Lesson 5: Defining equivalent ratios

Goals

- Generate equivalent ratios and justify that they are equivalent.
- Present (in words and through other representations) a definition of equivalent ratios, including examples and non-examples.

Learning Targets

- If I have a ratio, I can create a new ratio that is equivalent to it.
- If I have two ratios, I can decide whether they are equivalent to each other.

Lesson Narrative

Previously, students understood equivalent ratios through physical perception of different batches of recipes. In this lesson, they work with equivalent ratios more abstractly, both in the context of recipes and in the context of abstract ratios of numbers. They understand and articulate that all ratios that are **equivalent** to $a : b$ can be generated by multiplying both a and b by the same number.

By connecting concrete quantitative experiences to abstract representations that are independent of a context, students develop their skills in reasoning abstractly and quantitatively. They continue to use diagrams, words, or a combination of both for their explanations. The goal in subsequent lessons is to develop a general definition of **equivalent ratios**.

Building On

- Operations and Algebraic Thinking

Addressing

- Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2 : 1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

Instructional Routines

- Group Presentations
- Stronger and Clearer Each Time
- Discussion Supports

Required Materials

Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Student Learning Goals

Let’s investigate equivalent ratios some more.

5.1 Dots and Half Dots

Warm Up: 10 minutes

In this warm-up, students are asked to determine the number of dots in an image and explain how they arrived at that answer. The goal is to prompt students to visualise and articulate different ways in which they can decompose the dots, using what they know about arrays, symmetry, and multiplication to arrive at the total number of dots. To encourage students to refer to the image in their explanation but not count every dot, this image is flashed for a few seconds and then hidden. It is flashed once more for students to check their thinking. Ask students how they *saw* the dots instead of how they *found* the number of dots, so they focus on the structure of the dots in the image.

As students share how they saw the dots, ask how the expressions they used to describe the arrangements and grouping of the dots in the two problems are similar. This prompts students to make connections between the properties of multiplication.

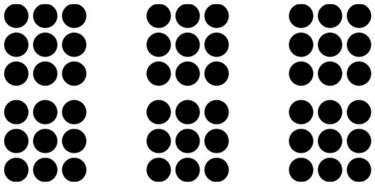
Launch

Tell students you will show them an image made up of dots for 3 seconds. Their job is to find how many dots are in the image and explain how they saw them. Display the image for all to see for 3 seconds and then hide it. Do this twice. Give students 1 minute of quiet think time between each flash of the image. Encourage students who have one way of seeing the dots to think of another way while they wait.

Representation: Internalise Comprehension. Guide information processing and visualisation. To support working memory, show the image for a longer period of time or repeat the image flash as needed. Students may also benefit from being explicitly told not to count the dots, but instead to look for helpful structure within the image.
Supports accessibility for: Memory; Organisation

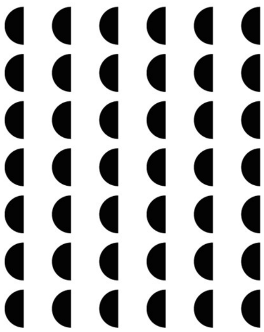
Student Task Statement

Dot Pattern 1:



Dot Pattern 2:

 = 1

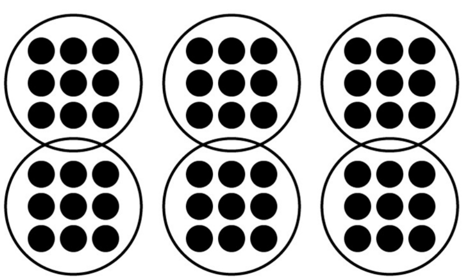


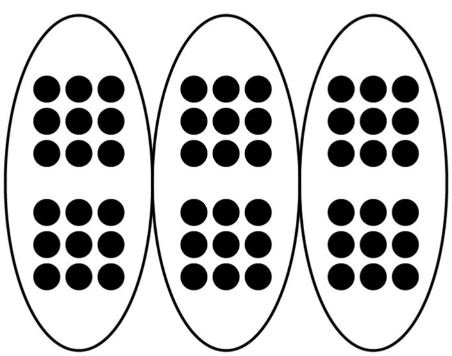
Student Response

Dot pattern 1: 54 dots. Answers vary. Possible strategies:

6 groups with a 3 by 3 array in each group $6 \times 3 \times 3 = 54$

3 groups with two group of 9 in each group $3 \times 2 \times 9 = 54$

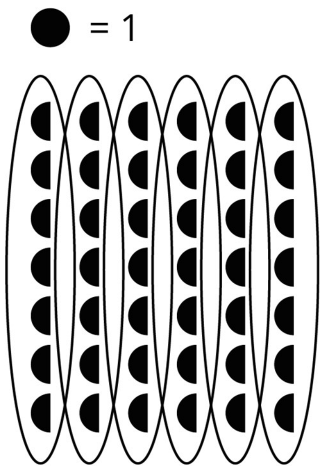


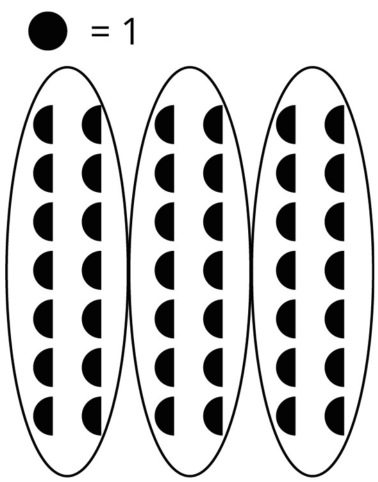


Dot pattern 2: 21 dots. Answers vary. Possible strategies:

6 groups with 3 and a half in each group $6 \times 3\frac{1}{2} = 21$

3 groups with 7 in each group $3 \times 7 = 21$





Activity Synthesis

Ask students to share how many dots they saw and how they saw them. Record and display student explanations for all to see. (Consider re-displaying the image for reference while students are explaining what they saw.) To involve more students in the conversation, consider asking:

- "Who can restate the way ___ saw the dots in different words?"
- "Did anyone see the dots the same way but would explain it differently?"
- "Does anyone want to add an observation to the way ____ saw the dots?"
- "Who saw the dots differently?"

- “Do you agree or disagree? Why?”

5.2 Tuna Casserole

15 minutes

Students use a realistic food recipe to find equivalent ratios that represent different numbers of batches. Students use the original recipe to form ratios of ingredients that represent double, half, five times, and one-fifth of the recipe. Then they examine given ratios of ingredients and determine how many batches they represent.

Instructional Routines

- Discussion Supports

Launch

Ask students if they have ever cooked something by following a recipe. If so, ask them what they made and what some of the ingredients were.

Ask: “How might we use ratios to describe the ingredients in your recipe?” (The ratios could associate the quantities of each ingredient being used.)

Explain to students that in this task, they will think about the ratios of ingredients for a tuna casserole and how to adjust them for making different numbers of batches.

Action and Expression: Internalise Executive Functions. To support development of organisational skills in problem solving, chunk this task into more manageable parts. For example, provide students with a task checklist which makes all the required components of the activity explicit.
Supports accessibility for: Memory; Organisation Listening: Discussion Supports. When asking the question, “How might we use ratios to describe the ingredients in your recipe?”, act out or use images that demonstrate the meaning of the terms "ratio", "recipe", and "ingredients" in the context of cooking. Demonstrate combining specific ingredients in their stated ratios. This will help students connect the language found in a recipe with the ratio reasoning needed for different batches of that recipe.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Students who are not yet fluent in fraction multiplication from KS2 may have difficulty understanding how to find half or one-fifth of the recipe ingredient amounts. Likewise, they may have difficulty identifying one-third of a batch. Suggest that they draw a picture of $\frac{1}{2}$ of 10, remind them that finding $\frac{1}{2}$ of a number is the same as dividing it by 2, or remind them that $\frac{1}{2}$ of a number means $\frac{1}{2}$ times that number.

Student Task Statement

Here is a recipe for tuna casserole.

Ingredients

- 3 cups cooked elbow-shaped pasta
- 6 ounce can tuna, drained
- 10 ounce can cream of chicken soup
- 1 cup grated cheddar cheese
- $1\frac{1}{2}$ cups French fried onions

Instructions

Combine the pasta, tuna, soup, and half of the cheese. Transfer into a 9 inch by 18 inch baking dish. Put the remaining cheese on top. Bake 30 minutes at 350 degrees. During the last 5 minutes, add the French fried onions. Let sit for 10 minutes before serving.

1. What is the ratio of the ounces of soup to the cups of grated cheese to the cups of pasta in one batch of casserole?

2. How much of each of these 3 ingredients would be needed to make:

a. twice the amount of casserole?

b. half the amount of casserole?

c. five times the amount of casserole?

d. one-fifth the amount of casserole?

3. What is the ratio of cups of pasta to ounces of tuna in one batch of casserole?

4. How many batches of casserole would you make if you used the following amounts of ingredients?

   a. 9 cups of pasta and 18 ounces of tuna?

   b. 36 ounces of tuna and 18 cups of pasta?

   c. 1 cup of pasta and 2 ounces of tuna?

Student Response

1. The ratio of the ounces of soup to cups of grated cheese to cups of pasta is $10 : 1 : 3$.

2. The ratio of these ingredients for different numbers of batches are:

   a. 20 ounces, 2 cups, 6 cups

   b. 5 ounces, $\frac{1}{2}$ cup, $1\frac{1}{2}$ cups

   c. 50 ounces, 5 cups, 15 cups

   d. 2 ounces, $\frac{1}{5}$ cup, $\frac{3}{5}$ cup

3. The ratio of cups of pasta to ounces of tuna is $3 : 6$.

4.

   a. 3 batches

   b. 6 batches

   c. $\frac{1}{3}$ batch

Are You Ready for More?

The recipe says to use a 9 inch by 18 inch baking dish. Determine the length and width of a baking dish with the same height that could hold:

1. Twice the amount of casserole

2. Half the amount of casserole

3. Five times the amount of casserole

4. One-fifth the amount of casserole

Student Response

Answers vary. Sample responses:

1. 18 inch by 18 inch

2. 9 inch by 9 inch

3. 45 inch by 18 inch

4. 9 inch by $\frac{18}{5}$ inch

Activity Synthesis

Display the recipe for all to see. Ask students to share and explain their responses. List their responses—along with the specified number of batches—for all to see. Ask students to analyse the list and describe how the ratio of quantities relate to the number of batches in each case. Draw out the idea that each quantity within the recipe was *multiplied* by a number to obtain each batch size, and that each ingredient amount is multiplied by the *same* value.

In finding one-half and one-fifth of a batch, students may speak in terms of dividing by 2 and dividing by 5. Point out that "dividing by 2" has the same outcome as "multiplying by one-half," and "dividing by 5" has the same outcome as "multiplying by one-fifth." Later, we will want to state our general definition of equivalent ratios as simply as possible: as *multiplying* both a and b in the ratio $a : b$ by the same number (not "multiplying or dividing").

5.3 What Are Equivalent Ratios?

15 minutes

In this activity, students identify what equivalent ratios have in common (a ratio equivalent to $a : b$ can be generated by multiplying both a and b by the same number) and generate equivalent ratios. It is at this point in the unit where students will explicitly define the term equivalent ratios.

Instructional Routines

- Group Presentations

- Stronger and Clearer Each Time

Launch

Arrange students in groups of 3–4. Provide each group with tools for creating a visual display.

Summarise what we know so far about equivalent ratios. When we double or triple a colour recipe, the ratios of the amount of ingredients in the mixtures are equivalent to those in the original recipe. For example, 24 : 9 and 8 : 3 are equivalent ratios, because we can think of 24 : 9 as a mixture that contains three batches of purple water where a single batch is 8 : 3.

When we make multiple batches of a food recipe, we say the ratios of the amounts of the ingredients are equivalent to the ratios in a single batch. For example, 3 : 6, 1 : 2, and 9 : 18 are equivalent ratios because they correspond to the amount of the ingredients in different numbers of batches of tuna casserole, and they all taste the same.

In this activity, we'll write a definition for **equivalent ratios**.

When students pause after question 5, have a whole class discussion about the first five questions. Then assign each group a different ratio to use as their example for their visual display. Some possibilities:

4 : 5

3 : 2

5 : 6

3 : 4

2 : 5

Writing and speaking: Maths Language Routine 1 Stronger and Clearer Each Time. This is the first time Maths Language Routine 1 is suggested as a support in this course. In this routine, students are given a thought-provoking question or prompt and asked to create a first draft response. Students meet together in 2–3 partners to share and refine their response through conversation. While meeting, listeners ask questions such as: "What did you mean by . . .?" and "Can you say that another way?" Finally, students write a second draft of their response that reflects ideas from their partners, and improvement on their writing. The purpose of this routine is to provide a structured and interactive opportunity for students to revise and refine their ideas through verbal and written means.
Design Principle(s): Optimise output (for generalisation)

How It Happens:

1. Use this routine to provide students a structured opportunity to revise and refine their response to "How do you know when ratios are equivalent and when they are not equivalent?" Allow students 2–3 minutes to individually create first draft responses.

2. Invite students to meet with 2–3 other partners for feedback.

   Instruct the speaker to begin by sharing their ideas without looking at their written draft, if possible. Provide the listener with these prompts for feedback that will help teams strengthen their ideas and clarify their language: "Can you explain how...?", "You should expand on….", "Can you give an example of equivalent ratios?", and "Could you justify that differently?" Be sure to have the partners switch roles. Allow 1–2 minutes to discuss.

3. Signal for students to move on to their next partner and repeat this structured meeting.

4. Close the partner conversations and invite students to revise and refine their writing in a second draft. Students can borrow ideas and language from each partner to strengthen the final product.

   Provide these sentence frames to help students organise their thoughts in a clear, precise way: "I know when ratios are equivalent/not equivalent when…." and "An example of this is…because…."

   Here is an example of a second draft:

   "I know when ratios are equivalent when I multiply both parts of one ratio by the same number and I get the other ratio. For example, I know that 5 : 3 is equivalent to 30 : 18 because when I multiply 5 by 6, I get 30, and when I multiply 3 by 6, I get 18, so 6 is the same number used to multiply both parts. But 5 : 3 is not equivalent to 15 : 12 because when I multiply 5 by 3, I get 15, and when I multiply 3 by 4, I get 12. So since a different number is used to multiply to get the second ratio, they're not equivalent."

5. If time allows, instruct students to compare their first and second drafts. If not, the students can continue on with the lesson by returning to their first partner and creating the visual.

Anticipated Misconceptions

Students may incorporate recipes, specific examples, or batch thinking into their definitions. These are important ways of thinking about equivalent ratios, but challenge them to come up with a definition that only talks about the numbers involved and not what the numbers represent.

If groups struggle to get started thinking generally about a definition, give them a head start with: "A ratio is equivalent to $a : b$ when . . ."

If students include "or divide" in their definition, remind them that, for example, dividing by 5 gives the same result as multiplying by one-fifth. Therefore, we can just use "multiply" in our definition.

Student Task Statement

The ratios 5 : 3 and 10 : 6 are **equivalent ratios**.

1. Is the ratio 15 : 12 equivalent to these? Explain your reasoning.

2. Is the ratio 30 : 18 equivalent to these? Explain your reasoning.

3. Give two more examples of ratios that are equivalent to 5 : 3.

4. How do you know when ratios are equivalent and when they are *not* equivalent?

5. Write a definition of *equivalent ratios*.

   Pause here so your teacher can review your work and assign you a ratio to use for your visual display.

6. Create a visual display that includes:

   - the title "Equivalent Ratios"
   - your best definition of *equivalent ratios*
   - the ratio your teacher assigned to you
   - at least two examples of ratios that are equivalent to your assigned ratio
   - an explanation of how you know these examples are equivalent
   - at least one example of a ratio that is *not* equivalent to your assigned ratio
   - an explanation of how you know this example is *not* equivalent

   Be prepared to share your display with the class.

Student Response

1. $15 : 12$ is not equivalent to $5 : 3$ because 15 is 5×3 but 12 is 3×4.

2. $30 : 18$ is equivalent to $5 : 3$ because 30 is 5×6 and 18 is 3×6.

3. Answers vary and might include $15 : 9$, $20 : 12$, and $50 : 30$.

4. Answers vary and should include some version of "multiply both parts by the same number."

5. Answers vary. Sample response: A ratio is equivalent to $a : b$ when both a and b are multiplied by the same number.

6. Answers vary.

Activity Synthesis

Each group will share their visual display as they explain their definitions. Highlight phrases or explanations that are similar in each display. Make one class display that incorporates all valid definitions. This display should be kept posted in the classroom for the remaining lessons within this unit. It should look something like:

Equivalent Ratio A ratio is equivalent to $a : b$ when both a and b are multiplied by the same number.

Lesson Synthesis

In this lesson you came to an understanding of what **equivalent ratios** are.

Discuss:

- How do you make different amounts of a coloured-water mixture that have the same colour? (The amount of each colour being mixed must be multiplied by the same value.)
- If you want to make a different amount of a food recipe, how can you ensure that the resulting food will taste the same? (Each ingredient in the recipe must be multiplied by the same value.)
- What are equivalent ratios and how are they generated? (Each number is multiplied by the same value.)

5.4 Why Are They Equivalent?

Cool Down: 5 minutes

Anticipated Misconceptions

If students are not clear about the meaning of equivalent ratios, refer them to the visual displays created in the previous activity.

Student Task Statement

1. Write another ratio that is equivalent to the ratio 4 : 6.
2. How do you know that your new ratio is equivalent to 4 : 6? Explain or show your reasoning.

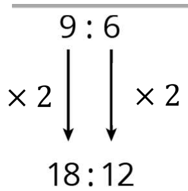
Student Response

1. Answers vary. Sample responses: 2 : 3, 16 : 24, 400 : 600.
2. Answers vary. 2 : 3 is equivalent to 4 : 6 because both 4 and 6 are multiplied by $\frac{1}{2}$. 16 : 24 is equivalent because both 4 and 6 are multiplied by 4. 400 : 600 is equivalent because both 4 and 6 are multiplied by 100.

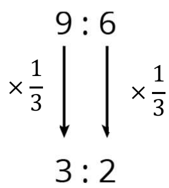
Student Lesson Summary

All ratios that are **equivalent** to $a : b$ can be made by multiplying both a and b by the same number.

For example, the ratio 18 : 12 is equivalent to 9 : 6 because both 9 and 6 are multiplied by the same number: 2.

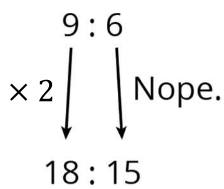


3 : 2 is also equivalent to 9 : 6, because both 9 and 6 are multiplied by the same number: $\frac{1}{3}$.



Is 18 : 15 equivalent to 9 : 6?

No, because 18 is 9 × 2, but 15 is *not* 6 × 2.



Glossary

- equivalent ratios

Lesson 5 Practice Problems

Problem 1 Statement

Each of these is a pair of equivalent ratios. For each pair, explain why they are equivalent ratios or draw a diagram that shows why they are equivalent ratios.

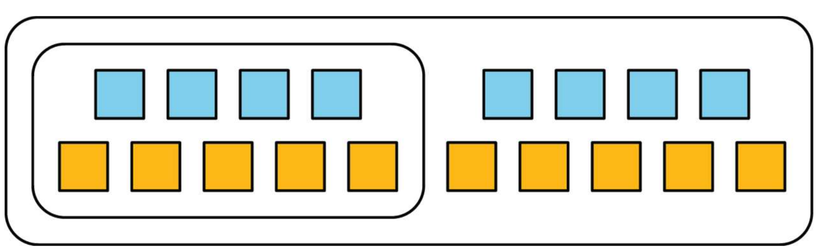
a. 4 : 5 and 8 : 10

b. 18 : 3 and 6 : 1

a. 2 : 7 and 10 000 : 35 000

Solution

Answers vary. Sample response:

a. 

The diagram shows that 8 to 10 is the same as 2 groups of 4 to 5 so these are equivalent ratios.

b. $18 \times \frac{1}{3} = 6$ and $3 \times \frac{1}{3} = 1$.

c. $2 \times (5\,000) = 10\,000$ and $7 \times (5\,000) = 35\,000$.

Problem 2 Statement

Explain why 6 : 4 and 18 : 8 are not equivalent ratios.

Solution

Answers vary. Sample response: 6 : 4 is not equivalent to 18 : 8 because 18 is 6×3, but 8 is not 4×3.

Problem 3 Statement

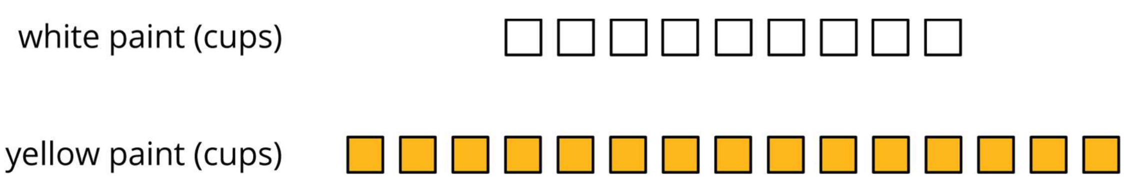
Are the ratios 3 : 6 and 6 : 3 equivalent? Why or why not?

Solution

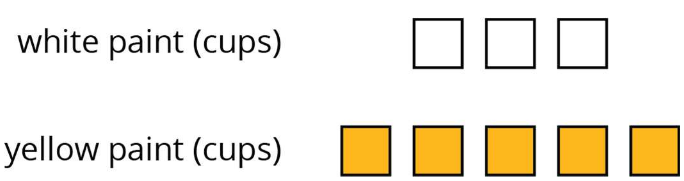
Answers vary. Sample response: No, the ratio 3 : 6 is not equivalent to 6 : 3. The ratio 3 : 6 represents 3 of one type of object for every 6 of another type of object while the ratio 6 : 3 represents 6 of the first type of object for every 3 of the second type of object.

Problem 4 Statement

This diagram represents 3 batches of light yellow paint. Draw a diagram that represents 1 batch of the same shade of light yellow paint.



Solution



Problem 5 Statement

In the fruit bowl there are 6 bananas, 4 apples, and 3 oranges.

a. For every 4 ________________, there are 3 ________________.

b. The ratio of ________________ to ________________ is 6 : 3.

c. The ratio of ________________ to ________________ is 4 to 6.

d. For every 1 orange, there are _____ bananas.

Solution

a. apples, oranges

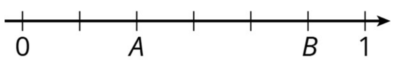
b. bananas, oranges

c. apples, bananas

d. 2

Problem 6 Statement

Write fractions for points A and B on the number line.



Solution

$A = \frac{2}{6}$ or $\frac{1}{3}$

$B = \frac{5}{6}$