Lesson 7: Revisit percentages

Goals

- State explicitly what the chosen variable represents when creating an equation.
- Use equations to solve problems involving percentages and explain (orally) the solution method.
- Write equations of the form $px = q$ or equivalent to represent situations where the amount that corresponds to 100% is unknown.

Learning Targets

- I can solve percent problems by writing and solving an equation.

Lesson Narrative

Students learned about what percentages are and how to solve certain problems in an earlier unit. At the time, they did not learn an efficient procedure for finding B in "A% of B is C" given A and C, because they didn't have an efficient way to solve an equation of the form $px = q$. Now they do, so we briefly revisit this type of problem.

Alignments

Building On

- Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Addressing

- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
- Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Instructional Routines

- Clarify, Critique, Correct
- Discussion Supports

- Number Talk
- Think Pair Share

Student Learning Goals

Let's use equations to find percentages.

7.1 Number Talk: Percentages

Warm Up: 5 minutes

The purpose of this warm-up is to rekindle anything students remember about percentages and representations they use to reason about them.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Solve each problem mentally.

1. Bottle A contains 4 ounces of water, which is 25% of the amount of water in Bottle B. How much water is there in Bottle B?
2. Bottle C contains 150% of the water in Bottle B. How much water is there in Bottle C?
3. Bottle D contains 12 ounces of water. What percentage of the amount of water in Bottle B is this?

Student Response

1. 16 ounces
2. 24 ounces
3. 75%

Activity Synthesis

Invite students to share different representations and ways of reasoning. Record student strategies and nonchalantly write an equation for each in the process.

Speaking: Discussion Supports. Provide sentence frames to support students with explaining their strategies. For example, "I noticed that _____." or "First, I _____ because _____." When students share their answers with a partner, prompt them to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to clarify their thinking.
Design Principle(s): Optimise output (for explanation)

7.2 Representing a Percentage Problem with an Equation

20 minutes

Students perform repeated calculations and then generalise with an algebraic expression. The purpose of this activity is to help students see that any basic percentage problem like "n percent of this is that" can be represented with an equation in the form $px = q$.

Instructional Routines

- Clarify, Critique, Correct
- Think Pair Share

Launch

Using any insights from the warm-up as an example, remind students of any efficient method they know to compute a percentage. For example, 25% of 16 can be computed using $\frac{25}{100} \times 16$.

Arrange students in groups of 2. Give 5–10 minutes of quiet work time and time to share responses with a partner, followed by a whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge about computing percentages. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Students might not understand that we are trying to find the whole that we know an amount is a certain percent of. Encourage them to draw a bar model or a double number line to visualise the relationship between the three quantities.

Student Task Statement

1. Answer each question and show your reasoning.

a. Is 60% of 400 equal to 87?

b. Is 60% of 200 equal to 87?

c. Is 60% of 120 equal to 87?

2. 60% of x is equal to 87. Write an equation that expresses the relationship between 60%, x, and 87. Solve your equation.

3. Write an equation to help you find the value of each variable. Solve the equation.

   60% of c is 43.2.

   38% of e is 190.

Student Response

a. No, 60% of 400 equals $\frac{60}{100} \times 400$ or 240

b. No, 60% of 200 equals $\frac{60}{100} \times 200$ or 120

c. No, 60% of 120 equals $\frac{60}{100} \times 120$ or 72

1. $\frac{60}{100}x = 87, x = \frac{87}{\frac{60}{100}}, x = 145$

2. $\frac{60}{100}c = 43.2, c = 72$

   $\frac{38}{100}e = 190, e = 500$

Activity Synthesis

The purpose of this discussion is for students to see how writing and solving an equation can be an efficient way to solve a problem about percentages. In the course of the discussion, they should see three equations written and solved. If any students used representations like bar models or double number lines to reason about the problem, it can be advantageous to display these alongside the equations so that students can make connections between strategies they understand well and the more abstract strategy of writing and solving an equation.

Writing, Representing: Clarify, Critique, Correct. Present an incorrect statement, "For 60% of c is 43.2, the value of c is 25.92 because 0.6 times 43.2 is 25.92." Invite students to ask clarifying questions about the statement to identify the error. Invite students to work with a partner to write a correct statement using a representation such as a bar model or double number line. This will help students to visualise the relationship between the three quantities and use language to critique and create viable mathematical arguments.
Design Principle(s): Maximise meta-awareness

7.3 Puppies Grow Up, Revisited

10 minutes

In this activity, students are asked to write an equation but are not given a letter to use. This is an opportunity to explain to students that when they decide to use a letter to represent something, they need to state what the letter represents.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Keep students in the same groups. Allow students 5 minutes of quiet work time and time to share responses with a partner, followed by a whole-class discussion.

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, invite students to draw a picture or bar model to help as an intermediate step before writing an equation.
Supports accessibility for: Social-emotional skills; Conceptual processing

Student Task Statement

1. Puppy A weighs 8 pounds, which is about 25% of its adult weight. What will be the adult weight of Puppy A?
2. Puppy B weighs 8 pounds, which is about 75% of its adult weight. What will be the adult weight of Puppy B?
3. If you haven't already, write an equation for each situation. Then, show how you could find the adult weight of each puppy by solving the equation.

Student Response

1. 32 pounds
2. $\frac{32}{3}$ pounds
3. Answers vary. Sample responses: Divide both sides of $8 = \frac{1}{4}x$ by $\frac{1}{4}$ to get $32 = x$. Divide both sides of $8 = \frac{3}{4}x$ by $\frac{3}{4}$ to get $y = \frac{32}{3}$ or about 10.7 pounds.

Are You Ready for More?

Diego wants to paint his room purple. He bought one gallon of purple paint that is 30% red paint and 70% blue paint. Diego wants to add more blue to the mix so that the paint mixture is 20% red, 80% blue.

1. How much blue paint should Diego add? Test the following possibilities: 0.2 gallons, 0.3 gallons, 0.4 gallons, 0.5 gallons.

2. Write an equation in which x represents the amount of paint Diego should add.

3. Check that the amount of paint Diego should add is a solution to your equation.

Student Response

1. 0.5 gallons is the correct amount of blue paint to add.
   0.2 gallons: 75% blue paint, because the amount of blue paint divided by the total amount of paint is $\frac{0.9}{1.2} = 0.75$.
   0.3 gallons: 77% blue paint, by similar reasoning
   0.4 gallons: 78.6% blue paint
   0.5 gallons: 80% blue paint

2. $0.7 + x = 0.8(1 + x)$

3. Yes.

Activity Synthesis

The focus of the discussion should be the selection of a variable to represent an unknown quantity. Invite students to share how they decided where to use a variable, what it represented in the story, what letter they used and why. Ask why it is important to state what the letter represents and where they made that statement in their solutions.

Speaking, Representing: Discussion Supports. To support whole-class discussion about selecting a variable to represent an unknown quantity, provide sentence frames to help students explain their reasoning. For example, "I knew I needed to use a variable to represent _____ because _____." or "The variable I chose to represent _____ is _____, because_____."
Design Principle(s): Support sense-making

Lesson Synthesis

Students have been solving equations with fraction coefficients in the past few lessons so these percent problems are an application of their prior work. Consider asking some of the following questions to guide the discussion and help students recognise this connection:

- "How are the equations we wrote today related to the equations we have previously written with fractions? How do solution strategies compare?"

- "Can equations be used to solve other types of problems with percentages? For example, where we know the part and the whole but not what percent the part is of the whole?" (Yes. For example, the equation $20p = 5$ and its solution $p = \frac{1}{4}$ or $\frac{25}{100}$ tells us that 5 is 25% of 20.)

- "Describe a situation where you know what percent a number is of another, but you don't know that second number. Explain to a partner how you would find the second number."

7.4 Fundraising for the Animal Shelter

Cool Down: 5 minutes

Student Task Statement

Noah raised £54 to support the animal shelter, which is 60% of his fundraising goal.

1. Write an equation to represent the situation.
2. What is Noah's fundraising goal? Show or explain how you found it.

Student Response

1. $54 = \frac{60}{100}x$ (or equivalent)
2. £90, divide both sides of the equation by $\frac{60}{100} = \frac{3}{5}$ to get $x = 54 \times \frac{5}{3} = 90$

Student Lesson Summary

If we know that 455 students are in school today and that number represents 70% attendance, we can write an equation to figure out how many students go to the school.

The number of students in school today is known in two different ways: as 70% of the students in the school, and also as 455. If s represents the total number of students who go to the school, then 70% of s, or $\frac{70}{100}s$, represents the number of students that are in school today, which is 455.

We can write and solve the equation:

$$\begin{aligned} \frac{70}{100}s &= 455 \\ s &= 455 \div \frac{70}{100} \\ s &= 455 \times \frac{100}{70} \\ s &= 650 \end{aligned}$$

There are 650 students in the school.

In general, equations can help us solve problems in which one amount is a percentage of another amount.

Lesson 7 Practice Problems

1. **Problem 1 Statement**

   A crew has paved $\frac{3}{4}$ of a mile of road. If they have completed 50% of the work, how long is the road they are paving?

   Solution

   $1\frac{1}{2}$ miles because $\frac{3}{4}$ is half (or 50%) of $\frac{6}{4}$ or $1\frac{1}{2}$.

2. **Problem 2 Statement**

   40% of x is 35.

   a. Write an equation that shows the relationship of 40%, x, and 35.

   b. Use your equation to find x. Show your reasoning.

   Solution

   a. $0.4x = 35$

   b. $x = 87.5$ $(35 \div 0.4 = 87.5)$

3. **Problem 3 Statement**

   Priya has completed 9 exam questions. This is 60% of the questions on the exam.

   a. Write an equation representing this situation. Explain the meaning of any variables you use.

   b. How many questions are on the exam? Show your reasoning.

   Solution

   a. Answers vary. Sample responses: $9 = \frac{60}{100}q$ or $9 = 0.6q$ where q is the number of questions on the exam.

   b. 15 because $9 \div (0.6) = 15$.

4. **Problem 4 Statement**

   Answer each question. Show your reasoning.

   20% of a is 11. What is a?

   75% of b is 12. What is b?

   80% of c is 20. What is c?

200% of d is 18. What is d?

Solution

a. 55

b. 16

c. 25

d. 9

Sample reasoning for "75% of b is 12":

- Using an equation: $\frac{75}{100}b = 12$, so $b = 12 \div \frac{75}{100}$, so $b = 12 \times \frac{100}{75}$, so $b = 16$.
- Using a table. To get from the first row to the second row, divide 75 and 12 each by 3. To get from the second to the third row, multiply the 25 and 4 each by 4.

percentage	number
75	12
25	4
100	16

5. **Problem 5 Statement**

For the equation $2n - 3 = 7$

a. What is the variable?

b. What is the coefficient of the variable?

c. Which of these is the solution to the equation? 2, 3, 5, 7, n

Solution

a. n

b. 2

c. 5

6. **Problem 6 Statement**

Which of these is a solution to the equation $\frac{1}{8} = \frac{2}{5} \times x$?

a. $\frac{2}{40}$

b. $\frac{5}{16}$

c. $\frac{11}{40}$

d. $\frac{17}{40}$

Solution B

7. **Problem 7 Statement**

Find the quotients.

a. $0.009 \div 0.001$

b. $0.009 \div 0.002$

c. $0.0045 \div 0.001$

d. $0.0045 \div 0.002$

Solution

a. 9

b. 4.5

c. 4.5

d. 2.25