Mathematics Education Innovation

Lesson 14: Using linear relations to solve problems

Goals

- Describe (orally) limitations of a graphical representation of a situation based on real-world constraints on the quantities.
- Interpret the graph of a linear equation in context, including gradient, intercept, and solution, in contexts using multiple representations of non-proportional linear relationships.

Learning Targets

- I can write linear equations to reason about real-world situations.

Lesson Narrative

In this culminating lesson for the unit, students put what they have learned to work in solving real-world problems, using all the different forms of equations they have studied.

Addressing

- Use similar triangles to explain why the gradient m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + c$ for a line intercepting the vertical axis at c.
- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

Instructional Routines

- Collect and Display
- Discussion Supports
- Think Pair Share

Student Learning Goals

Let's write equations for real-world situations and think about their solutions.

14.1 Buying Fruit

Warm Up: 5 minutes

Students write expressions and equations representing total cost. The purpose of this activity is to support students in writing the equation for the "Ordering Fish" activity.

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Display questions for all to see. Give 2 minutes quiet think time, followed by 2 minutes partner discussion then whole-class discussion.

Anticipated Misconceptions

Some students may not be sure how to approach writing the scenario as an equation. For these students, suggest that they make a table of possible prices based on the amount of fruit purchased.

Student Task Statement

For each relationship described, write an equation to represent the relationship.

1. Grapes cost £2.39 per pound. Bananas cost £0.59 per pound. You have £15 to spend on g pounds of grapes and b pounds of bananas.

2. A savings account has £50 in it at the start of the year and £20 is deposited each week. After x weeks, there are y pounds in the account.

Student Response

1. $2.39g + 0.59b = 15$

2. $y = 20x + 50$

Activity Synthesis

The purpose of this discussion is to have students explain strategies for writing equations for real-world scenarios. Ask students to share what they discussed with their partners by asking:

- "What did each of the variables mean in the situations?" (Since we want a price based on the number of pounds of fruit, b and g represent the amount of bananas and grapes purchased. For the savings account, the x was the number of weeks, and the y was number of pounds.)

- "Was the gradient for each of these equations positive or negative? Why does that make sense with the scenario?" (For the fruit, the gradient was negative, which makes sense because if you buy more of one fruit, you have to buy less of the other. For the savings account, the gradient is positive, which makes sense because the more weeks go by, the more money will be in the account.)

14.2 Five Savings Accounts

25 minutes

Given a graph with five lines representing changes in bank account balance over time, students write equations and interpret how points represent solutions. The activity also connects to and contextualises students' prior understanding of gradient and intercepts, and lays the foundation for future learning on systems of equations by considering what points of intersection of lines and non-intersecting lines represent.

Instructional Routines

- Discussion Supports

Launch

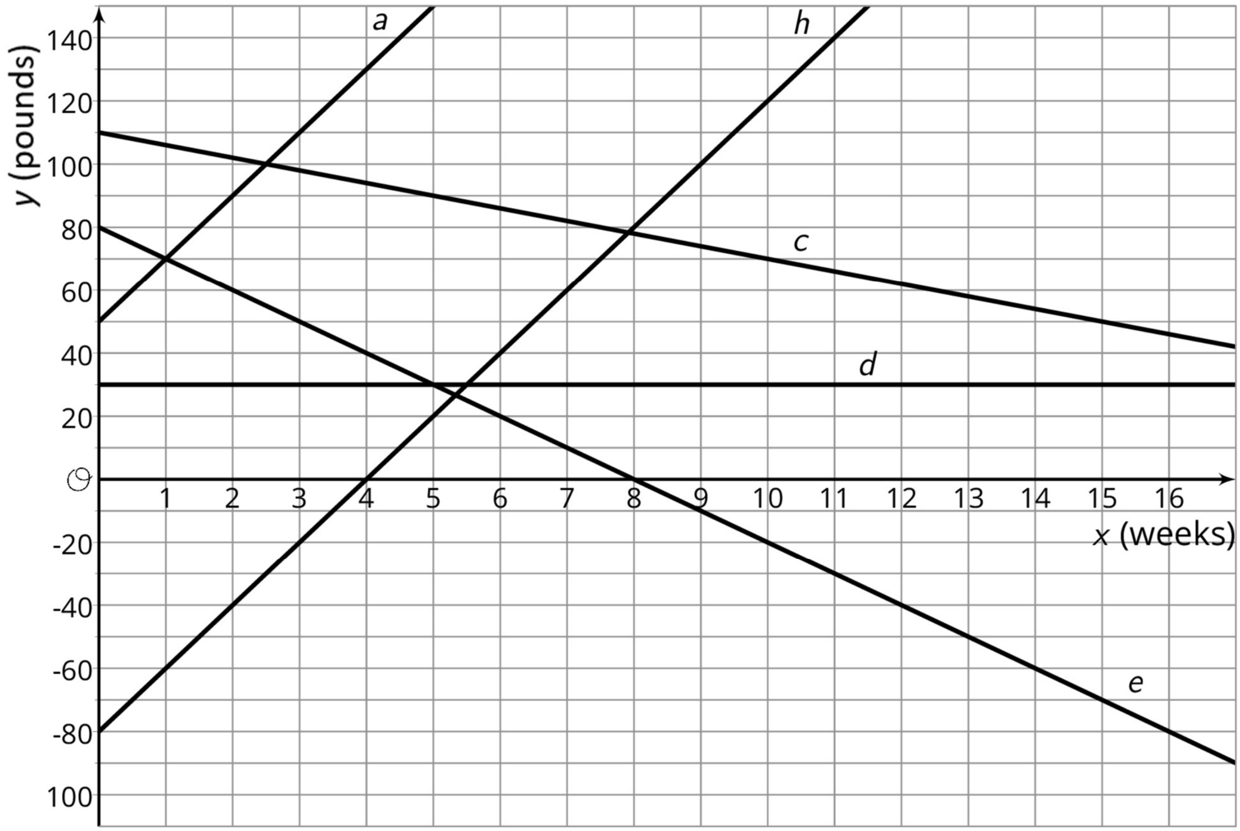
Display the image from the lesson for all to see and ask the students to consider line a. Invite 2–3 students to describe in words what line a shows. If no students bring it up, tell students that they saw this line before in the warm-up, and they wrote an equation for it. Instruct students that for #1, they should not choose line a.

Arrange students in groups of 3–4. Groups work for about 10 minutes, followed by a whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing; Conversing: Discussion Supports.
Use this routine to support discussion about the question, “What can we say about the points where two lines cross?” Invite students work with a partner to select a few of the intersection points to discuss (between lines a and c, a and e, h and c, h and d, h and e, and d and e). Consider providing these sentence frames for pairs to use: “Lines _ and _ cross at the point _ and this tells me that . . . . I know this because . . . .” Encourage students to make explicit references to the number of weeks and pound amounts that are represented. This will help students communicate about the point where two lines intersect in the context of a real-world situation.
Design Principle(s): Support sense-making; Cultivate conversation

Student Task Statement

Each line represents one person’s weekly savings account balance from the start of the year.



1. Choose one line and write a description of what happens to that person's account over the first 17 weeks of the year. Do not tell your group which line you chose.

2. Share your story with your group and see if anyone can guess your line.

3. Write an equation for each line on the graph. What do the gradient, m, and vertical intercept, c, in each equation mean in the situation?

4. For which equation is $(1,70)$ a solution? Interpret this solution in terms of your story.

5. Predict the balance in each account after 20 weeks.

Student Response

1. Answers vary. Sample responses: Person a starts with £50 and is saving money at the rate of £20 per week. Person h owed £80 and is paying it back at the rate of £20 per week, then saving once the debt is paid off. Person c starts with £110 and is spending money at the rate of £20 every 5 weeks, or £4 per week. Person d has £30 and is neither saving or spending. Person e starts with £80 and spends at the rate of £10 per week.

2. Responses vary.

3. a: $y = 20x + 50$; h: $y = 20x - 80$; c: $y = -4x + 110$; d: $y = 30$; e: $y = -10x + 80$; For each equation, the gradient tells the rate of change of saving (positive) or spending

(negative). The value of c indicates the amount of money they started with, positive represents a saved balance, negative represents money they owe. Person d shows a gradient of zero—neither saving or spending, so that they remain over time with the same amount that they start with.

4. We can see from the graph that lines a and e share a common point, or solution, at $x =$ 1 week, $y =$£70 for both. Sample explanation: at 1 week, each of these people had £70 in their accounts.

5. Person a will have £450. Person h will have £320. Person c will have £30. Person d will have £30. Person e will have -£120.

Activity Synthesis

Students should understand that points on a line show solutions to the equation of the line. Discuss with students:

- "What can we say about the points where two lines cross?" (The accounts had the same amount of money at the same time.)
- "How do the gradients of the lines help to tell the story from the graph?" (The gradient tells us whether a person is spending or saving each week.)
- "What does your answer to question 3 tell us about their rates of saving?" (By knowing the value of the gradient, we can compare who is spending or saving more quickly or more slowly.)

14.3 Fabulous Fish

20 minutes

Students represent a scenario with an equation and use the equation to find solutions. They create a graph (either with a table of values or by using two intercepts), interpret points on the graph, and interpret points not on the graph.

Instructional Routines

- Collect and Display

Launch

Allow about 10 minutes quiet think time for questions 1 through 4, then have students work with a partner to discuss questions 4 and 5. Look for students who define the variables or label the axes differently. This can be an opportunity to discuss the importance of defining what quantities your variables represent and that different graphs can represent the same information.

Speaking, Listening: Collect and Display. Listen for and record the language students use to discuss the statement "List two ways that you can tell if a pair of numbers is a solution to an equation." Organise and group similar strategies in the display for students to refer back to

throughout the lesson. For example, group strategies that reference the equation in one area and strategies that reference the graph in another area. This will help students solidify their understanding about solutions to an equation.
Design Principle(s): Support sense-making; Maximise meta-awareness

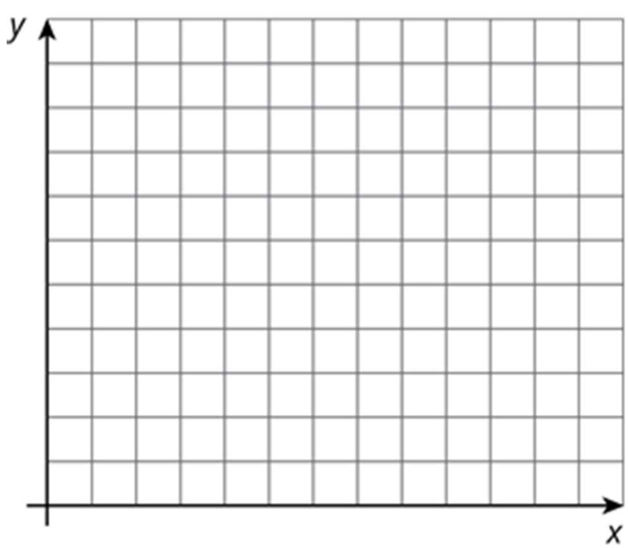
Anticipated Misconceptions

If students let x be pounds of salmon, then the equation would be $5x + 3y = 210$ and the coordinates would be reversed. The intercepts of the graph would be $(0,70)$ and $(42,0)$. This is a good place to mention the importance of defining what quantities your variables represent.

Student Task Statement

The Fabulous Fish Market orders tilapia, which costs £3 per pound, and salmon, which costs £5 per pound. The market budgets £210 to spend on this order each day.

1. What are five different combinations of salmon and tilapia that the market can order?
2. Define variables and write an equation representing the relationship between the amount of each fish bought and how much the market spends.
3. Sketch a graph of the relationship. Label your axes.



4. On your graph, plot and label the combinations A—F.

	A	B	C	D	E	F
pounds of tilapia	5	19	27	25	65	55
pounds of salmon	36	30.6	25	27	6	4

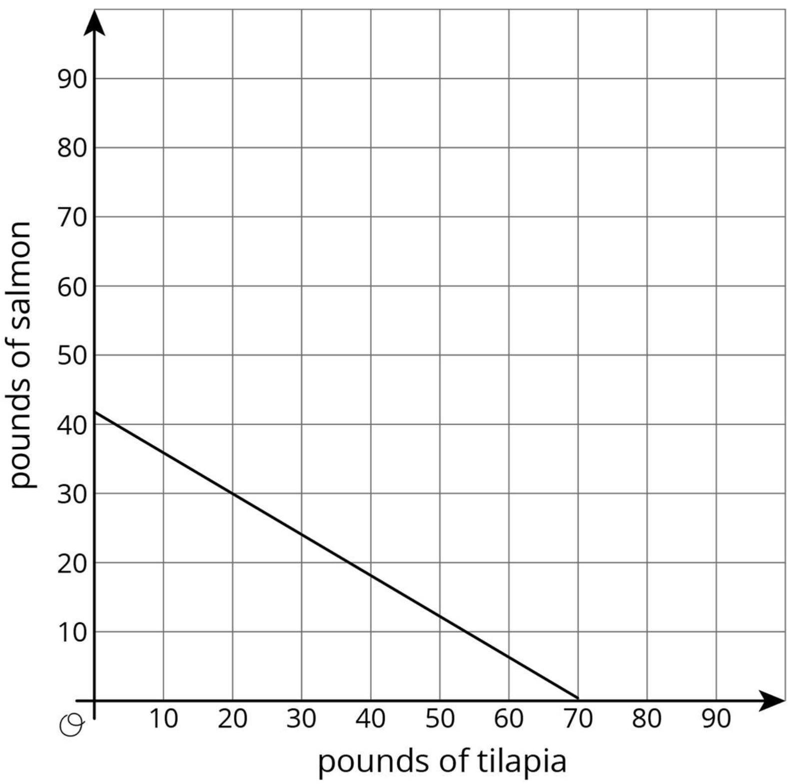
5. Which of these combinations can the market order? Explain or show your reasoning.
6. List two ways you can tell if a pair of numbers is a solution to an equation.

Student Response

1. Answers vary. Sample responses: (0,42), (10,36), (20,30), (30,24), (50,12), (70,0)

2. Answers vary. Sample response: Let x be number of pounds of tilapia, let y be number of pounds of salmon: $3x + 5y = 210$.

3. Descriptions vary. Sample response: graph is a line that begins at (0,42) and gradients downward until reaching (70,0). It will not continue on indefinitely because negative pounds of fish does not make sense in the situation.



4. A does not work because $3(5) + 5(36)$ is 195, not 210.
   B works because $3(19) + 5(30.6)$ is 210.
   C does not work because $3(27) + 5(25)$ is 206, not 210.
   D works because $3(25) + 5(27)$ is 210.
   E does not work because $3(65) + 5(6)$ is 225, not 210.
   F does not work because $3(55) + 5(4)$ is 185, not 210.

5. Responses vary. Sample response: Solutions make the equation true and can be found on the graph of the equation.

Activity Synthesis

Ask students to share some strategies for graphing and features of their graphs. Invite 2–3 students to display their graphs. Consider asking:

- "Was the gradient of the line positive or negative? Why does that makes sense in this situation?" (In this relationship, as one quantity increases the other must decrease in order to keep the sum $3x + 5y$ constant.)
- "What was your strategy for graphing the relationship?" (Plotting points from question 1, using the table, figuring out the intercepts and connecting the line)
- "Why does it make sense for the graph to be only in quadrant I?" (You cannot purchase a negative amount of fish, so the x and y values cannot be negative.)
- "How is this situation different from the apples and oranges problem in a previous lesson?" (Buying $\frac{1}{2}$ pound of fish is reasonable while buying $\frac{1}{2}$ of an apple probably is not.)

Briefly reiterate key concepts: If a point is not on the graph of the equation then it is not a solution. The ordered pairs that are solutions to the equation all make the equation true and are all found on the line that is the graph of the equation.

A discussion could also include the detail that orders that are less than £210 can also be considered to work, because there is money left over. That gives an opportunity to discuss the shape of the graph of $3x + 5y \leq 210$.

Lesson Synthesis

Ask students to consider the real-world situations described in this lesson. Discuss:

- "Give an example of a solution to an equation that doesn't make sense in the context it represents." (Some values might not make sense in the context, like negative values. A length cannot be negative, for example.)
- "If some values make sense in the equation but not in the context, how could this impact the graph?" (We might only draw part of the line or draw some of the points that lie on the line.)