Lesson 7: Reasoning about solving equations (Part 1)

Goals

- Compare and contrast (orally) different strategies for solving an equation of the form $px + q = r$.
- Explain (orally and in writing) how to use a balanced balance diagram to solve an equation of the form $px + q = r$.
- Interpret a balanced balance diagram, and write an equation of the form $px + q = r$ to represent the relationship shown.

Learning Targets

- I can explain how a balanced balance and an equation represent the same situation.
- I can find an unknown weight on a balance diagram and solve an equation that represents the diagram.
- I can write an equation that describes the weights on a balanced balance.

Lesson Narrative

The goal of this lesson is for students to understand that we can generally approach equations of the form $px + q = r$ by subtracting q from each side and dividing each side by p (or multiplying by $\frac{1}{p}$). Students only work with examples where p, q, and r are specific numbers, not represented by letters. This is accomplished by considering what can be done to a balance to keep it balanced.

Students are solving equations in this lesson in a different way than they did in the previous lessons. They are reasoning about things one could "do" to balances while keeping them balanced alongside an equation that represents a balance, so they are thinking about "doing" things to each side of an equation, rather than simply thinking "what value would make this equation true" or reasoning with situations or diagrams.

Addressing

- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Building Towards

- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- Compare and Connect
- Discussion Supports
- Notice and Wonder
- Think Pair Share

Student Learning Goals

Let's see how a balanced balance is like an equation and how moving its weights is like solving the equation.

7.1 Balance Diagrams

Warm Up: 10 minutes

Students encounter and reason about a concrete situation, balances with equal and unequal weights on each side. They then see diagrams of balanced and unbalanced balances and think about what must be true and false about the situations. In subsequent activities, students will use the balance diagrams to develop general strategies for solving equations.

Instructional Routines

- Notice and Wonder

Launch

Display the photo of socks and ask students, "What do you notice? What do you wonder?"

Give students 1 minute to think about the picture. Record their responses for all to see.

Things students may notice:

- There are two pink socks and two blue socks.
- The socks are clipped to either ends of two clothes balances. The balances are hanging from a rod.
- The balance holding the pink socks is level; the balance holding the blue socks is not level.

Things students may wonder:

- Why is the balance holding the blue socks not level?
- Is something inside one of the blue socks to make it heavier than the other sock?
- What does this picture have to do with maths?

Use the word "balanced" to describe the balance on the left and "unbalanced" to describe the balance on the right. Tell students that the balance on the left is balanced because the two pink socks have an equal weight, and the balance on the right is unbalanced because one blue sock is heavier than the other. Tell students that they will look at a diagram that is like the photo of socks, except with more abstract shapes, and they will reason about the weights of the shapes.

Give students 3 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

In the two diagrams, all the triangles weigh the same and all the squares weigh the same.

For each diagram, come up with . . .

1. One thing that *must* be true
2. One thing that *could* be true
3. One thing that *cannot possibly* be true

Student Response

Answers vary. Possible responses:

1. Triangle is heavier than square; 1 triangle weighs same as 3 squares and a circle.
2. Triangle weighs 32 ounces, square weighs 10 ounces, and circle weighs 2 ounces.
3. Triangle and square weigh the same.

Activity Synthesis

Ask students to share some things that must be true, could be true, and cannot possibly be true about the diagrams. Ask them to explain their reasoning. The purpose of this discussion is to understand how the balance diagrams work. When the diagram is balanced, there is equal weight on each side. For example, since diagram B is balanced, we know that one triangle weighs the same as three squares. When the diagram is unbalanced, one side is heavier than the other. For example, since diagram A is unbalanced, we know that one triangle is heavier than one square.

7.2 Balance and Equation Matching

15 minutes

Students are presented with four balance diagrams and are asked to match an equation to each balance. They analyse relationships and find correspondences between the two representations. Then students use the diagrams and equations to find the unknown value in each diagram. This value is a solution of the equation.

Instructional Routines

- Compare and Connect
- Think Pair Share

Launch

Display the diagrams and explain that each square labelled with a 1 weighs 1 unit, and each shape labelled with a letter has an unknown weight. Shapes labelled with the same letter have the same weight.

Arrange students in groups of 2. Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Representation: Internalise Comprehension. Demonstrate and encourage students to use colour coding and annotations to highlight connections between representations in a problem. For example, use the same colour to highlight the variables in the balance with the same variables in its corresponding equation.
Supports accessibility for: Visual-spatial processing

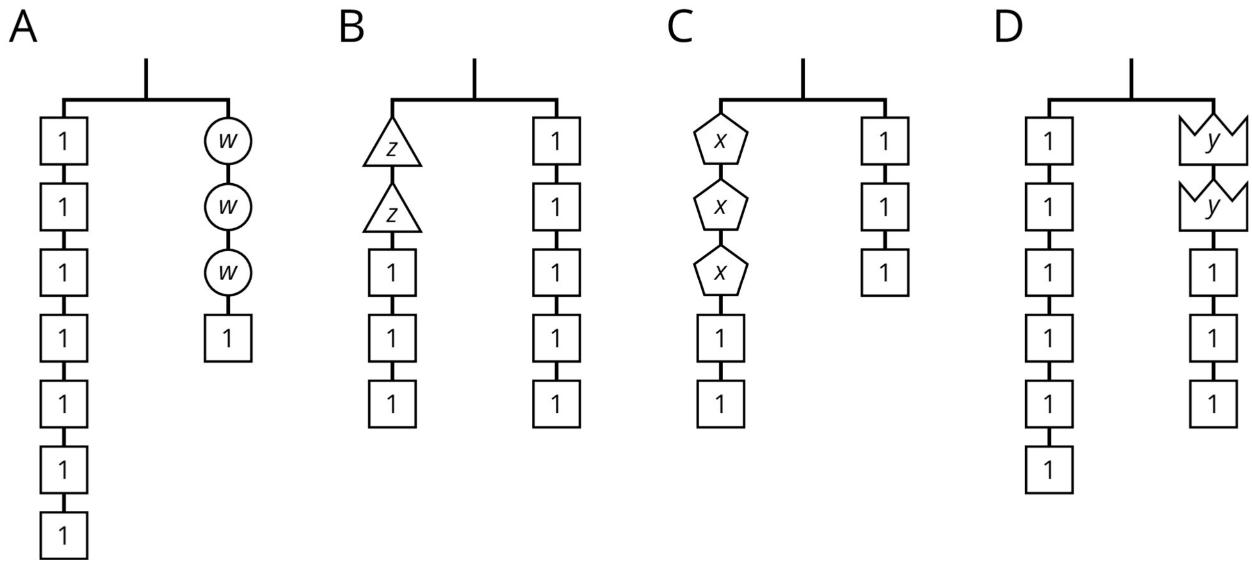
Student Task Statement

On each balanced balance, figures with the same letter have the same weight.

1. Match each balance to an equation. Complete the equation by writing x, y, z, or w in the empty box.

   - $2\square + 3 = 5$
   - $3\square + 2 = 3$
   - $6 = 2\square + 3$
   - $7 = 3\square + 1$

2. Find the solution to each equation. Use the balance to explain what the solution means.



Student Response

a. $7 = 3w + 1$

b. $2z + 3 = 5$

c. $3x + 2 = 3$

d. $6 = 2y + 3$

a. $w = 2$, because 1 circle weighs the same as 2 squares.

b. $z = 1$, because 1 triangle weighs the same as 1 square.

c. $x = \frac{1}{3}$, because 3 pentagons weigh the same as 1 square.

d. $y = \frac{3}{2}$, because 2 crowns weigh the same as 3 squares.

Activity Synthesis

Demonstrate one of the balances alongside its equation, removing the same number from each side, and then dividing each side by the same thing. Show how these moves correspond to doing the same thing to each side of the equation. (See the student lesson summary for an example of this.)

Representing, Speaking: Compare and Connect. After students have discussed what the solutions to the four equations mean, invite students to compare approaches to finding unknown values through different representations (e.g., visual balance, equation). Help students make connections between the representations by asking questions such as, "Where do you see division in both the balance diagram and the equation?" This will help

students reason about the ways to find unknown values in balanced balances and to explain the meaning of a solution to an equation.
Design Principle(s): Maximise meta-awareness; Cultivate conversation

7.3 Use Balances to Understand Equation Solving

15 minutes

This activity continues the work of using a balanced balance to develop strategies for solving equations. Students are presented with balanced balances and are asked to write equations that represent them. They are then asked to explain how to use the diagrams, and then the equations, to reason about a solution. Students notice the structure of equations and diagrams and find correspondences between them and between solution strategies.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

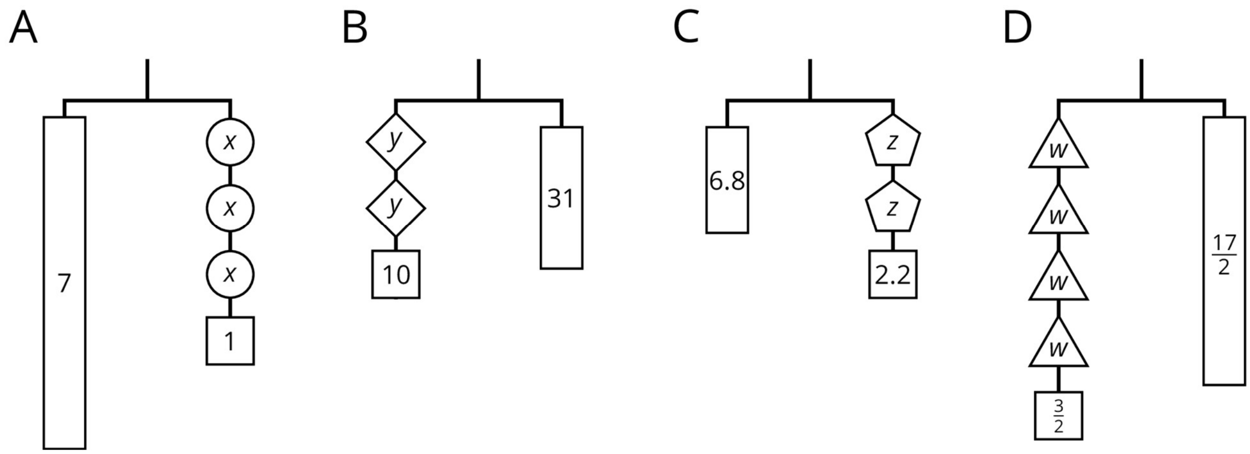
Draw students' attention to the diagrams in the task statement. Ensure they notice that the balances are balanced and that each object is labelled with its weight. Some weights are labelled with numbers but some are unknown, so they are labelled with a variable.

Keep students in the same groups. Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Student Task Statement

Here are some balanced balances where each piece is labelled with its weight. For each diagram:

1. Write an equation.
2. Explain how to figure out the weight of a piece labelled with a letter by reasoning about the diagram.
3. Explain how to figure out the weight of a piece labelled with a letter by reasoning about the equation.



Student Response

1. A: $7 = 3x + 1$ B: $2y + 10 = 31$ C: $6.8 = 2z + 2.2$ D: $4w + \frac{3}{2} = \frac{17}{2}$

2. Sample reasoning for diagram A: remove 1 unit of weight from each side of the balance, leaving 6 units on the left and 3 x's on the right. Split each side into three equal groups, showing that $x = 2$.

3. Sample reasoning for $7 = 3x + 1$: Subtract 1 from each side, leaving $6 = 3x$. Divide each side by 3, leaving $2 = x$.

Activity Synthesis

Invite students to demonstrate, side by side, how they reasoned with both the diagram and the equation. For example, diagram A can be shown next to the equation $7 = 3x + 1$. Cross out a piece representing 1 from each side, and write $7 - 1 = 3x + 1 - 1$, followed by $6 = 3x$. Encircle 3 equal groups on each side, and write $6 \div 3 = 3x \div 3$, followed by $2 = x$. Repeat for as many diagrams as time allows. If diagrams A and B did not present much of a challenge for students, spend most of the time on diagrams C and D.

We want students to walk away with two things:

1. An instant recognition of the structure of equations of the form $px + q = r$ where p, q, and r are specific, given numbers.

2. A visual representation in their minds that can be used to support understanding of why for equations of this type, you can subtract q from each side and then divide each side by p to find the solution.

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: balance diagram. For example, display an example of a balanced balance. With class participation, create step-by-step instructions on how to write and solve an equation based on the balance.

Supports accessibility for: Memory; Language Representing, Speaking: Discussion Supports. To invite participation in the whole-class discussion, use sentence frames to support students' explanations for how they found the weights by reasoning about the balances and equations. For example, provide the frame "First, I ___ because . . .", "Then I ___ because . . ." Be sure to verbalise and amplify mathematical language in the students' explanations (e.g., "subtracting the constant", and "dividing by the coefficient"). This will help students explain their reasoning with the diagram and the equation.
Design Principle(s): Optimise output (for explanation); Cultivate conversation

Lesson Synthesis

Display the equation $4x + 6 = 9.2$. Ask students to work with their partner to draw a corresponding balance diagram. Then, one partner solves by reasoning about the equation, the other solves by reasoning about the diagram. Ask students to compare the two strategies and discuss how they are alike and how they are different.

7.4 Solve the Equation

Cool Down: 5 minutes

Student Task Statement

Solve the equation. If you get stuck, try using a diagram.

$$5x + \frac{1}{4} = \frac{61}{4}$$

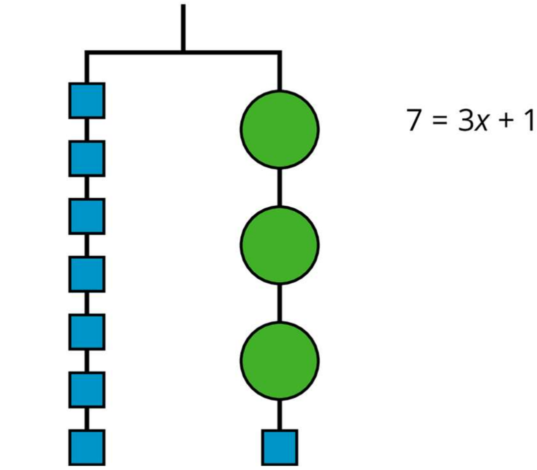
Student Response

$x = 3$

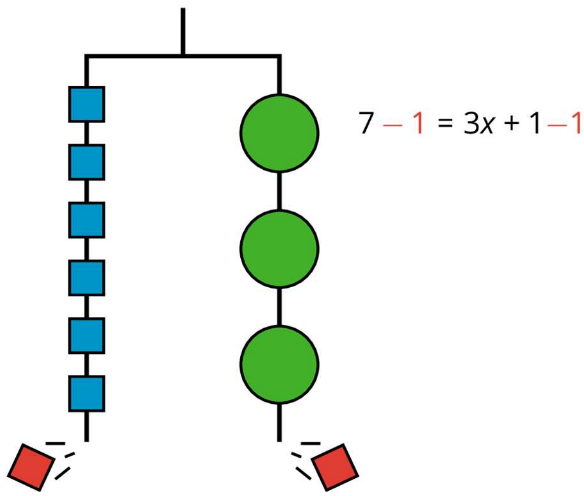
Student Lesson Summary

In this lesson, we worked with two ways to show that two amounts are equal: a balanced balance and an equation. We can use a balanced balance to think about steps to finding an unknown amount in an associated equation.

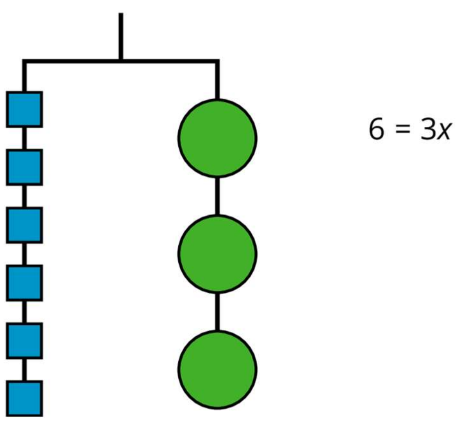
The balance shows a total weight of 7 units on one side that is balanced with 3 equal, unknown weights and a 1-unit weight on the other. An equation that represents the relationship is $7 = 3x + 1$.



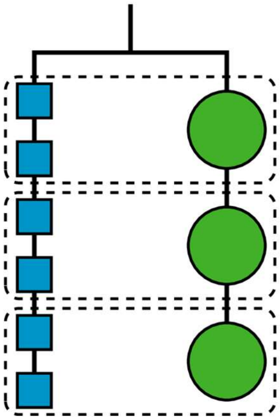
We can remove a weight of 1 unit from each side and the balance will stay balanced. This is the same as subtracting 1 from each side of the equation.



An equation for the new balanced balance is $6 = 3x$.

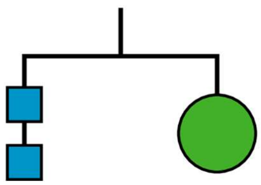


So the balance will balance with $\frac{1}{3}$ of the weight on each side: $\frac{1}{3} \times 6 = \frac{1}{3} \times 3x$.



$6 = 3x$

The two sides of the balance balance with these weights: 6 1-unit weights on one side and 3 weights of unknown size on the other side.



$2 = x$

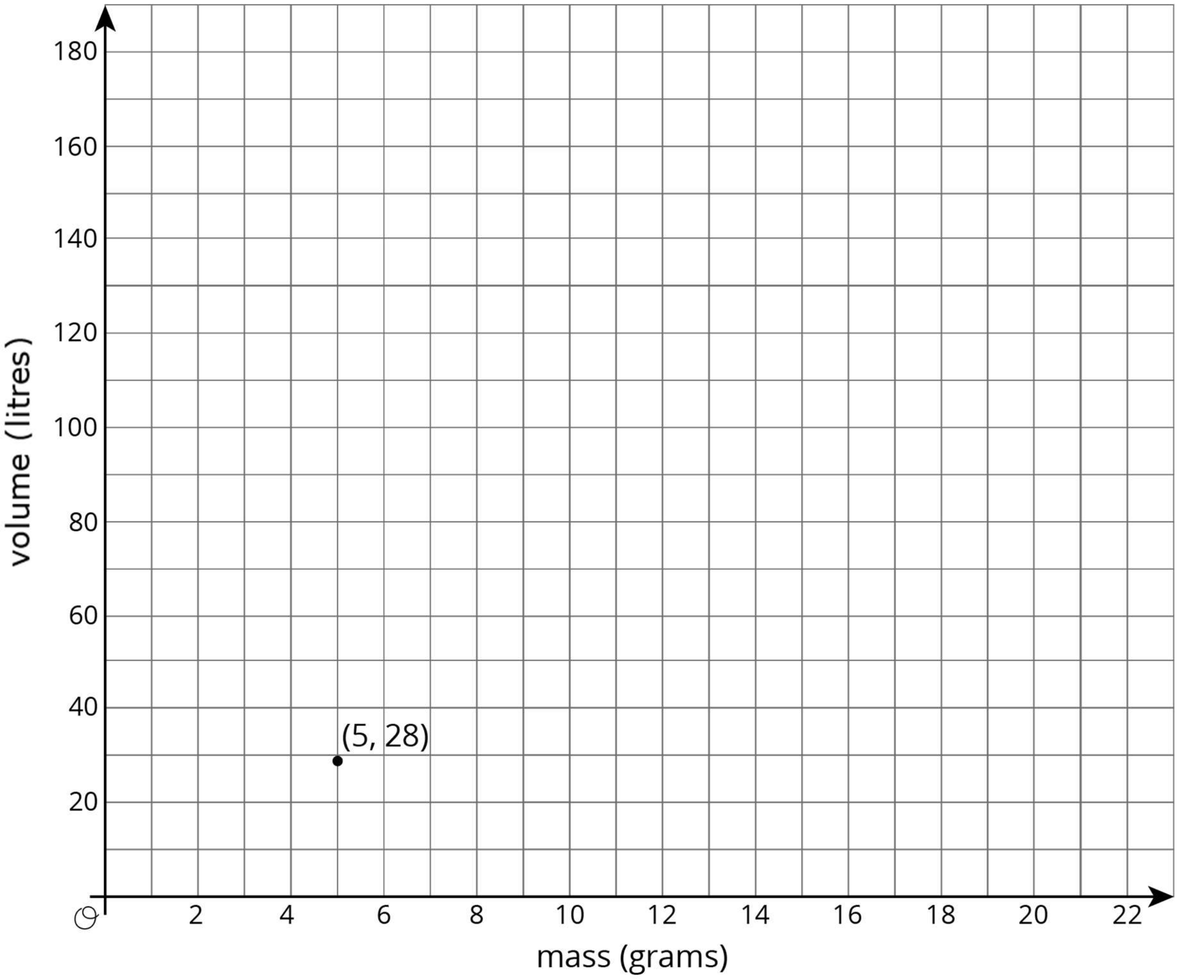
Here is a concise way to write the steps above:

$7 = 3x + 1$
$6 = 3x$ after subtracting 1 from each side
$2 = x$ after multiplying each side by $\frac{1}{3}$

Lesson 7 Practice Problems

1. **Problem 1 Statement**

   There is a proportional relationship between the volume of a sample of helium in litres and the mass of that sample in grams. If the mass of a sample is 5 grams, its volume is 28 litres. (5, 28) is shown on the graph below.



a. What is the constant of proportionality in this relationship?

b. In this situation, what is the meaning of the number you found in part a?

c. Add at least three more points to the graph above, and label with their coordinates.

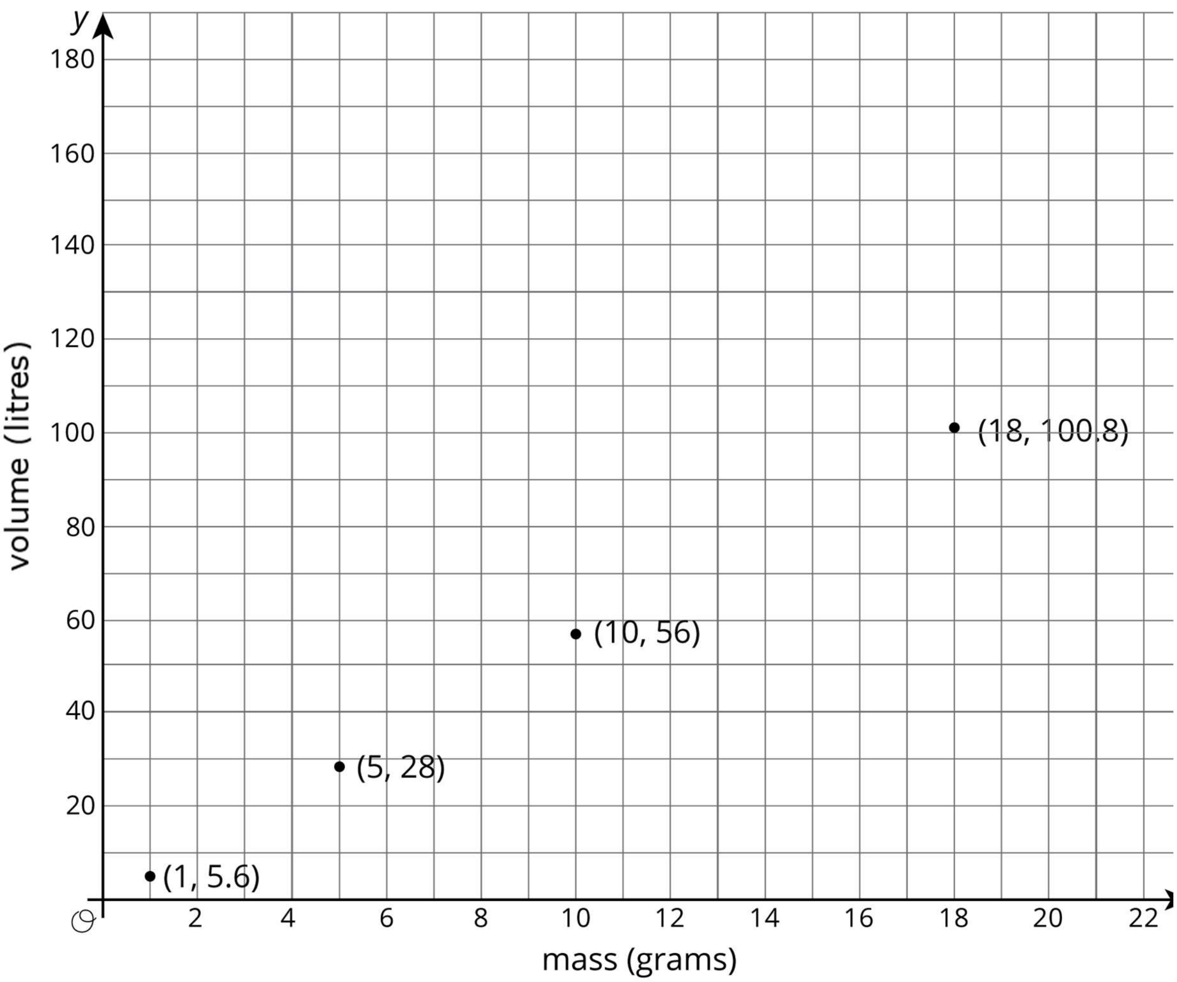
d. Write an equation that shows the relationship between the mass of a sample of helium and its volume. Use m for mass and v for volume.

Solution

a. 5.6 litres per gram

b. The volume of 1 gram of helium is 5.6 litres.

c. Answers vary. Sample answer:

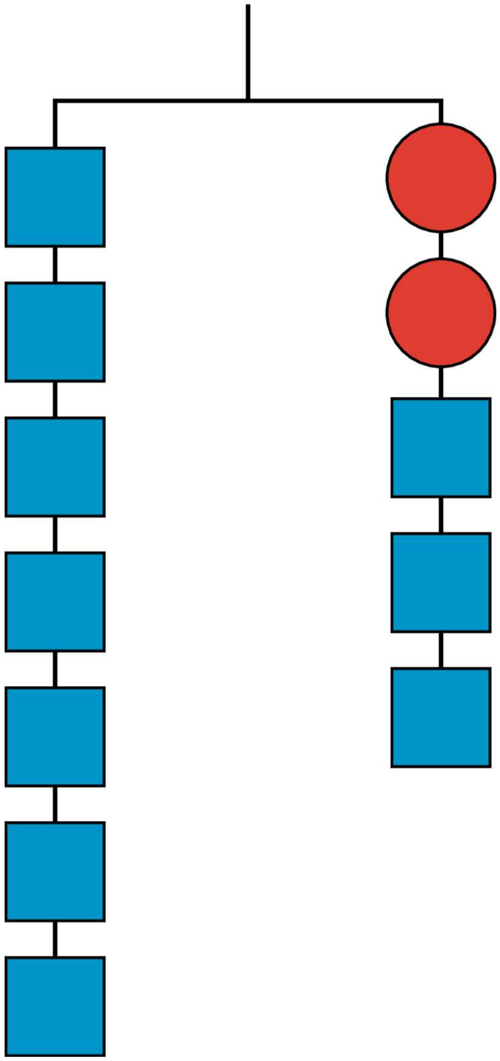


d. $v = 5.6m$

2. **Problem 2 Statement**

Explain how the parts of the balanced balance compare to the parts of the equation.

$7 = 2x + 3$



Solution

Responses vary. Sample response: The fact that the balance is balanced (equal weights on each side) matches the equal sign in the equation (equal expressions on each side). On the left of the balance there are 7 equal weights. The equation shows 7 on the left side, so we can assume that each square represents 1 unit. The right side of the balance has 2 circles of unknown weight, which matches the $2x$ in the equation - twice an unknown amount. The right side of the balance also has 3 squares of unit weight, which matches the 3 on the right side of the equation. The weight of the 2 circles and 3 squares added together (the plus sign in the equation) is the same as (equals sign) the weight of the 7 squares.

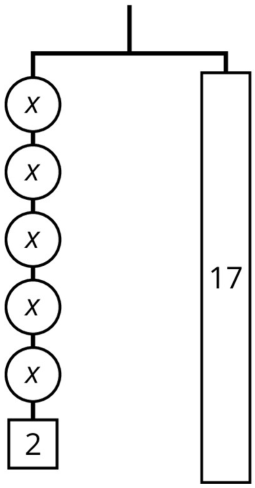
3. **Problem 3 Statement**

For the balance below:

a. Write an equation to represent the balance.

b. Draw more balances to show each step you would take to find x. Explain your reasoning.

c. Write an equation to describe each balance you drew. Describe how each equation matches its balance.



Solution

a. $5x + 2 = 17$

b. Subtract 2 from each side to get a balance with 5 circles on the left and a rectangle labelled 15 on the right. Then divide both sides by 5 to get a balance with one circle on the left and a rectangle labelled 3 on the right.

c. $5x = 15, x = 3$