Voronoi Diagrams

We will look at the concepts of Modelling and Relationships by asking some basic practical questions:

- If I am in a park and have a choice of n ice-cream stands around me to go to, how do I know which one is the closest to me?
- What would be the best place to put an additional ice-cream stand so it takes up the most available space?

Inherently this has to do with much bigger questions such as: "How do navigation apps determine the shortest route" or "Where best to store toxic waste as far as possible from surrounding cities".

Introduction activity (class discussion)

You are in New York's Central Park and there are two ice-cream stands that you could go to for some refreshments. You are to research how Voronoi diagrams are used to determine mathematically which one is closest.

The first task is to find out how Voronois are constructed. Watch the first animation that will be shown for the group on the board.

Think and discuss in your groups how the general problem "which point is closest" is solved in this animation. Guiding questions are:

- How are the boundaries (cell edges) between the points (sites) determined?
- What is the mathematical term for those cell edges?
- How can you determine which of the two points you are closer to?
- What could we do if there were more than two points?

Now your group can think about how this would work for multiple points. Open the Geogebra workbook: https://ggbm.at/Df6h3Kgh. Watch the 4 point animation (page 2 of the workbook)

- Explain in words how the boundary lines are constructed
- What is the significance of point **J** in the animation in terms of distance to the ice cream stalls? Point **J** appears when the blue red and green circles form their intersection.

Kangaroo Island part 1

Kangaroo Island is an island off the coast of South Australia. It is known for its abundant flora and fauna eg a large kangaroo population. On Kangaroo Island, The authorities decide to appoint a number of *caretakers* who are to monitor and report on the flora and fauna on the island.

The caretakers will be provided with their own monitoring stations in various locations on the island. Initially it is decided that three stations will be set-up. One in Kingscote, one in Seal Bay and one in Flinders Chase.

Use the #3 applet (kangaroo island caretakers 2) in the workbook to complete the following.

- Draw by hand on the screen how the island can best be divided in order for both caretakers to monitor the region of the Island that is closest to their station. Use the 'line' or 'pen' tool in the tools menu to draw your line.
- Check your answer using the *voronoi* command. In the Algebra menu *input box* type: Voronoi (A,B,C)
- Where you close?

Algebra and Tools menu icons

A closer look at Edges

Now we will take a break from kangaroo island and look at voronoi diagrams a bit closer from a mathematical perspective with the #4 applet (voronoi student worksheet)

On a map two sites A and B are given. The coordinates of A and B are A(5,3) and B(-2,0).

1. Discuss a possible way to graph the voronoi edge between A and B. What is the equation of that edge (the line separating site A and B)? Hint, once you have graphed the line in the *Tools* menu you can see it's equation in the *Algebra* menu
2. Add a site C at the coordinates C(0,-2). Find the equations of the additional Voronoi edges formed between A, B and C .
3. Find the point D that is equidistant to A, B and C and show in your applet that D is equidistant to A,B and C.
4. State the *domain* of the voronoi edges. Hint: think back to the *functions* chapter.

In a new situation, given is a Voronoi diagram with the line segment 2x + 3y = 6 which separates site B(-2,0) from a site D of which the exact location is unknown.

5. Find the coordinates of point D.

Extension or out of class work:

6. If you are at the location P(3,-2) use the graph to determine whether you are closest to B or D. You can plot the line and points using the *Algebra* and *Tools* menus
7. Now suppose you can't use the graph and you don't know the location of the voronoi sites. How can you use the equation of the voronoi edge and the coordinates of your location to show in which cell you are.
8. How can you express this algebraically.

Kangaroo Island Part 2

Kangaroo island also has a large agricultural community. Soil testing is an important tool for landholders on the island to monitor soil fertility levels and it provides valuable information for making correct fertiliser and liming decisions. Soil fertility refers to the ability of a soil to sustain agricultural plant growth, i.e. to provide plant habitat and result in sustained and consistent yields of high quality. The soil's PH-value provides important information regarding the fertility. Use the #5 applet (Kangaroo Island PH)

- According to the "nearest neighbour" method, what is the soil PH value in Karatta
- Using a *linear model* between the two nearest sites, what would be the PH value? hint; you will need to measure distances using the measurement tool
- Is it important to know the real-life distance between these places for the purpose of calculating the PH value?

Kangaroo Island Part 3

On Kangaroo Island it is decided that 11 caretakers will be appointed to monitor and take care of the flora and fauna on the island. The caretakers are located in monitoring stations in various locations.

- They each take care of the land that is closest to their station. Show how the island can be divided in regions to illustrate this (page 6)

A new caretaker is appointed and will be joining the others somewhere on the island in such a way that they take the largest portion of the island available without moving the existing stations.

- How can we find the location where can they best setup-their station?