Lesson 16: Finding the percentage

Goals

- Critique or justify (orally) statements about percentages and equivalent numerical expressions.
- Generalise a process for finding the percentage that C is of B and justify (orally) why this can be abstracted as $\frac{C}{B} \times 100$.

Learning Targets

- I can solve different problems like "60 is what percentage of 40?" by dividing and multiplying.

Lesson Narrative

While students have found percentages with easy numbers before now, in this lesson they will develop a general structure that will work for any numbers.

Addressing

- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percentage.

Instructional Routines

- Compare and Connect
- Discussion Supports
- True or False

Student Learning Goals

Let's find percentages in general.

16.1 True or False: Percentages

Warm Up: 10 minutes

The purpose of this warm-up is to encourage students to reason about properties of operations in equivalent expressions. While students may evaluate each expression to determine if the statement is true or false, encourage students to think about the properties of arithmetic operations in their reasoning.

Instructional Routines

- True or False

Launch

Display one problem at a time. Tell students to give a signal when they have decided if the equation is true or false. Give students 1 minute of quiet think time followed by a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Is each statement true or false? Be prepared to explain your reasoning.

1. 25% of 512 is equal to $\frac{1}{4} \times 500$.

2. 90% of 133 is equal to $(0.9) \times 133$.

3. 30% of 44 is equal to 3% of 440.

4. The percentage 21 is of 28 is equal to the percentage 30 is of 40.

Student Response

Student explanations will vary.

1. False; 25% is equal to $\frac{1}{4}$, but 512 is not equal to 500.

2. True; These are equal, because 90% is equal to 0.9

3. True; These are equal because $\frac{3}{10} \times 44 = \frac{3}{100} \times 440$

4. True; These are equal, because $\frac{21}{28} = \frac{3}{4} = \frac{30}{40}$

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- Do you agree or disagree? Why?
- Who can restate ___'s reasoning in a different way?
- Does anyone want to add on to _____'s reasoning?

After each true equation, ask students if they could rely on the same reasoning to determine if other similar problems are equivalent. After each false equation, ask students how the problem could be changed to make the equation true.

16.2 Skipping

15 minutes

The purpose of this activity is for students to see that to find what percentage one number is of another, divide them and then multiply by 100. First they find what percentage of 20 various numbers are, then they organise everything into a table. Using the table, students then describe the relationship they see between dividing the numbers by 20 and finding the percentage.

Instructional Routines

- Discussion Supports

Launch

Depending on students' strengths in KS2 work, it may be beneficial to offer some strategies for writing a fraction in an equivalent decimal form before students start working. Display the fraction $\frac{15}{25}$ and ask students how they might think about writing it in decimal form. Possible strategies are writing the equivalent fraction $\frac{60}{100}$ and knowing that "60 hundredths" is written 0.60 or 0.6. Another possibility is to write the equivalent fraction $\frac{3}{5}$, which is equal to $\frac{6}{10}$ and knowing that "6 tenths" can be written as 0.6.

Give students quiet think time to complete the activity and then time to share their explanations with a partner.

Representation: Internalise Comprehension. Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems and other text-based content.
Supports accessibility for: Language; Conceptual processing Speaking: Discussion Supports. To amplify mathematical uses of language to communicate about equivalent fractions and percentages (e.g., 15 minutes is 75% of 20 minutes, 75% of 20 minutes is equivalent to $\frac{15}{20}$), invite students to use these phrases when stating their ideas, revoicing and rephrasing as necessary.
Design Principle(s): Support sense-making, Optimise output (for explanation)

Student Task Statement

A school held a skipping contest. Diego skipped for 20 minutes.

1. Jada skipped for 15 minutes. What percentage of Diego's time is that?
2. Lin skipped for 24 minutes. What percentage of Diego's time is that?
3. Noah skipped for 9 minutes. What percentage of Diego's time is that?
4. Record your answers in this table. Write the quotients in the last column as decimals.

	time (minutes)	percentage	time ÷ 20
Diego	20	100	$\frac{20}{20} = 1$
Jada	15		$\frac{15}{20} =$
Lin	24		$\frac{24}{20} =$
Noah	9		$\frac{9}{20} =$

5. What do you notice about the numbers in the last two columns of the table?

Student Response

1. 15 minutes is 75% of 20 minutes.

2. 24 minutes is 120% of 20 minutes.

3. 9 minutes is 45% of 20 minutes.

4. Here is the table:

time (minutes)	percentage	time/20
20	100	$\frac{20}{20} = 1$
15	75	$\frac{15}{20} = 0.75$
24	120	$\frac{24}{20} = 1.2$
9	45	$\frac{9}{20} = 0.45$

5. The percentages in the second to last column are 100 times the decimals in the last column. To find what percentage a number is of 20, divide it by 20, and then multiply by 100.

Activity Synthesis

Have students share their strategies for the first three problems. Then ask what they noticed about the last two columns. There are many ways to formulate the relationship. Make sure everyone sees that to find what percentage a number is of 20, divide it by 20 and then multiply by 100. A table showing the percentage for 1 minute might also help:

	time (minutes)	percentage
Diego	20	100
unit rate	1	5
Jada	15	75
Lin	24	120
Noah	9	45

Note that to find the percentage for 1 minute, we divide 100 by 20, and to find the percentage for any number of minutes, we multiply the result by that number of minutes. That is the same as dividing the number of minutes by 20 and multiplying by 100.

16.3 Restaurant Capacity

10 minutes

This activity gives students a chance to practice their new-found insight about how to find percentages. It also gives them an opportunity to practice dividing whole numbers in preparation for the next unit on base-ten numbers.

Note that it is expected and perfectly okay for students to revert to less-efficient methods that they trust. Monitor for students with more- and less-efficient methods. A less-efficient representation can be used to make sense of a more-efficient method, and these connections can be made in the discussion that follows the task.

Instructional Routines

- Compare and Connect

Launch

Give students quiet think time to complete the activity and then time to share their explanations with a partner.

Action and Expression: Internalise Executive Functions. Provide students with a graphic organiser to organise their problem solving. The graphic organiser should ask students to identify what they need to find out, what information is provided, how they solved the problem, and why their answer is correct.
Supports accessibility for: Language; Organisation

Student Task Statement

A restaurant has a sign by the front door that says, "Maximum occupancy: 75 people." Answer each question and explain or show your reasoning.

1. What percentage of its capacity is 9 people?
2. What percentage of its capacity is 51 people?
3. What percentage of its capacity is 84 people?

Student Response

Reasonings vary.

1. 9 people is 12% of 75 people. Sample reasoning:
   - $\frac{9}{75}$ is equivalent to $\frac{3}{25}$, which is equivalent to $\frac{12}{100}$.

- $\frac{9}{75} \times 100 = \frac{900}{75}$, which is 12.

2. 51 people is 68% of 75 people. Sample reasonings:

   - $\frac{51}{75}$ is equivalent to $\frac{17}{25}$, which is equivalent to $\frac{68}{100}$.
   - $\frac{51}{75} \times 100 = \frac{5{,}100}{75}$, which is 68.
   - If 9 people is 12% of 75, then 45 people, which is 5×9, is 5×12 or 60%. 6 more people is $\frac{6}{9} \times 12$ or 8%. That means 51 people is $(60 + 8)$ (or 68%).

3. 84 people is 112% of 75 people. Sample reasoning:

   - 84 is 9 more than 75. We already know that 9 people is 12% and 75 people is 100%, so 84 people is $(12 + 100)$ or 112%.
   - $\frac{84}{75} \times 100 = \frac{8\,400}{75}$, which is 112.

Are You Ready for More?

Water makes up about 71% of Earth's surface, while the other 29% consists of continents and islands. 96% of all Earth's water is contained within the oceans as salt water, while the remaining 4% is fresh water located in lakes, rivers, glaciers, and the polar ice caps.

If the total volume of water on Earth is 1 386 million cubic kilometres, what is the volume of salt water? What is the volume of fresh water?

Student Response

1 330 560 000 cubic kilometres of salt water

55 440 000 cubic kilometres of fresh water

Activity Synthesis

Invite selected students to share various approaches to the three problems. Sequence less-efficient methods before more-efficient ones. Make sure all students have a chance to see the approach that is the focus of this lesson. For example, to find "9 is what percentage of 75?" we can divide 9 by 75 and then multiply by 100.

Speaking, Listening: Compare and Connect. For the first question, "What percentage of its capacity is 9 people?", display a range of student approaches to share with the class. Some students may use a double number line or table and reason that 3 people is 4% of 75 people because 100% divided by 25 is 4%, and 75 people divided by 25 is 3 people. Then multiply 4% by 3 to find what percentage of 75 people is 9 people. Other students may use a more efficient method and reason that $\frac{9}{75}$ is equivalent to $\frac{3}{25}$, which is equivalent to $\frac{12}{100}$. As students investigate each other's work, ask questions such as, "What is especially clear in

this approach or representation?" and "Where do you see $\frac{9}{75}$ or 12% represented in the diagram?" This will foster students' meta-awareness and support constructive conversations as they compare and connect the various ways to find what percentage one number is of another.
Design Principles(s): Cultivate conversation; Maximise meta-awareness

Lesson Synthesis

The main idea in this lesson is that to find what percentage C is of B, multiply: $\frac{C}{B} \times 100$ Ask students to describe a procedure for finding what percentage one number is of another number. If they struggle to describe a general method, ask about some specific examples, like "How could you find what percentage of 80 is 56?" (You could multiply 100 by $\frac{56}{80}$.)

16.4 Jet Fuel

Cool Down: 5 minutes

Student Task Statement

A jet plane can carry up to 200 000 litres of fuel. It used 130 000 litres of fuel during a flight. What percentage of the fuel capacity did it use on this flight?

Student Response

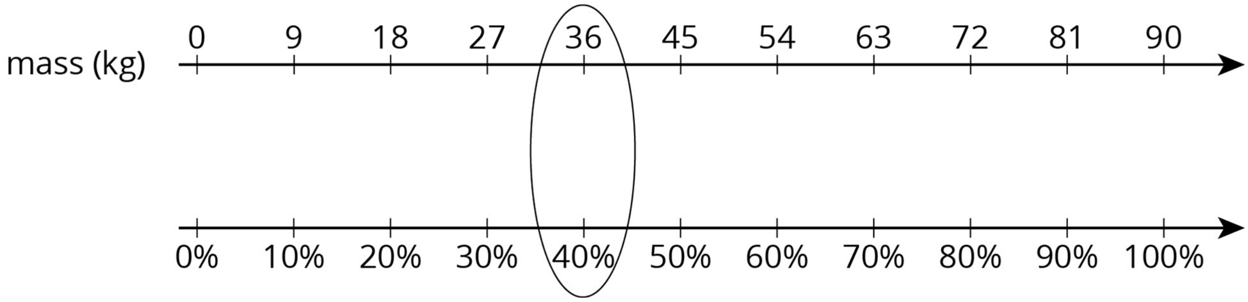
$130\,000 \div 200\,000 = 0.65$, so it burned 65% of its fuel capacity on the flight.

Student Lesson Summary

What percentage of 90 kg is 36 kg? One way to solve this problem is to first find what percentage 1 kg is of 90, and then multiply by 36.

	mass (kg)	percentage	
	90	100	
$\times \frac{1}{90}$	1	$\frac{1}{90} \times 100$	$\times \frac{1}{90}$
$\times 36$	36	$\frac{36}{90} \times 100$	$\times 36$

From the table we can see that 1 kg is $\left(\frac{1}{90} \times 100\right)$ %, so 36 kg is $\left(\frac{36}{90} \times 100\right)$ % or 40% of 90. We can confirm this on a double number line:



In general, to find what percentage a number C is of another number B is to calculate $\frac{C}{B}$ of 100%. We can find that by multiplying: $\frac{C}{B} \times 100$

Suppose a school club has raised £88 for a project but needs a total of £160. What percentage of its goal has the club raised?

To find what percentage £88 is of £160, we find $\frac{88}{160}$ of 100% or $\frac{88}{160} \times 100$, which equals $\frac{11}{20} \times 100$ or 55. The club has raised 55% of its goal.

Lesson 16 Practice Problems

Problem 1 Statement

A sign in front of a roller coaster says "You must be 40 inches tall to ride." What percentage of this height is:

a. 34 inches?

b. 54 inches?

Solution

a. 85%

b. 135%

Problem 2 Statement

At a hardware store, a tool set normally costs £80. During a sale this week, the tool set costs £12 less than usual. What percentage of the usual price is the savings? Explain or show your reasoning.

Solution

Reasoning varies. Sample response: 15%, because $12 \div 80 = \frac{3}{20} = \frac{15}{100}$.

Problem 3 Statement

A bathtub can hold 80 gallons of water. The tap flows at a rate of 4 gallons per minute. What percentage of the tub will be filled after 6 minutes?

Solution

30%, because the tub will hold 24 gallons after 6 minutes, and 24 is 30% of 80.

Problem 4 Statement

The sale price of every item in a store is 85% of its usual price.

a. The usual price of a backpack is £30, what is its sale price?

b. The usual price of a sweatshirt is £18, what is its sale price?

c. The usual price of a football is £24.80, what is its sale price?

Solution

a. £25.50

b. £15.30

c. £21.08

Problem 5 Statement

A shopper needs 48 hot dogs. The store sells identical hot dogs in 2 differently sized packages. They sell a six-pack of hot dogs for £2.10, and an eight-pack of hot dogs for £3.12. Should the shopper buy 8 six-packs, or 6 eight-packs? Explain your reasoning.

Solution

He should buy 8 six-packs. The hot dogs in the six-pack are being sold at a rate of 35 pence each, because $2.10 \div 6 = 0.35$. The hot dogs in the eight-pack are being sold at a rate of 39 pence each, because $3.12 \div 8 = 0.39$. The six-packs are a better deal, because the hot dogs have a cheaper unit rate.

Problem 6 Statement

Elena is 56 inches tall.

a. What is her height in centimetres? (Note: 100 inches = 254 centimetres)

b. What is her height in metres?

Solution

a. 142.24 centimetres

b. 1.42 metres