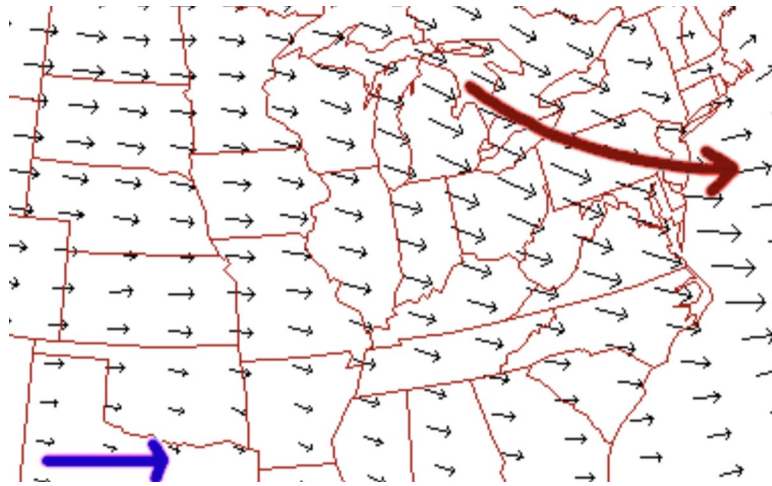**Phase portraits (AI HL 5.17)**
Title: The Weather Map - Understanding Phase Portraits

Concept: Phase Portraits in Differential Equations

Intuition Pump: Think of a phase portrait as similar to a weather map that shows wind patterns across a region. Just like a weather map uses arrows to indicate wind directions and strengths at various points, a phase portrait uses arrows to show the direction and speed of solutions to a differential equation at different points in the phase space.

1. Visual Analogy:
   - Weather Patterns: Imagine a weather map with vectors showing wind directions and speeds. Each vector's length and direction represent how the wind (or a system's state) changes over space at that point. Similarly, in a phase portrait, arrows (vectors) illustrate how the state of a system evolves over time from different starting conditions.
   - Predicting Changes: Just as meteorologists predict weather changes by analyzing patterns on a map, mathematicians and scientists use phase portraits to predict how dynamical systems will behave based on their initial conditions and underlying rules.

2. Interactive Activity:
   - Use simulation software that allows students to input different differential equations and initial conditions to generate phase portraits. Students can observe how changing parameters affects the system's behavior.
   - Conduct a hands-on activity with flow visualization in a water tank by adding dye and observing how it moves. This physical model helps students visualize how phase portraits represent the flow of the system over time.

3. Real-life Example:
   - Discuss how engineers use phase portraits to analyze the stability of electrical circuits and mechanical systems. For instance, they might use phase portraits to predict how a pendulum swings under various forces or how an electrical circuit responds to different voltages.
   - Explore the application of phase portraits in ecology to predict population dynamics under various environmental conditions.

4. Mathematical Connection:
   - Explain that a phase portrait graphically represents solutions to a system of first-order differential equations. Each point in the phase space corresponds to a state of the system, and the arrows show the direction in which the system will evolve from that state.
   - Discuss key features of phase portraits, such as fixed points (where the arrows converge or diverge), which indicate system equilibria, and trajectories, which show how the system evolves from various initial conditions.

Using the "Weather Map" analogy helps students visualize phase portraits as dynamic systems' "forecast maps," making a complex mathematical concept tangible and relatable. This method highlights how abstract mathematical tools are applied to predict and analyze real-world phenomena, enhancing students' understanding and appreciation of the practical applications of mathematics.