

## Lesson 2: Corresponding parts and scale factors

### Goals

- Comprehend the phrase “scale factor” and explain (orally) how it relates corresponding lengths of a shape and its scaled copy.
- Explain (orally) what it means to say one part in a shape “corresponds” to a part in another shape.
- Identify and describe (orally and in writing) corresponding points, corresponding line segments, or corresponding angles in a pair of shapes.

### Learning Targets

- I can describe what the scale factor has to do with a shape and its scaled copy.
- In a pair of shapes, I can identify corresponding points, corresponding line segments, and corresponding angles.

### Lesson Narrative

This lesson develops the vocabulary for talking about scaling and scaled copies more precisely, and identifying the structures in common between two shapes.

Specifically, students learn to use the term **corresponding** to refer to a pair of points, line segments, or angles in two shapes that are scaled copies. Students also begin to describe the numerical relationship between the corresponding lengths in two shapes using a **scale factor**. They see that when two shapes are scaled copies of one another, the same scale factor relates their corresponding lengths. They practise identifying scale factors.

A look at the angles of scaled copies also begins here. Students use tracing paper to trace and compare angles in an original shape and its copies. They observe that in scaled copies the sizes of corresponding angles are equal.

### Building On

- Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
- Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

### Addressing

- Solve problems involving scale drawings of geometric shapes, including calculating actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

### Building Towards

- Recognise and represent proportional relationships between quantities.

### Instructional Routines

- Discussion Supports
- Notice and Wonder
- Number Talk
- Think Pair Share

### Required Materials

#### Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

### Required Preparation

Prepare to display the images of the railroad crossing sign for the Corresponding Parts activity. Make sure students have access to their geometry toolkits, especially tracing paper and graph paper.

### Student Learning Goals

Let's describe features of scaled copies.

## 2.1 Number Talk: Multiplying by a Unit Fraction

### Warm Up: 5 minutes

This number talk allows students to review multiplication strategies, refreshing the idea that multiplying by a unit fraction is the same as dividing by its whole number reciprocal. It encourages students to use the structure of base ten numbers and the properties of operations to find the product of two whole numbers. Each problem was chosen to elicit different approaches, so as students share theirs, ask how the factors in each problem impacted their strategies.

Before students begin, consider establishing a small, discreet hand signal (such as a thumbs-up) students can display to indicate they have an answer that they can support by reasoning. Discreet signalling is a quick way for teachers to gather feedback about timing. It also keeps students from being distracted or rushed by raised hands around the class.

### Instructional Routines

- Discussion Supports
  - Number Talk
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## Launch

Display one problem at a time. Give students up to 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a brief whole-class discussion.

*Representation: Internalise Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organisation*

## Student Task Statement

Find each product mentally.

$$\frac{1}{4} \times 32$$

$$(7.2) \times \frac{1}{9}$$

$$\frac{1}{4} \times (5.6)$$

## Student Response

- $\frac{1}{4} \times 32 = 8$ . Possible strategy:  $32 \div 4 = 8$
- $(7.2) \times \frac{1}{9} = 0.8$ . Possible strategy:  $72 \div 9 = 8$  so  $(7.2) \div 9 = 0.8$
- $\frac{1}{4} \times (5.6) = 1.4$ . Possible strategy:  $(5.6) \div 4 = 1.4$

## Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. If students express strategies in terms of division, ask if that strategy would work for any multiplication problem involving fractions. Highlight that these problems only involve unit fractions and division by the denominator is a strategy that works when multiplying by a unit fraction.

To involve more students in the conversation, consider asking:

- Who can restate \_\_\_'s reasoning in a different way?
  - Did anyone solve the problem the same way but would explain it differently?
  - Did anyone solve the problem in a different way?
  - Does anyone want to add on to \_\_\_'s strategy?
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- Do you agree or disagree? Why?

*Speaking: Discussion Supports:* Display sentence frames to support students when they explain their strategy. For example, "First, I \_\_\_\_ because . . ." or "I noticed \_\_\_\_ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimise output (for explanation)*

## 2.2 Corresponding Parts

### 15 minutes (there is a digital version of this activity)

This activity introduces important language students will apply to describe scaled copies. In particular, it introduces the important idea of corresponding parts. Students have previously analysed corresponding sides in shapes. Here they will begin to examine angles explicitly as well, understanding that corresponding angles in a shape and its scaled copy have the same size.

#### Instructional Routines

- Discussion Supports
- Notice and Wonder

#### Launch

Tell students that in this lesson, they will look more closely at copies of shapes and describe specific parts in them.

Display the designs (the three images in the activity statement) and the following descriptions for all to see. Ask students what they notice and what they wonder. After discussion, explain that the original design and its two copies have parts that correspond to one another. Point out some of their corresponding parts:

- The X-pattern going across each shape
- The curved outline of each shape
- The points  $K$  in the original sign,  $A$  in copy 1, and  $U$  in copy 2

Arrange students in groups of 2 and provide access to their geometry toolkits (especially tracing paper). Give students 2–3 minutes to complete the first two questions and another 2 minutes to discuss their responses with their partner. Ask students to pause their work for a quick class discussion afterwards.

Have a few students name a set of corresponding points, line segments, and angles.

Then, ask students to indicate whether they think either copy is a scaled copy. Invite a couple of students to share their reasoning. When the class reaches an agreement that copy

1 is a scaled copy and copy 2 is not, ask students to complete the remaining questions individually and to use tracing paper as a tool.

Consider demonstrating to the class how to use tracing paper to compare angles. Tell or show students that the line segments forming an angle could be extended for easier tracing and comparison.

For classrooms using the digital version of the activity, the applet has a moveable angle tool to compare the angles in the copies with the angles in the original.

*Action and Expression: Develop Expression and Communication.* Maintain a display of important terms and vocabulary. During the launch take time to review the following terms from previous lessons that students will need to access for this activity: corresponding points, corresponding line segments, and corresponding angles.

*Supports accessibility for: Memory; Language Speaking: Discussion Supports.* Use this routine to amplify mathematical uses of language to communicate about corresponding points, line segments, and angles. As students share what they noticed between the three images, revoice their statements using the term “corresponding.” Then, invite students to use the term “corresponding” when describing what they noticed. Some students may benefit from chorally repeating the phrases that include the word “corresponding” in context.

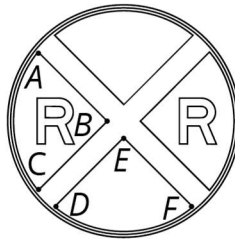
*Design Principle(s): Optimise output (for explanation)*

### Student Task Statement

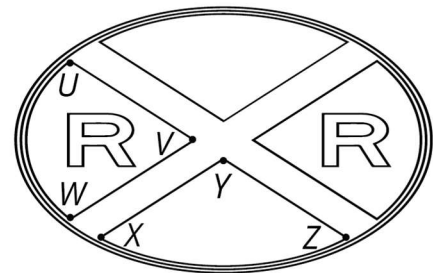
Here is a shape and two copies, each with some points labelled.



ORIGINAL



COPY 1



COPY 2

- Complete this table to show **corresponding parts** in the three shapes.

original	copy 1	copy 2
point $P$		
line segment $LM$		
	line segment $EF$	
		point $W$
angle $KLM$		
		angle $XYZ$

2. Is either copy a scaled copy of the original shape? Explain your reasoning.
3. Use tracing paper to compare angle  $KLM$  with its corresponding angles in copy 1 and copy 2. What do you notice?
4. Use tracing paper to compare angle  $NOP$  with its corresponding angles in copy 1 and copy 2. What do you notice?

### Student Response

1.

original	copy 1	copy 2
point $P$	point $F$	point $Z$
line segment $LM$	line segment $BC$	line segment $VW$
line segment $OP$	line segment $EF$	line segment $YZ$
point $M$	point $C$	point $W$
angle $KLM$	angle $ABC$	angle $UVW$
angle $NOP$	angle $DEF$	angle $XYZ$

2. Copy 1 is a scaled copy, but copy 2 is not. Sample explanation: The original sign is a circle. Copy 1 is also a circle, only smaller. Copy 2 has been stretched sideways and shrunken vertically; its shape has changed into an oval, so it is not a scaled copy.
3. Angle  $ABC$  in copy 1 corresponds to and has the same size as angle  $KLM$ . Angle  $UVW$  in copy 2 also corresponds to angle  $KLM$  but is smaller in size than the original angle.
4. Angle  $DEF$  in copy 1 corresponds to and has the same size as angle  $NOP$ . Angle  $XYZ$  in copy 2 also corresponds to angle  $NOP$  but is larger in size than the original angle.

### Activity Synthesis

Select a few students to share their observations about angles. Discuss the size of corresponding angles in shapes that are scaled copies and those that are not. Ask questions such as:

- In the scaled copy, copy 1, did the size of any angle change compared to its corresponding angle in the original sign? (No)
- In copy 2, did the size of any angle change relative to its corresponding angle in the original sign? (Yes) Which ones? (Angle  $UVW$  has a different size than angle  $KLM$ , for example.)
- What can you say about corresponding angles in two shapes that are scaled copies of one another? (They have the same size.)
- What can you say about corresponding angles in two shapes that are *not* scaled copies? (They *might* not have the same size.)

## 2.3 Scaled Triangles

### 15 minutes

In this activity, students continue to practise identifying corresponding parts of scaled copies. By organising corresponding lengths in a table, students see that there is a single factor that relates each length in the original triangle to its corresponding length in a copy. They learn that this number is called a **scale factor**.

As students work on the first question, listen to how they reason about which triangles are scaled copies. Identify groups who use side lengths and angles as the basis for deciding. (Students are not expected to reason formally yet, but should begin to look to lengths and angles for clues.)

As students identify corresponding sides and their lengths in the second and third questions, look out for confusion about corresponding parts. Notice how students decide which sides of the right-angled triangles correspond.

If students still have access to tracing paper, monitor for students who use this tool strategically.

### Instructional Routines

- Think Pair Share

### Launch

Arrange students into groups of 4. Assign each student one of the following pairs of triangles in the first question.

- A and E
- B and F
- C and G
- D and H

Give students 2 minutes of quiet think time to determine if their assigned triangles are scaled copies of the original triangle. Give another 2–3 minutes to discuss their responses and complete the first question in groups.

Discuss briefly as a class which triangles are scaled copies and select a couple of groups who reasoned in terms of lengths and angles to explain their reasoning. Some guiding questions:

- What information did you use to tell scaled copies from those that are not?
  - How were you able to tell right away that some shapes are not scaled copies?
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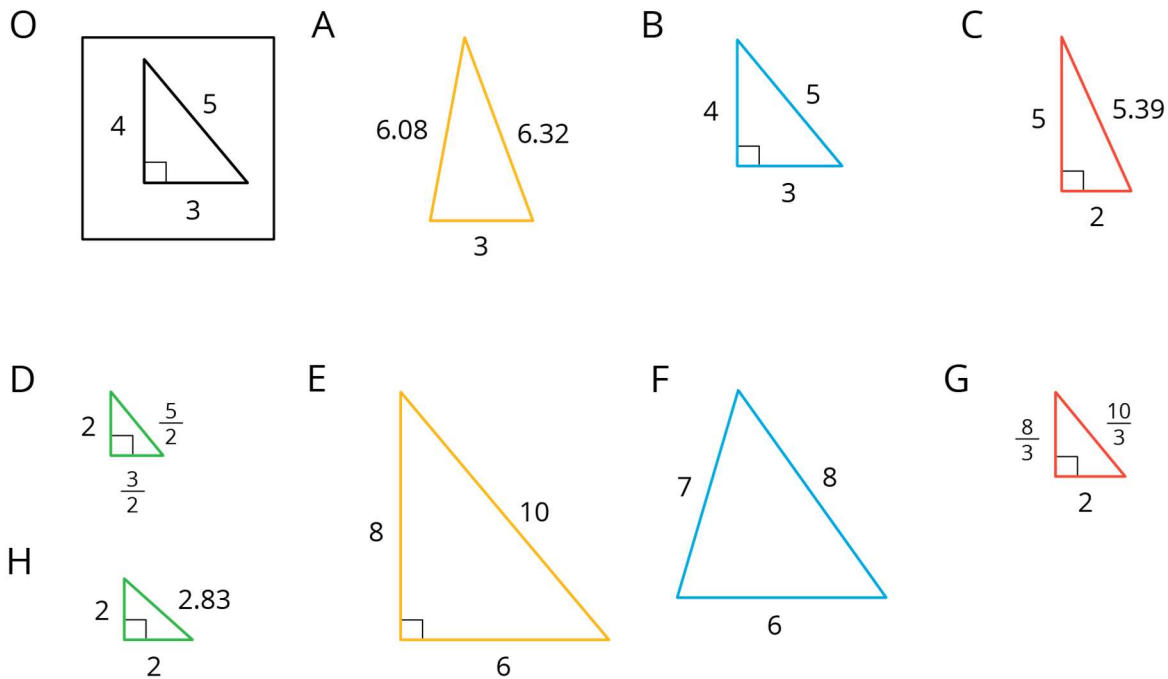
Give students quiet work time to complete the rest of the task after the class recognises that A, C, F, and H are not scaled copies.

### Anticipated Misconceptions

Students may think that triangle F is a scaled copy because just like the 3-4-5 triangle, the sides are also three consecutive whole numbers. Point out that corresponding angles are not equal.

### Student Task Statement

Here is triangle O, followed by a number of other triangles.



Your teacher will assign you two of the triangles to look at.

1. For each of your assigned triangles, is it a scaled copy of triangle O? Be prepared to explain your reasoning.
2. As a group, identify *all* the scaled copies of triangle O in the collection. Discuss your thinking. If you disagree, work to reach an agreement.
3. List all the triangles that are scaled copies in the table. Record the side lengths that correspond to the side lengths of triangle O listed in each column.

Triangle O	3	4	5




4. Explain or show how each copy has been scaled from the original (triangle O).

### Student Response

- Answers vary depending on the pair of triangles students have. Triangles B, D, E, and G are scaled copies.
- Triangles B, D, E, and G are scaled copies. Sample reasoning: B, D, E, and G have not changed in shape (they are still right-angled triangles). Each of their sides are the same number of times as long as the corresponding sides in the original triangle. Triangles A and F do not have the same shape as triangle O (their angles are all different), so they are not scaled copies. Triangles C, G, and H are right-angled triangles but their sides are not the same number of times as long as the corresponding sides in the original triangle.

Triangle O	3	4	5
Triangle B	3	4	5
Triangle D	$\frac{3}{2}$	2	$\frac{5}{2}$
Triangle E	6	8	10
Triangle G	2	$\frac{8}{3}$	$\frac{10}{3}$

- Explanations vary. Sample explanations:
  - Triangle B is a same-size copy of the original. All the lengths stay the same.
  - In triangle D, all the lengths are half of the original ones.
  - In triangle E, all sides double in length.
  - In triangle G, the lengths are  $\frac{2}{3}$  times the corresponding lengths in the original triangle.

### Are You Ready for More?





Choose one of the triangles that is not a scaled copy of triangle O. Describe how you could change at least one side to make a scaled copy, while leaving at least one side unchanged.

### Student Response

Answers vary. Sample response: on triangle F, the side of length 7 could be extended to have length 10.

## Activity Synthesis

Display the image of all triangles and invite a couple of students to share how they knew which sides of the triangles correspond. Then, display a completed table in the third question for all to see. Ask each group to present its observations about one triangle and how the triangle has been scaled from the original. Encourage the use of “corresponding” in their explanations. As students present, record or illustrate their reasoning on the table, e.g., by drawing arrows between rows and annotating with the operation students are describing, as shown here.

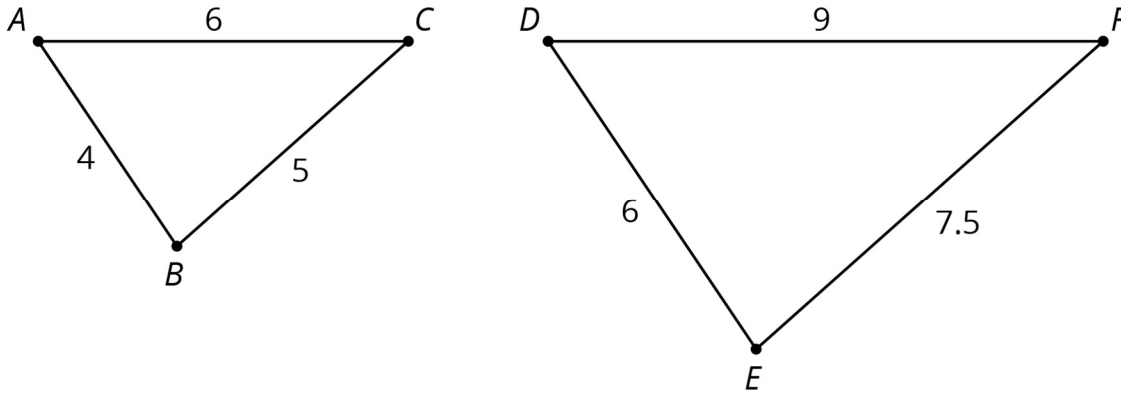
Triangle O		3	4	5
Triangle D		$\frac{3}{2}$	2	$\frac{5}{2}$
Triangle E		6	8	10
Triangle B		3	4	5
Triangle G		2	$\frac{8}{3}$	$\frac{10}{3}$

Use the language that students use to describe the side lengths and the numerical relationships in the table to guide students toward **scale factor**. For example: “You explained that the lengths in triangle F are all twice those in the original triangle, so we can write those as “2 times” the original numbers. Lengths in triangle A are half of those in the original; we can write “ $\frac{1}{2}$  times” the original numbers. We call those multipliers—the 2 and the  $\frac{1}{2}$ —scale factors. We say that scaling triangle O by a scale factor of 2 produces triangle F, and that scaling triangle O by  $\frac{1}{2}$  produces triangle A.”

## Lesson Synthesis

- What do we mean by **corresponding parts**?
- What is a **scale factor**? How does it work?

Students can use informal language to describe corresponding parts, and recognise a scale factor as a common ratio between the lengths of corresponding side lengths. In the shape, triangle  $DEF$  is a scaled copy of triangle  $ABC$ . We call parts that have the same position within each shape *corresponding parts*. For example, we refer to vertex  $E$  in triangle  $DEF$  and vertex  $B$  in triangle  $ABC$  as *corresponding points*; line segment  $BC$  and line segment  $EF$  as *corresponding line segments*; and angle  $C$  (or angle  $BCA$ ) and angle  $F$  (or angle  $EFD$ ) as *corresponding angles*.



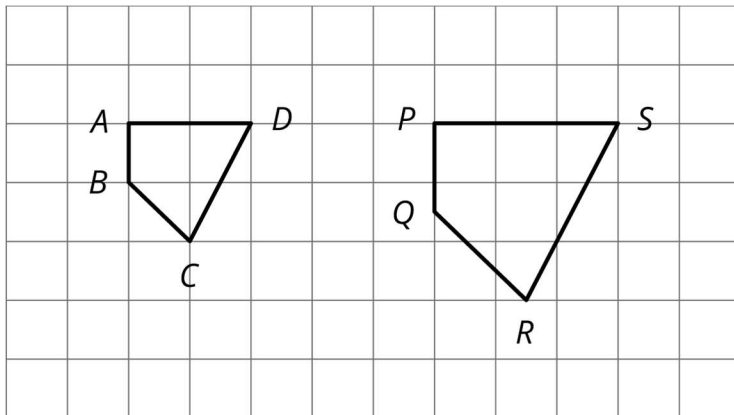
The line segments in a scaled copy are always a certain number of times as long as the corresponding line segments in the original shape. We call that number the *scale factor*. For example, the scale factor between  $ABC$  and its copy triangle  $DEF$  is  $\frac{3}{2}$  or 1.5 because all lengths in triangle  $DEF$  are 1.5 times as long as the corresponding lengths in triangle  $ABC$ .

## 2.4 Comparing Polygons $ABCD$ and $PQRS$

**Cool Down: 5 minutes**

### Student Task Statement

Polygon  $PQRS$  is a scaled copy of polygon  $ABCD$ .



1. Name the angle in the scaled copy that corresponds to angle  $ABC$ .
2. Name the line segment in the scaled copy that corresponds to line segment  $AD$ .
3. What is the scale factor from polygon  $ABCD$  to polygon  $PQRS$ ?

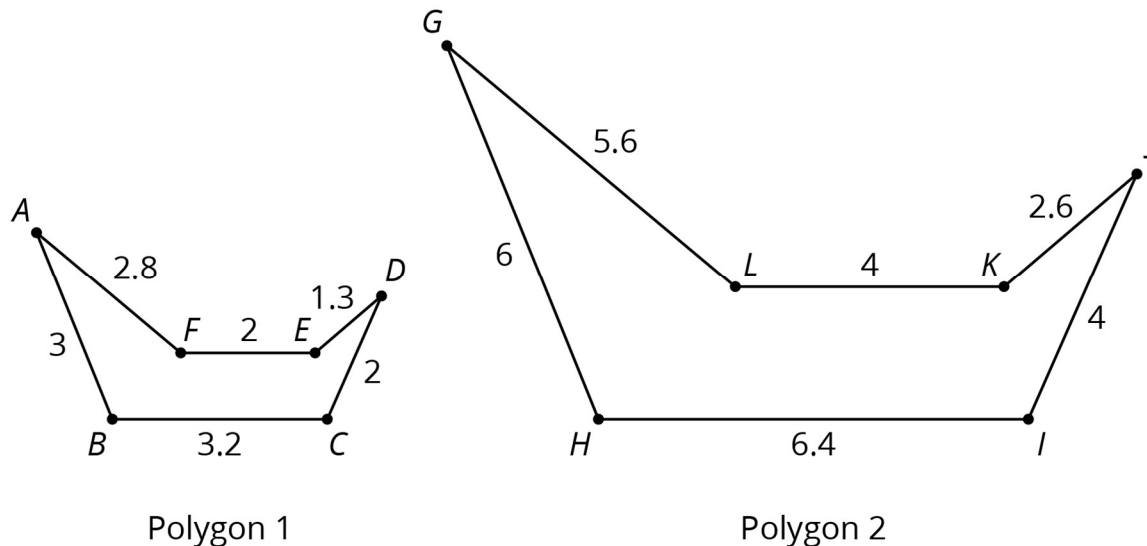
### Student Response

1. Angle  $PQR$  corresponds to angle  $ABC$ .
2. Line segment  $PS$  corresponds to line segment  $AD$ .

3. The scale factor is  $\frac{3}{2}$  since  $PS = 3$  and  $AD = 2$ .

### Student Lesson Summary

A shape and its scaled copy have **corresponding parts**, or parts that are in the same position in relation to the rest of each shape. These parts could be points, line segments, or angles. For example, polygon 2 is a scaled copy of polygon 1.



- Each point in polygon 1 has a *corresponding point* in polygon 2. For example, point  $B$  corresponds to point  $H$  and point  $C$  corresponds to point  $I$ .
- Each line segment in polygon 1 has a *corresponding line segment* in polygon 2. For example, line segment  $AF$  corresponds to line segment  $GL$ .
- Each angle in polygon 1 also has a *corresponding angle* in polygon 2. For example, angle  $DEF$  corresponds to angle  $JKL$ .

The **scale factor** between polygon 1 and polygon 2 is 2, because all of the lengths in polygon 2 are 2 times the corresponding lengths in polygon 1. The angles in polygon 2 are the same as the corresponding angles in polygon 1. For example, angle  $JKL$  is the same as angle  $DEF$ .

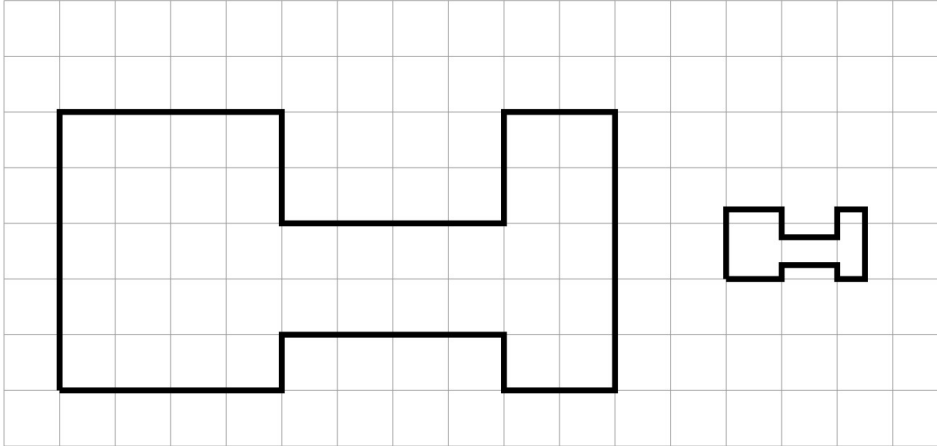
### Glossary

- corresponding
- scale factor

## Lesson 2 Practice Problems

### Problem 1 Statement

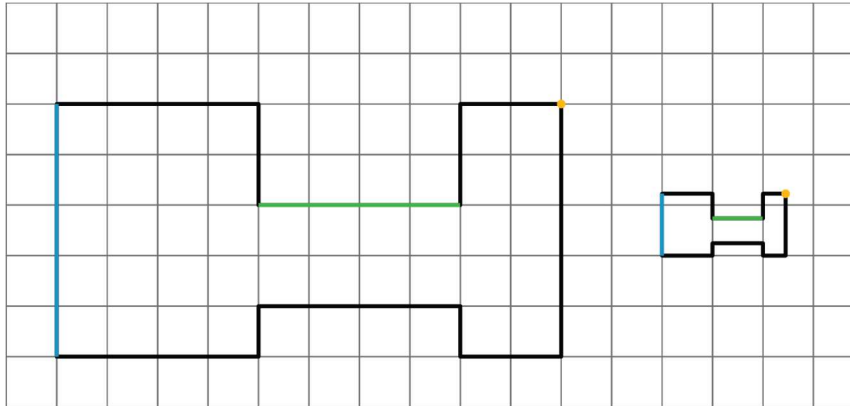
The second H-shaped polygon is a scaled copy of the first.



- Show one pair of corresponding points and two pairs of corresponding sides in the original polygon and its copy. Consider using coloured pencils to highlight corresponding parts or labelling some of the vertices.
- What scale factor takes the original polygon to its smaller copy? Explain or show your reasoning.

### Solution

- Answers vary. Sample markings:



- $\frac{1}{4}$  or 0.25. Sample explanation: The sides that are 4 units long in the original polygon are 1 unit long in the copy, which is one fourth of the original length.

### Problem 2 Statement

Shape B is a scaled copy of shape A. Select **all** of the statements that must be true:

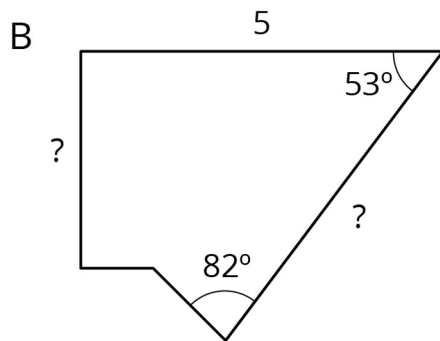
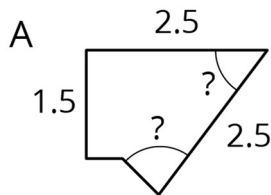
- Shape B is larger than shape A.
- Shape B has the same number of edges as shape A.
- Shape B has the same perimeter as shape A.
- Shape B has the same number of angles as shape A.
- Shape B has angles with the same measurements as shape A.

**Solution** ["B", "D", "E"]

**Problem 3 Statement**

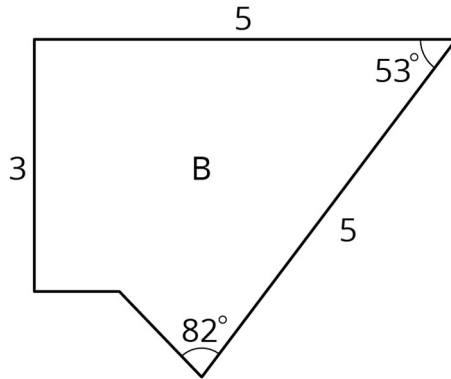
Polygon B is a scaled copy of polygon A.

- What is the scale factor from polygon A to polygon B? Explain your reasoning.
- Find the missing length of each side marked with ? in polygon B.
- Determine the size of each angle marked with ? in polygon A.

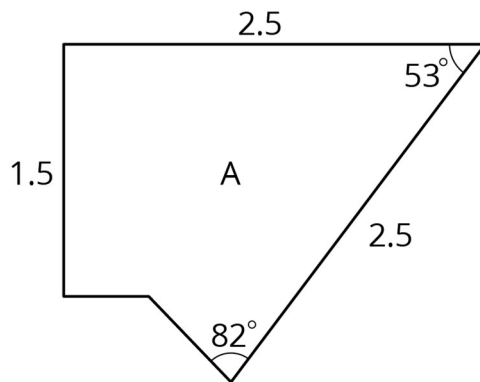


**Solution**

- 2 because the top horizontal side has length 2.5 units in polygon A and 5 units in polygon B
- All sides scale by the same factor of 2, so the side that is 2.5 units in polygon A is 5 units in the copy, and the 1.5-unit-long one is 3 units in the copy



- c.  $53^\circ$  and  $82^\circ$  because scaled copies have the same corresponding angles



**Problem 4 Statement**

Complete each equation with a number that makes it true.

- a.  $8 \times \underline{\hspace{1cm}} = 40$
- b.  $8 + \underline{\hspace{1cm}} = 40$
- c.  $21 \div \underline{\hspace{1cm}} = 7$
- d.  $21 - \underline{\hspace{1cm}} = 7$
- e.  $21 \times \underline{\hspace{1cm}} = 7$

**Solution**

- a. 5
- b. 32
- c. 3
- d. 14
- e.  $\frac{1}{3}$



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