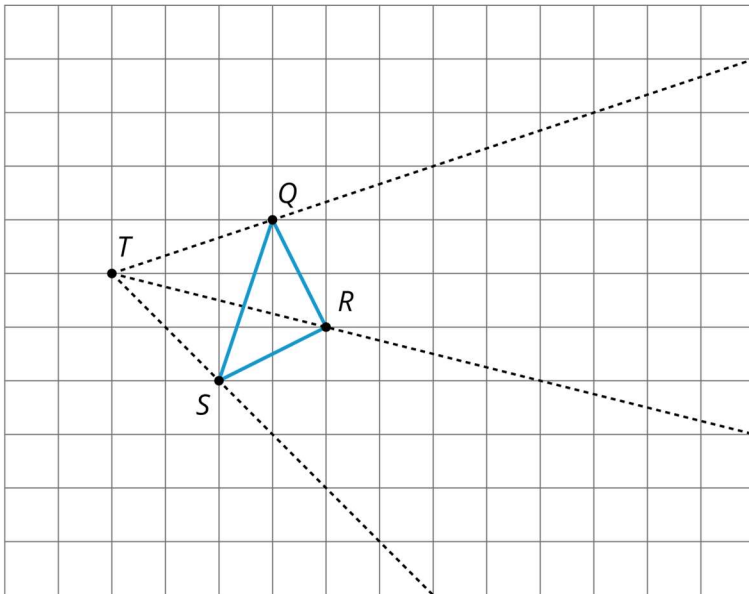*Supports accessibility for: Social-emotional skills; Conceptual processing*
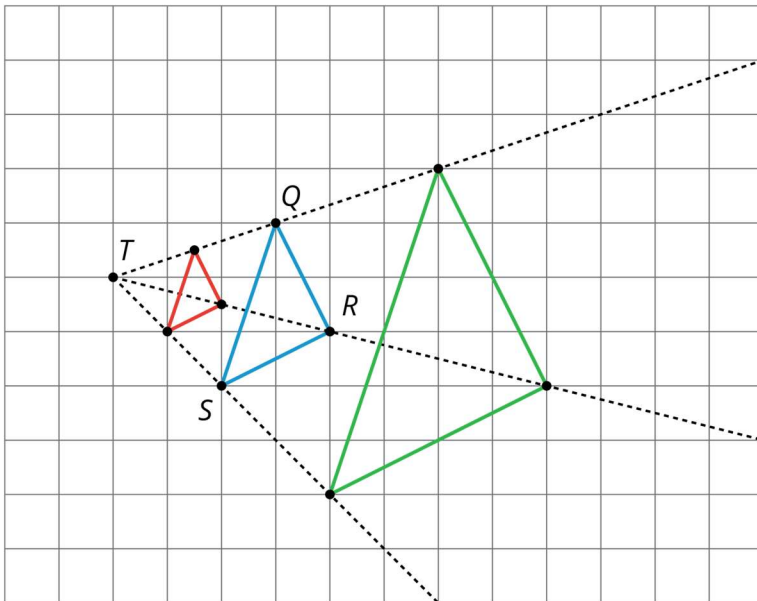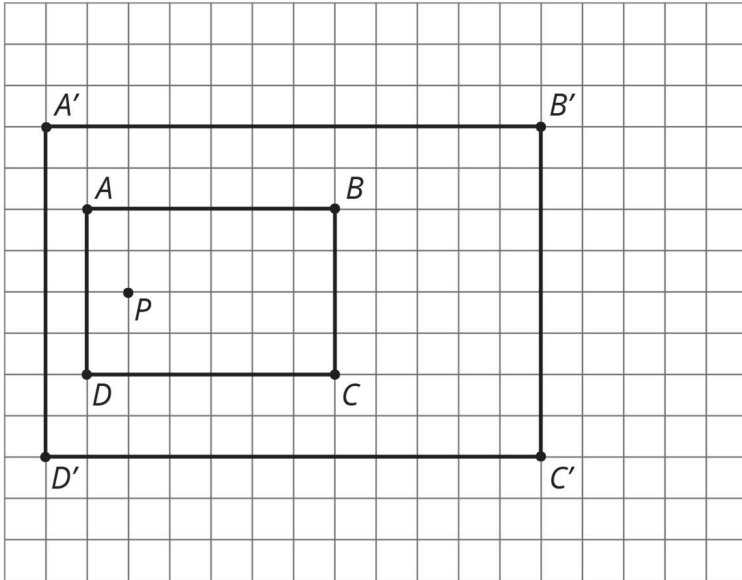
Student Task Statement
1. Find the enlargement of quadrilateral *ABCD* with centre *P* and scale factor 2.
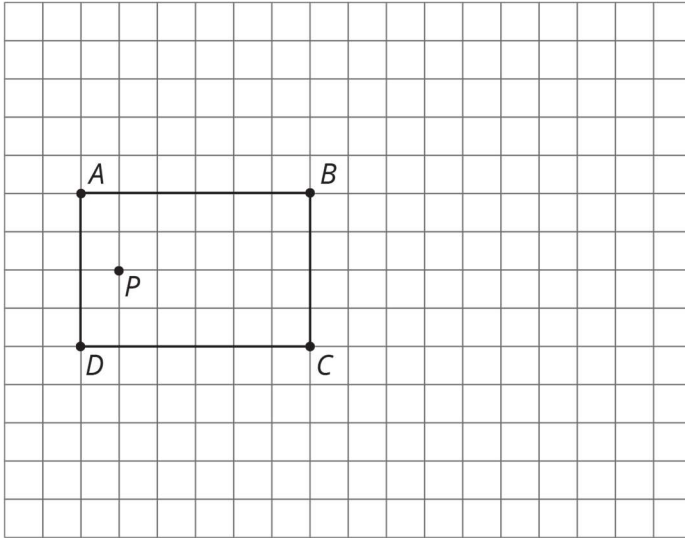
2. Find the enlargement of triangle QRS with centre T and scale factor 2.

3. Find the enlargement of triangle QRS with centre T and scale factor $\frac{1}{2}$.

Student Response

Activity Synthesis

Select students to show how they found the enlargements. First, select any students who used the same methods as when there was no grid, followed by students who noticed they could use the structure of the grid. Draw connections between these two methods—show that when you measure with a ruler or by making markings on an index card, the enlarged point ends up in the same place as by reasoning about the grid.

Expect students to use expressions like "moving over two and up one." These measurements can be multiplied by the scale factor in order to find the location of the enlarged point.

Tell students that moving forward they will do work on the grid with the added structure of coordinates. The method of performing enlargements is the same. The only change is that the coordinates give a concise way to *name* points.

4.3 Card Sort: Matching Enlargements on a Coordinate Grid

15 minutes

In the previous task, students worked on a square grid without coordinates. This activity adds the structure of coordinates and this extra structure plays a key role, allowing students to name points. Students match shapes with their enlarged images, using coordinates to describe the centre of enlargement and the vertices. The same strategies that were used previously in enlarging images, on a circular grid and with no grid, will be useful here.

Monitor for students who identify that the enlargement of a circle is a circle and similarly for triangles and quadrilaterals. This will help them eliminate certain possibilities for each match. Because there is one card that does not match, students should verify the other matches by performing the enlargements. Once the card without a match has been identified, reasoning based on eliminating possibilities (without performing the enlargements) is correct. Monitor for students who systematically perform the enlargements to help identify a match versus those who reason by structure and elimination of possibilities. Invite both to share during the discussion.

Instructional Routines
- Collect and Display

Launch

Students practise matching an original shape and enlargement description to information about the enlarged images using the coordinate plane. Distribute one set (numbers 1 through 6 and letters A through E) of cards to each student.

There is one extra option that does not have a match. Students should draw the enlarged image for that option themselves.

Representation: Internalise Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual processing; Organisation Conversing, Reading: Collect and Display. As students work in pairs on the task, circulate and listen to pairs as they decide whether two cards match. Write down the words and phrases students use to justify why an original shape card matches with an enlarged shape card. As students review the

language collected in the visual display, encourage students to clarify the meaning of a word or phrase. For example, a phrase such as "card 1 matches with card C because they are both trapeziums" can be clarified by asking students to explain why card 1 does not match with card A even though both are trapeziums. Listen for students who state that the scale factor and centre of enlargement must also be considered when matching the cards. Write down the language students use to describe how the scale factor and centre of enlargement affect the enlarged shape. This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.

Design Principle(s): Support sense-making; Maximise meta-awareness

Anticipated Misconceptions

If students are having trouble finding accurate matches, suggest that they identify the centre of enlargement and consider if the enlargement will result in a smaller or larger sized image.

Student Task Statement

Your teacher will give you some cards. Each of cards 1 through 6 shows a shape in the coordinate grid and describes an enlargement.

Each of cards A through E describes the image of the enlargement for one of the numbered cards.

Match number cards with letter cards. One of the number cards will not have a match. For this card, you'll need to draw an image.

Student Response

1. C. Answers vary. Sample response: The centre of enlargement is the point B, so the enlargement also contains point B, suggesting this card. The scale factor of $\frac{3}{2}$ works for the two trapeziums which are plotted together.
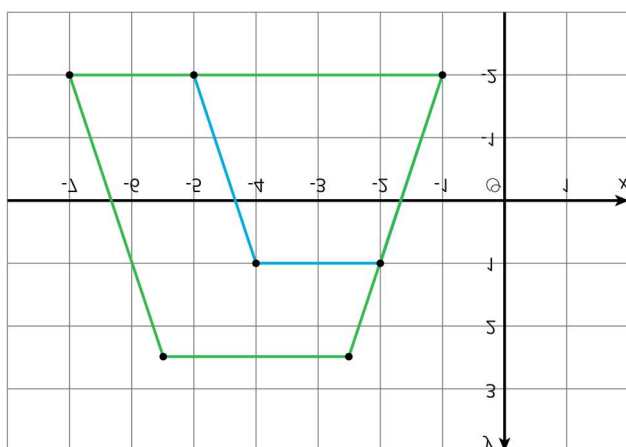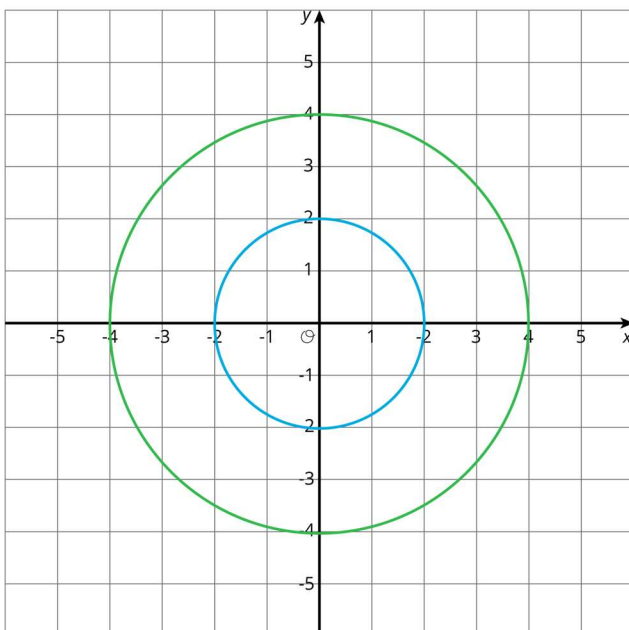

2. A. Answers vary. Sample response: The scale factor was less than one, so the enlargement will be closer to the centre of enlargement. Card A is plotted and shows

the enlargement since each vertex on the green trapezium is the midpoint between the centre of enlargement and the corresponding vertex on the blue trapezium.
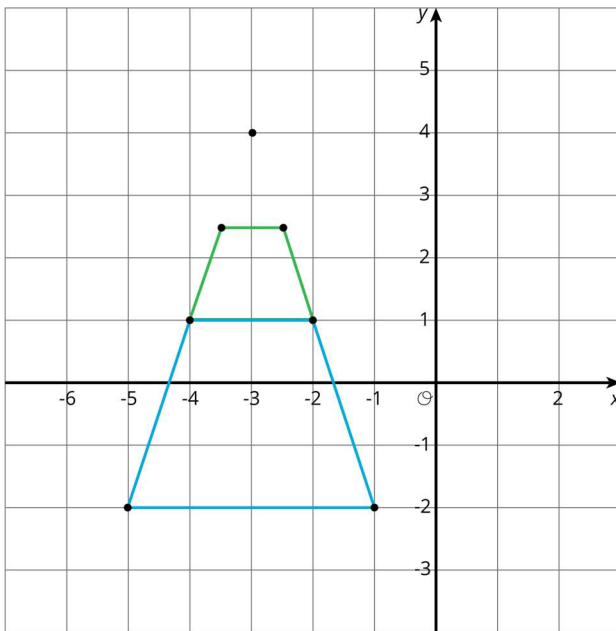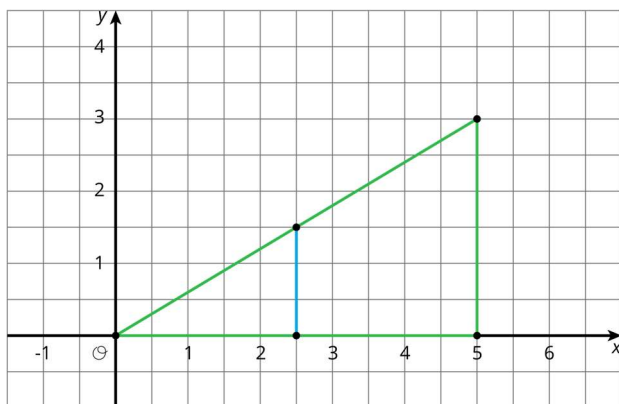

3. B. Answers vary. Sample response: The enlargement scale factor was greater than one, so the enlarged image will be a larger circle. The image is correct as both circles have the same centre and the radius of the green circle is twice the radius of the blue circle.


4. E. Answers vary. Sample response: This scale factor is less than one, so the image of the enlargement is a circle that is smaller than the original one. The image is correct because the circles have the same centre and the radius of the green circle is half the radius of the blue circle.
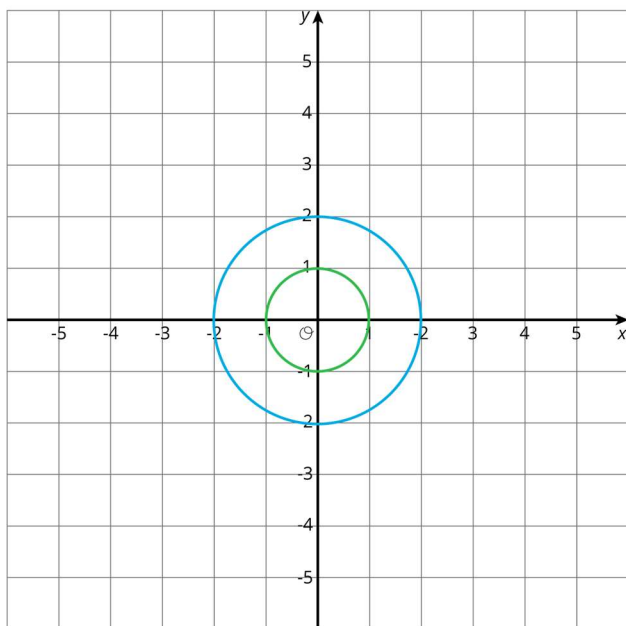
5.   F. The centre of enlargement is (0,0) so the enlarged image is a triangle containing (0,0). This does not match any of the lettered cards.

6.   D. Answers vary. Sample response: the enlargement of the triangle will be a triangle and it will be larger than $\triangle ABC$ since the scale factor is larger than 1. This suggests card D. The two are plotted together and $\triangle CDE$ is the enlargement of $\triangle ABC$ with centre P and scale factor $\frac{3}{2}$.
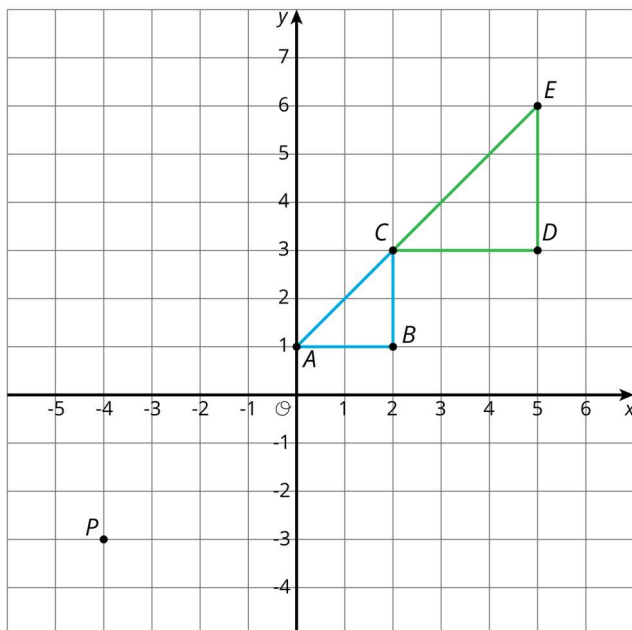
Are You Ready for More?

The image of a circle under enlargement is a circle where the centre of the enlargement is the centre of the circle. What happens if the centre of enlargement is a point on the circle? Using centre of enlargement (0,0) and scale factor 1.5, enlarge the circle shown on the diagram. This diagram shows some points to try enlarging.
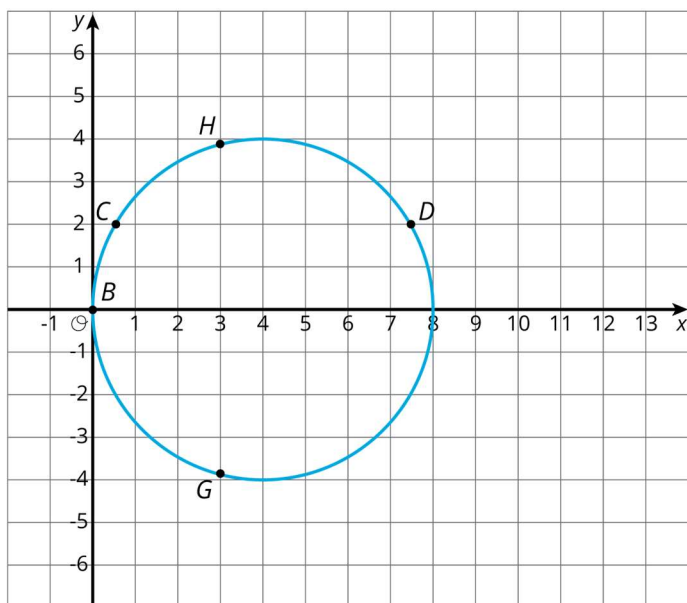
Student Response

Original has centre (4,0) and radius 4. Image has centre (6,0) and radius 6.
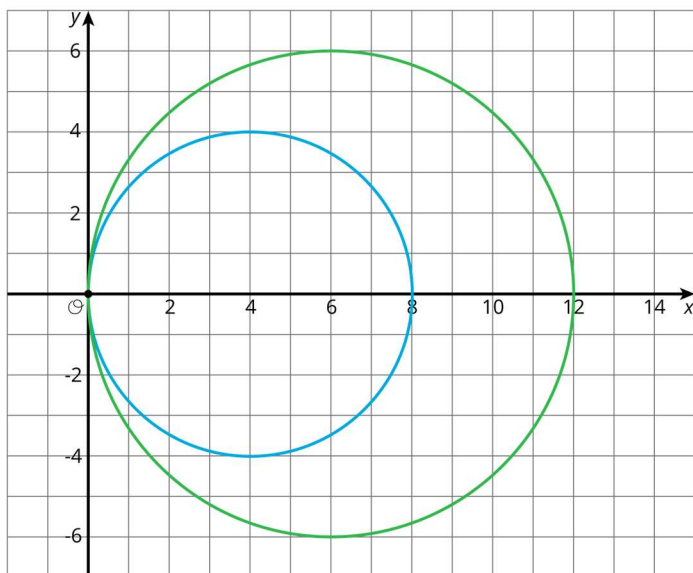
Activity Synthesis

Share the correct answers and invite selected students to share the strategies they used to solve the problems. This is a matching problem, so students may not have enlarged the entire image to find the correct answer among the choices. Important points to bring out include:

- An enlargement maps a circle to a circle, a quadrilateral to a quadrilateral, and a triangle to a triangle.

- If the centre of enlargement for a polygon is one of the vertices, then that vertex is on the enlarged polygon.

- If the scale factor is less than 1 then the enlarged image is smaller than the original shape.

- If the scale factor is larger than 1 then the enlarged image is larger than the original shape.

Lesson Synthesis
- "How do we perform enlargements on a square grid?"

- "How do coordinates help describe and perform enlargements?"
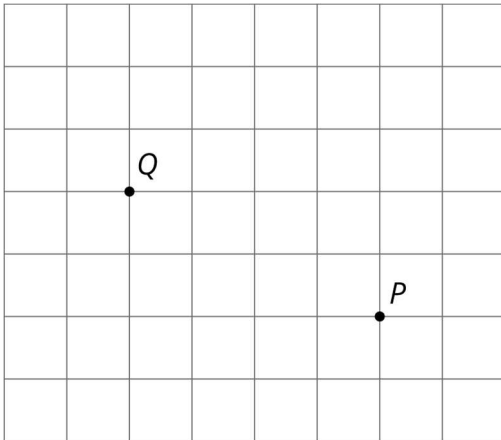
Just like the circular grid, a square grid is useful for performing enlargements. The grid lines can be used as a way to measure distance and direction between points. How can you enlarge Q with centre P and scale factor $\frac{1}{2}$? The image of Q will be half as many grid lines to the left and half has many grid lines up—that is, 2 grid lines to the left and 1 grid line up from P.
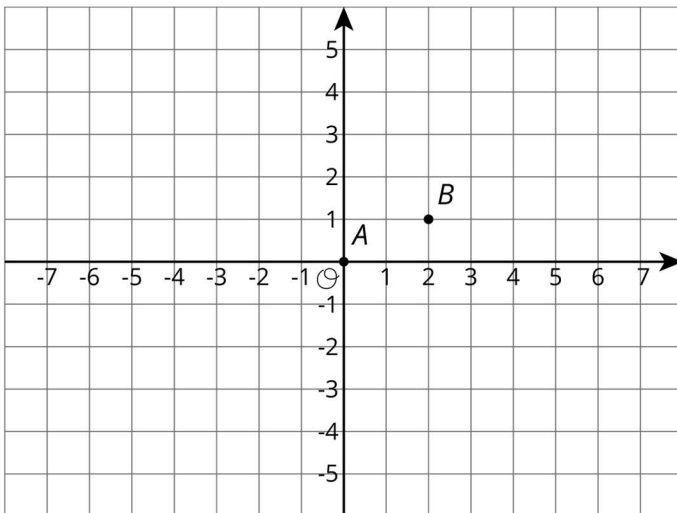
When the grid has coordinates, it is easier to communicate the location of new points. In the shape, we have $A = (0,0)$ and $B = (2,1)$. What is the enlargement of B with centre A and scale factor 3? To communicate the answer, we can just say $(6,3)$. It is three times as far to the right and 3 times as far up from A as B so it is the desired point.

4.4 A Enlarged Image

Cool Down: 5 minutes

Students apply an enlargement to a polygon where the centre of enlargement is on the interior of the shape. The polygon is on a grid without coordinates and the structure of the grid can be efficiently used to find the enlargement.

Student Task Statement

Draw the image of rectangle $ABCD$ under enlargement using centre P and scale factor $\frac{1}{2}$.
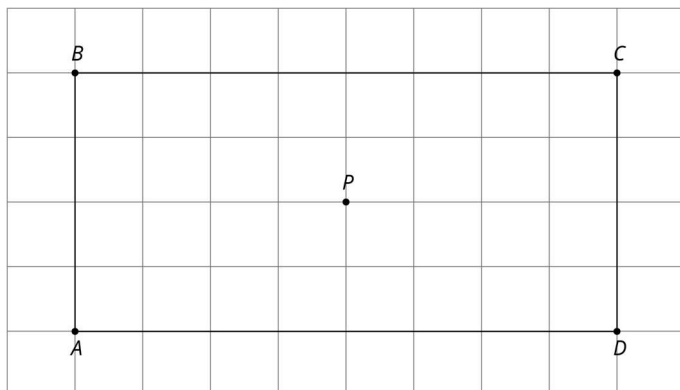

Student Response
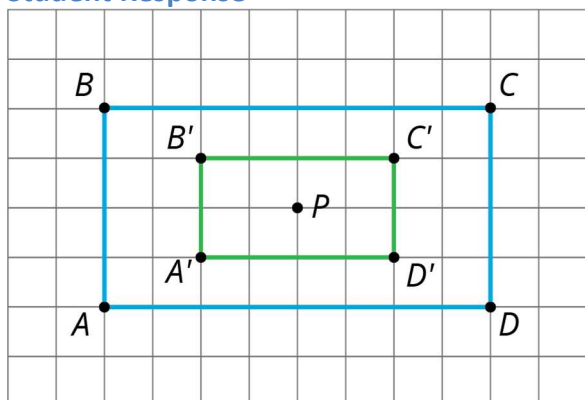

Student Lesson Summary

Square grids can be useful for showing enlargements. The grid is helpful especially when the centre of enlargement and the point(s) being enlarged lie at grid points. Rather than using a ruler to measure the distance between the points, we can count grid units.

For example, suppose we want to enlarge point Q with centre of enlargement P and scale factor $\frac{3}{2}$. Since Q is 4 grid squares to the left and 2 grid squares down from P, the enlargement will be 6 grid squares to the left and 3 grid squares down from P (can you see why?). The enlarged image is marked as Q' in the picture.
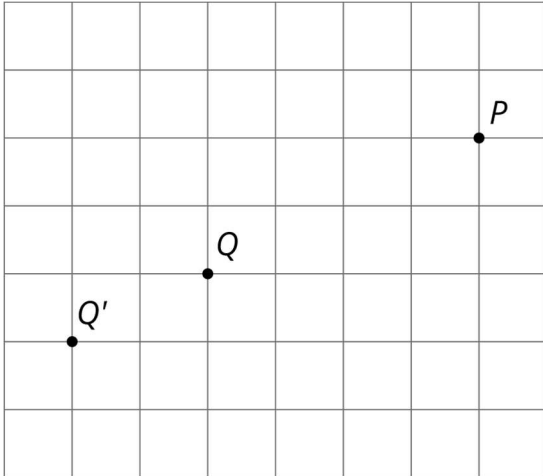
Sometimes the square grid comes with coordinates. The coordinate grid gives us a convenient way to *name* points, and sometimes the coordinates of the image can be found with just arithmetic.

For example, to make an enlargement with centre (0,0) and scale factor 2 of the triangle with coordinates (-1,-2), (3,1), and (2,-1), we can just double the coordinates to get (-2,-4), (6,2), and (4,-2).
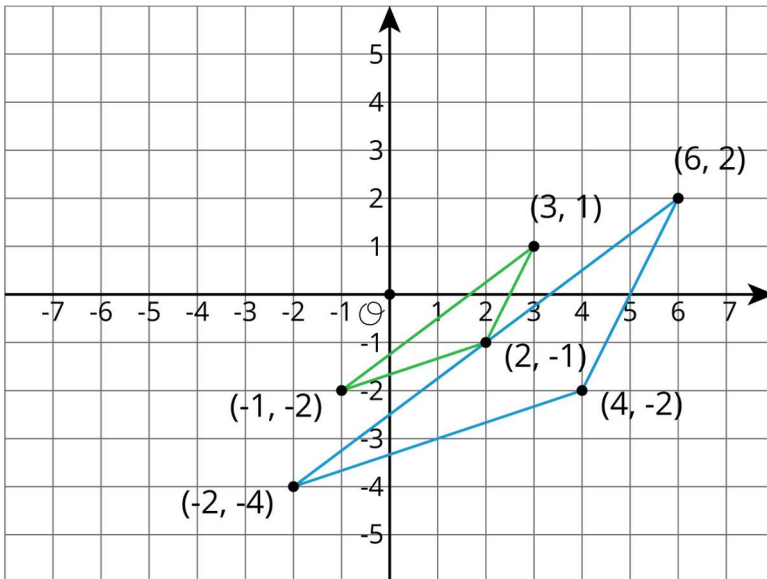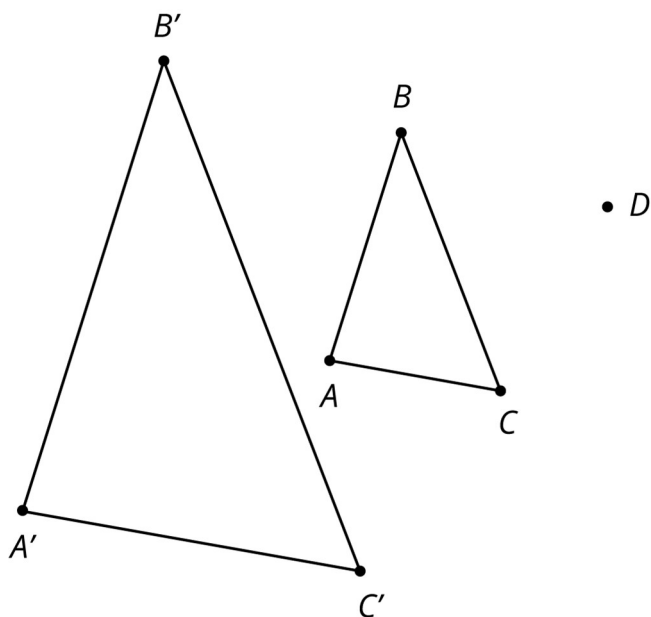
Lesson 4 Practice Problems

1. **Problem 1 Statement**

   Triangle ABC is enlarged using D as the centre of enlargement with scale factor 2.

   The image is triangle $A'B'C'$. Clare says the two triangles are congruent, because their angles are the same. Do you agree? Explain how you know.

   Solution

   No. The triangles are not congruent because their side lengths are different.

2. **Problem 2 Statement**

   On graph paper, sketch the image of quadrilateral PQRS under the following enlargements:

   a.   The enlargement centred at R with scale factor 2.

   b.   The enlargement centred at O with scale factor $\frac{1}{2}$.

   c.   The enlargement centred at S with scale factor $\frac{1}{2}$.
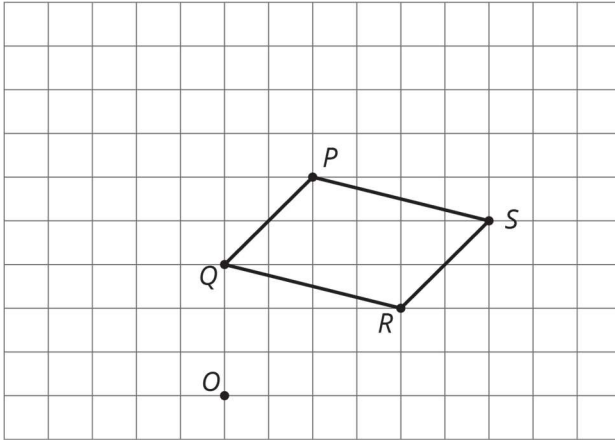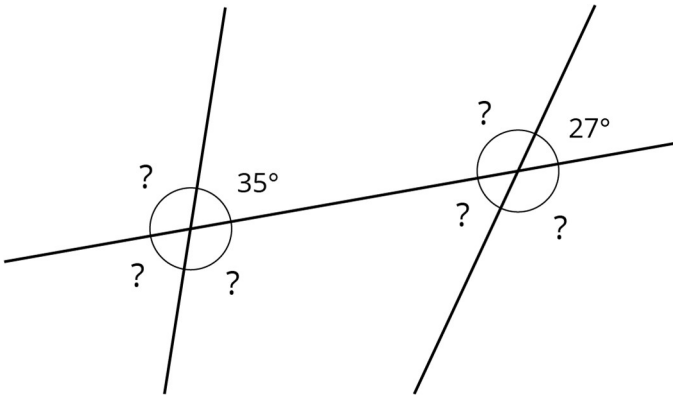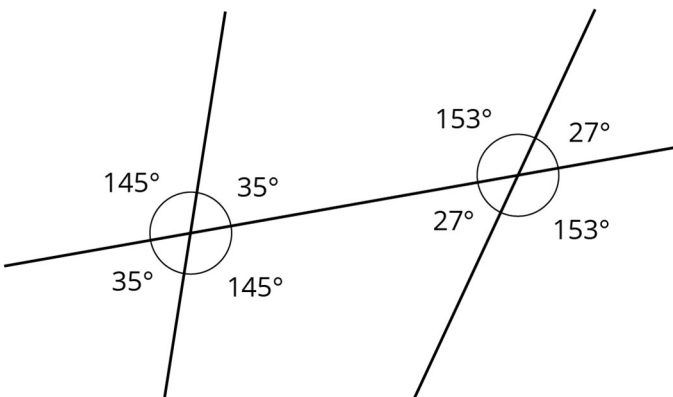
Solution
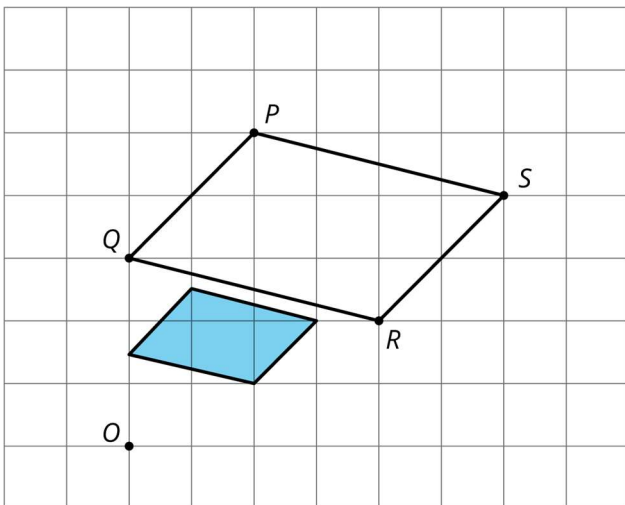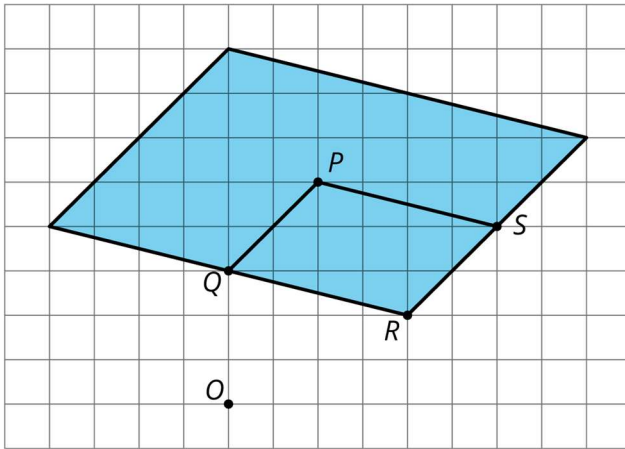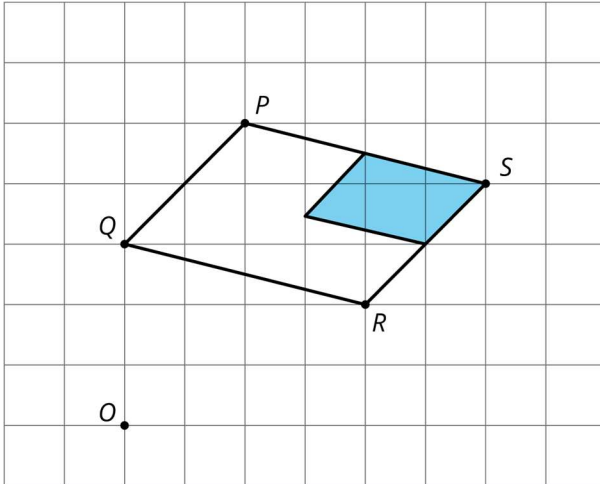
3. **Problem 3 Statement**

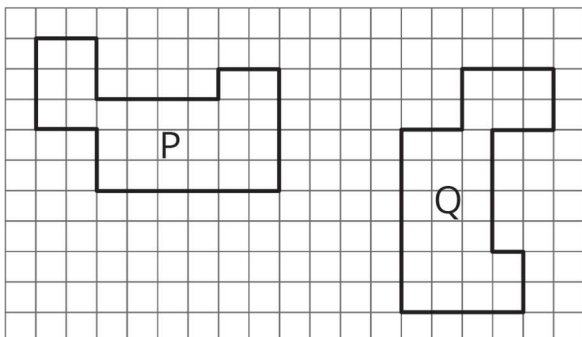The diagram shows three lines with some marked angles.

Find the missing angles marked with question marks.

Solution

4. **Problem 4 Statement**

    Describe a sequence of translations, rotations, and reflections that takes polygon P to polygon Q.

    Solution

    Answers vary. Sample response: P is rotated 90 degrees clockwise and translated until the corresponding vertices match up.

5. **Problem 5 Statement**

    Point B has coordinates (-2,-5). After a translation 4 units down, a reflection in the y-axis, and a translation 6 units up, what are the coordinates of the image?

    Solution

    (2,-3)