Lesson 15: Adding the angles in a triangle

Goals

- Comprehend that a straight angle can be decomposed into 3 angles to construct a triangle.
- Justify (orally and in writing) that the sum of angles in a triangle is 180 degrees using properties of rigid motions.

Learning Targets

- If I know two of the angles in a triangle, I can find the third angle.

Lesson Narrative

In this lesson, the focus is on the interior angles of a triangle. What can we say about the three interior angles of a triangle? Do they have special properties?

The lesson opens with an optional activity looking at different types of triangles with a particular focus on the angle combinations of specific acute-angled, right-angled, and obtuse-angled triangles. After being given a triangle, students form groups of 3 by identifying two other students with a triangle congruent to their own. After collecting some class data on all the triangles and their angles, they find that the sum of the angles in all the triangles turns out to be 180 degrees.

In the next activity, students observe that if a **straight angle** is decomposed into three angles, it appears that the three angles can be used to create a triangle. Together the activities provide evidence of a close connection between three positive numbers adding up to 180 and having a triangle with those three numbers as angle sizes.

A new argument is needed to justify this relationship between three angles making a line and three angles being the angles of a triangle. This is the topic of the following lesson.

Building On

- Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three sizes of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Addressing

- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of

the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

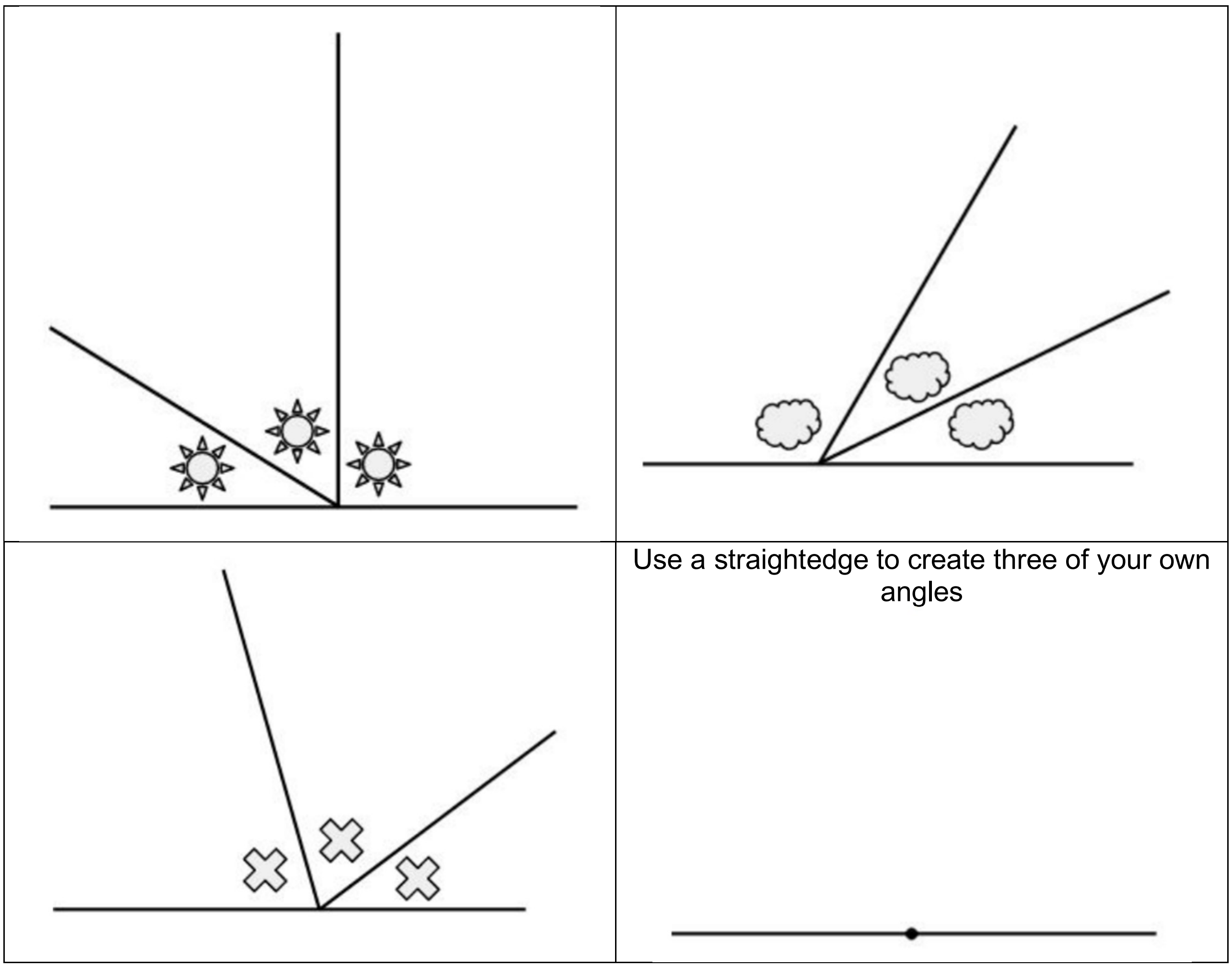
Instructional Routines

- Group Presentations
- Compare and Connect

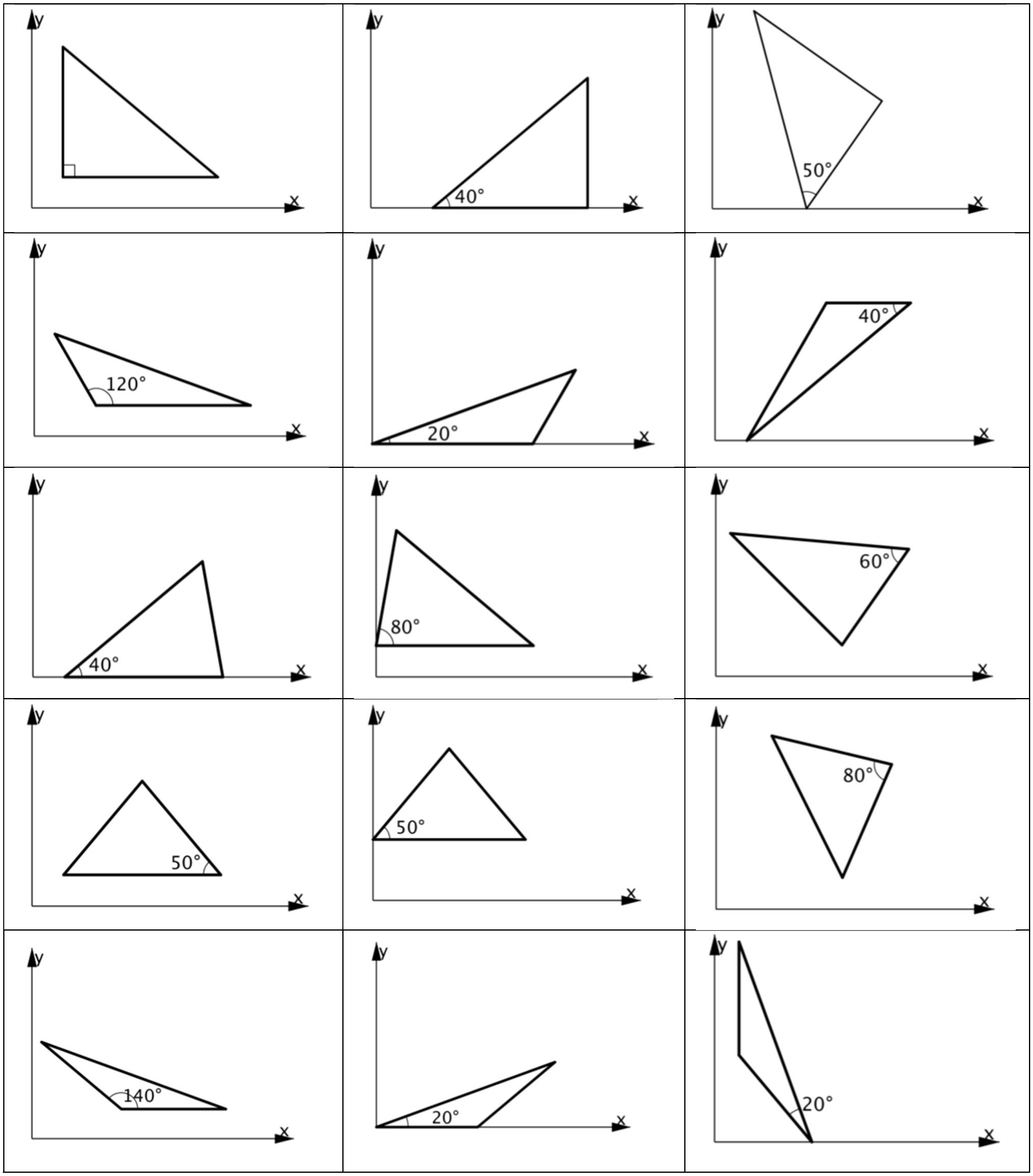
Required Materials

Copies of blackline master

Tear it up



Find all three



Geometry toolkits
tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Print copies of the Tear it Up blackline master. Prepare 1 copy for every group of 4 students. From the geometry toolkit, students will need scissors.

If you are doing the optional Find All Three activity, prepare 1 copy of the Find All Three blackline master for every 15 students. Cut these up ahead of time.

Student Learning Goals

Let's explore angles in triangles.

15.1 Can You Draw It?

Warm Up: 10 minutes

Students try to draw triangles satisfying different properties. They complete the table and then check with a partner whether or not they agree that the pictures are correct or that no such triangle can be drawn. The goals of this warm-up are:

- Reviewing different properties and types of triangles.
- Focusing on individual angle sizes in triangles in preparation for studying their sum.

Note that we use the inclusive definition of isosceles triangle having at least two equal sides. It is possible that in students' earlier experiences, they learned that an isosceles triangle has exactly two equal sides. This issue may not even come up, but be aware that students may be working under a different definition of isosceles than what is written in the task statement.

Launch

Arrange students in groups of 2. Quiet work time for 3 minutes to complete the table followed by partner and whole-class discussion.

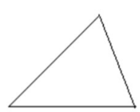
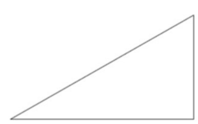
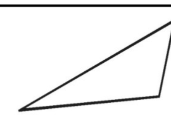
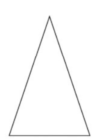
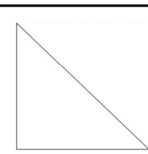
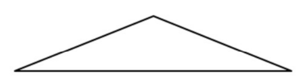
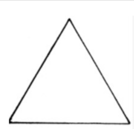
Student Task Statement

1. Complete the table by drawing a triangle in each cell that has the properties listed for its column and row. If you think you cannot draw a triangle with those properties, write "impossible" in the cell.
2. Share your drawings with a partner. Discuss your thinking. If you disagree, work to reach an agreement.

	acute (all angles acute)	right (has a right angle)	obtuse (has an obtuse angle)
scalene (side lengths all different)			
isosceles (at least two side lengths are equal)			
equilateral (three side lengths equal)			

Student Response

Sample response:

	acute (all angles acute)	**right** (has a right angle)	**obtuse** (has an obtuse angle)
scalene (side lengths all different)			
isosceles (at least two side lengths are equal)			
equilateral (three equal side lengths)		Impossible	Impossible

Activity Synthesis

Invite students to share a few triangles they were able to draw such as:

- Right-angled and isosceles
- Equilateral and acute-angled

Ask students to share which triangles they were unable to draw and why. For example, there is no right-angled equilateral or obtuse-angled equilateral triangle because the side opposite the right (or obtuse) angle is longer than either of the other two sides.

15.2 Find All Three

Optional: 15 minutes

This is a matching activity where each student receives a card showing a triangle and works to form a group of three. Each card has a triangle with the size of only one of its angles given. Students use what they know about transformations and estimates of angles to find partners with triangles congruent to theirs. Each unique triangle's three interior angles are then displayed for all to see. Students notice that the sum of the angles in each triangle is 180 degrees.

You will need the Find All Three blackline master for this activity.

Launch

Provide access to geometry toolkits. Distribute one card to each student, making sure that all three cards have been distributed for each triangle. (If the number of students in your class is not a multiple of three, it's okay for one or two students to take ownership over two cards showing congruent triangles.) Explain that there are two other students who have a triangle congruent to theirs that has been re-oriented in the coordinate grid through combinations of translations, rotations, and reflections. Instruct students to look at the triangle on their card and estimate the other two angles. With these estimates and their triangle in mind, students look for the two triangles congruent to theirs with one of the missing angles labelled.

Prepare and display a table for all to see with columns angle 1, angle 2, angle 3 and one row for each group of three students. It should look something like this:

student groups	angle 1	angle 2	angle 3

Once the three partners are together, they complete one row in the posted table for their triangle's angles. Whole-class discussion to follow.

Students might ask if they can use tracing paper to find congruent triangles. Ask how they would use it and listen for understanding of transformations to check for congruence. Respond that this seems to be a good idea.

Representation: Provide Access for Perception. Display or provide students with a physical copy of the written directions and read them aloud. Check for understanding by inviting students to rephrase directions in their own words. Consider keeping the display of directions visible throughout the activity.
Supports accessibility for: Language; Memory

Student Task Statement

Your teacher will give you a card with a picture of a triangle.

1. The measurement of one of the angles is labelled. Mentally estimate the other two angles.

2. Find two other students with triangles congruent to yours but with a different angle labelled. Confirm that the triangles are congruent, that each card has a different angle labelled, and that the angle sizes make sense.

3. Enter the three angles for your triangle on the table your teacher has posted.

Student Response

The angle combinations are: 40, 50, 90; 40, 60, 80; 50, 50, 80; 20, 20, 140; 20, 40, 120

Activity Synthesis

Begin the discussion by asking students:

- "What were your thoughts as you set about to find your partners?"
- "How did you know that you found a correct partner?"

Expect students to discuss estimates for angles and their experience of how different transformations influence the position and appearance of a polygon.

Next look at the table of triangle angles and ask students:

- "Is there anything you notice about the combinations of the three angle sizes?"
- "Is there something in common for each row?"

Guide students to notice that the sum in each row is the same, 180 degrees. Ask whether they think this is always be true for any triangle. Share with students that in the next activity, they will work towards considering whether this result is true for all triangles.

15.3 Tear It Up

25 minutes

In the optional activity, students found that the sum of the angles of all the triangles on the cards was 180 degrees and questioned if all triangles have the same angle sum. In this activity, students experiment with the converse: If we know three angles sum to 180 degrees, can these three angles be the interior angles in a triangle?

Students cut out three angles that form a line, and then try to use these three angles to make a triangle. Students also get to create their own three angles from a line and check whether they can construct a triangle with their angles.

Watch for students who successfully make triangles out of each set of angles and select them to share (both the finished product and how they worked to arrange the angles) during the discussion. Watch also for how students divide the blank line into angles. It is helpful if the rays all have about the same length as in the pre-made examples.

Instructional Routines

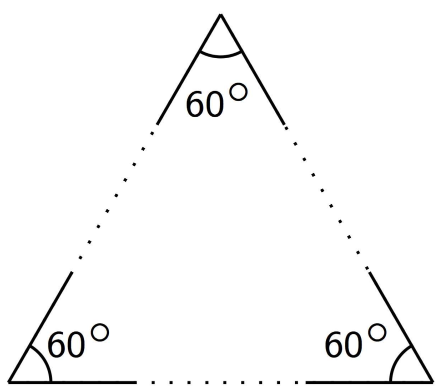
- Group Presentations
- Compare and Connect

Launch

Arrange students in groups of 4. Provide access to geometry toolkits, especially scissors. Distribute 1 copy of the black line master to each group.

Instruct students to cut the four individual pictures out of the black line master. Each student will work with one of these. Instruct the student with the blank copy to use a straightedge to divide the line into three angles (different from the three angles that the other students in the group have). Demonstrate how to do this if needed.

If needed, you may wish to demonstrate "making a triangle" part of the activity so students understand the intent. With three cut-out 60 degree angles, for example, you can build an equilateral triangle. Here is a picture showing three 60 degree angles arranged so that they can be joined to form the three angles of an equilateral triangle. The students will need to arrange the angles carefully, and they may need to use a straightedge in order to add the dotted lines to join the angles and create a triangle.



Student Task Statement

Your teacher will give you a page with three sets of angles and a blank space. Cut out each set of three angles. Can you make a triangle from each set that has these same three angles?

Student Response

Answers vary. Sample response: We were all able to build triangles with the given sets of angles. One is a right-angled triangle, one acute-angled, and one obtuse-angled. The three angles we chose also made a triangle.

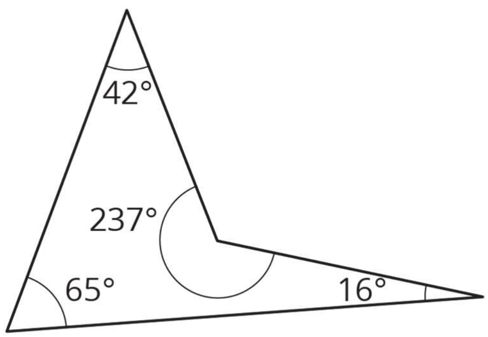
Are You Ready for More?

1. Draw a quadrilateral. Cut it out, tear off its angles, and line them up. What do you notice?

2. Repeat this for several more quadrilaterals. Do you have a conjecture about the angles?

Student Response

The sum of interior angles in any quadrilateral is 360°. This is pretty clear with rectangles. Parallelograms have 2 pairs of equal supplementary angles, so they work too. In fact, it works for anything, even non-convex quadrilaterals.



Activity Synthesis

If time allows, have students do a "gallery walk" at the start of the discussion. Ask students to compare the triangle they made to the other triangles made from the same angles and be prepared to share what they noticed. (For example, students might notice that all the other triangles made with their angles looked pretty much the same, but were different sizes.) If students do not bring it up, direct students to notice that all of the "create three of your own angles" students were able to make a triangle, not just students with the ready-made angles.

Ask previously selected students to share their triangles and explain how they made the triangles. To make the triangles, some trial and error is needed. A basic method is to line up the line segments from two angles (to get one side of the triangle) and then try to place the third angle so that it lines up with the rays coming from the two angles already in place. Depending on the length of the rays, they may overlap, or the angles may need to be moved further apart. Ask questions to make sure that students see the important connection:

- "How do you know the three angles you were given sum to 180 degrees?" (They were adjacent to each other along a line.)
- "How do you know these can be the three angles of a triangle?" (We were able to make a triangle using these three angles.)
- "What do you know about the three angles of the triangle you made and why?" (They sum to 180 because they were the same three angles that made a line.)

Ask students if they think they can make a triangle with *any* set of three angles that form a line and poll the class for a positive or negative response. Tell them that they will investigate this in the next lesson and emphasise that while experiments may lead us to believe this statement is true, the methods used are not very accurate and were only applied to a few sets of angles.

If time permits, perform a demonstration of the converse: Start with a triangle, tear off its three corners, and show that these three angles when placed adjacent each other sum to a line.

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as "To arrange the angles, first, I _____ because..." or "I noticed _____ so I…."
Supports accessibility for: Language; Social-emotional skills Representing, Conversing, Listening: Compare and Connect. As students prepare their work for discussion, look for those who successfully construct triangles out of each set of angles and for those who successfully create their own three angles from a line and create triangles. Encourage students to explain how they worked to arrange the angles. Emphasise language used to make sense of strategies used to find that the sum of the angles in a triangle is 180 degrees.
Design Principle(s): Maximise Meta-awareness, Support sense-making

Lesson Synthesis

Some guiding questions for the discussion include:

- "What did we observe about the sum of the angles inside a triangle?" (The sum of the angles inside a triangle seem to always add up to 180 degrees.)
- "When you know two angles inside a triangle, how can you find the third angle?" (If all three angles add up to 180 degrees, then subtracting two of the angles from 180 will give the size of the third angle.)
- "Are there pairs of angles that cannot be used to make a triangle?" (Yes. If the two angles are both bigger than or equal to 90 degrees, then you cannot make a triangle.)

Emphasise that we were able to see for multiple triangles that the sum of their angles is 180° *and* that using several sets of three angles adding to 180° we were able to build triangles with those angles. In the next lesson we will investigate and explain this interesting relationship.

15.4 Missing Angles

Cool Down: 5 minutes

Students have experimented to see that the sum of the angles in a triangle is 180 degrees. While they will prove this in the next lesson, here they apply the concept in order to give examples of different kinds of triangles with one given angle.

Launch

If needed, tell students to use their conjecture that the sum of the angles in a triangle is always 180 degrees as they work on these problems.

Student Task Statement

In triangle ABC, the size of angle B is 50 degrees.

1. Give possible values for the sizes of angles A and C if ABC is an acute-angled triangle.
2. Give possible values for the sizes of angles A and C if ABC is an obtuse-angled triangle.
3. Give possible values for the sizes of angles A and C if ABC is a right-angled triangle.

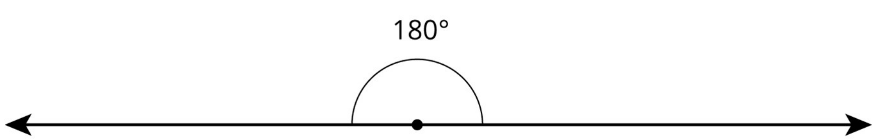
Student Response

Answers vary. Sample responses:

1. To make an acute-angled triangle, the other two angles must measure less than 90 degrees (for example: 60, 70).
2. To make an obtuse-angled triangle, one of the two angles must be greater than 90 degrees (100, 30).
3. There is only one way to make a right-angled triangle (90, 40).

Student Lesson Summary

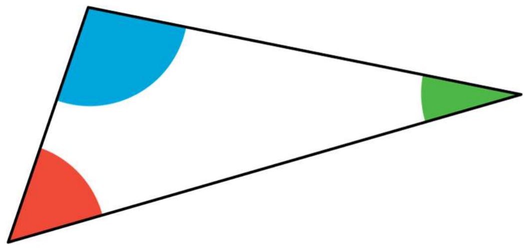
A 180° angle is called a **straight angle** because when it is made with two rays, they point in opposite directions and form a straight line.

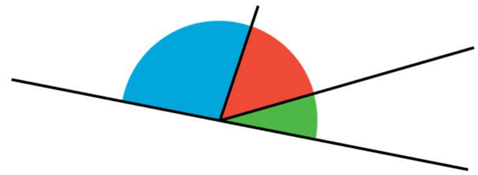


If we experiment with angles in a triangle, we find that the sum of the three angles in each triangle is 180°—the same as a straight angle!

Through experimentation we find:

- If we add the three angles of a triangle physically by cutting them off and lining up the vertices and sides, then the three angles form a straight angle.
- If we have a line and two rays that form three angles added to make a straight angle, then there is a triangle with these three angles.





Glossary

- straight angle

Lesson 15 Practice Problems

1. **Problem 1 Statement**

   In triangle ABC, the size of angle A is 40°.

   a. Give possible sizes for angles B and C if triangle ABC is isosceles.

   b. Give possible sizes for angles B and C if triangle ABC is right-angled.

   Solution

   a. There are two possibilities: Angles B and C each measure 70°, or one of angles B and C measures 40° and the other measures 100°.

   b. One of angles B and C measures 50°, and the other angle measures 90°.

2. **Problem 2 Statement**

   For each set of angles, decide if there is a triangle whose angles have these sizes in degrees:

   a. 60, 60, 60

   b. 90, 90, 45

   c. 30, 40, 50

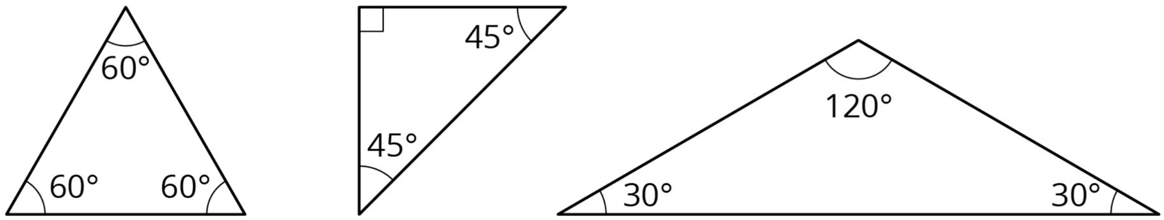
   d. 90, 45, 45

   e. 120, 30, 30

   If you get stuck, consider making a line segment. Then use a protractor to measure angles with the first two angle sizes.

   Solution

   Triangles can be made with the sets of angles in a, d, and e but not with b, and c.



3. **Problem 3 Statement**

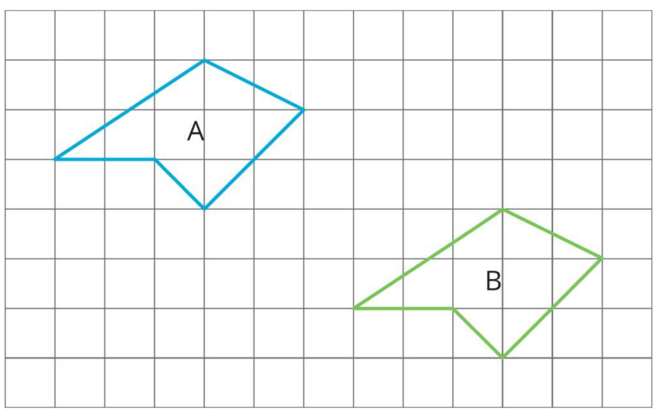
   Angle A in triangle ABC is obtuse. Can angle B or angle C be obtuse? Explain your reasoning.

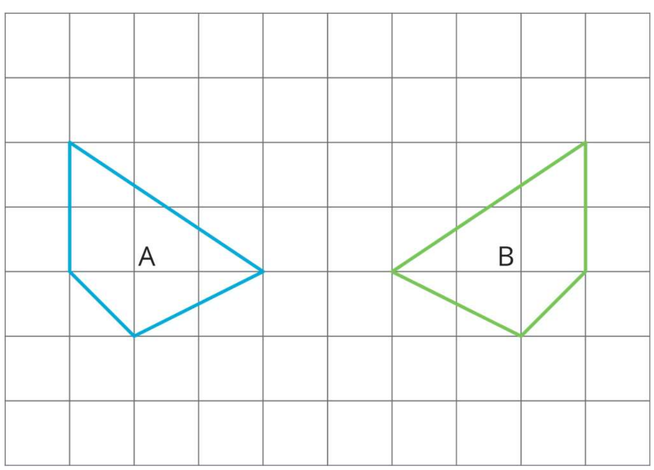
   Solution

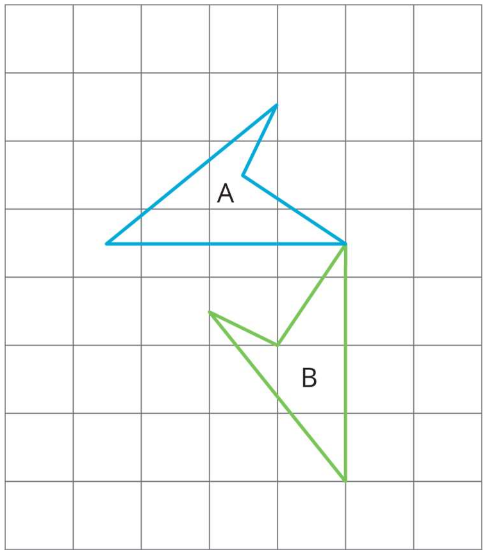
   No, a triangle cannot have two obtuse angles. If the obtuse angles were at vertices A and B, for example, then those angles do not meet at any point C.

4. **Problem 4 Statement**

   For each pair of polygons, describe the transformation that could be applied to polygon A to get polygon B.

   a. 

   b. 

c. 

Solution

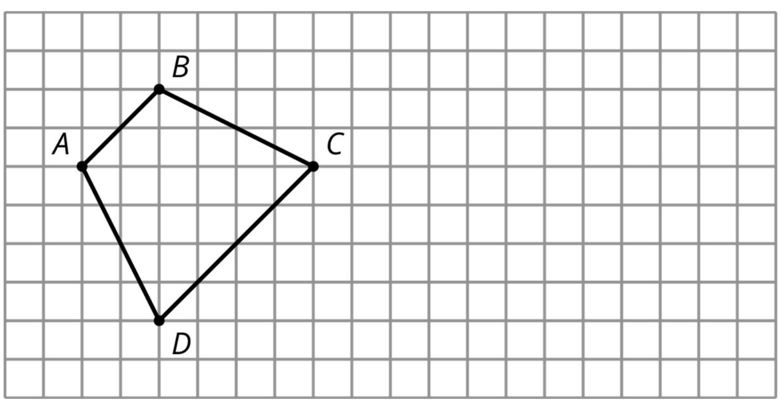
a. Translation down 3 units and right 6 units

b. Reflection in a vertical line of reflection halfway between the two polygons

c. Rotation by 90 degrees anti-clockwise with the vertex shared by the two polygons as the centre of rotation

5. **Problem 5 Statement**

On the grid, draw a scaled copy of quadrilateral $ABCD$ using a scale factor of $\frac{1}{2}$.



Solution

Answers vary. Each side is $\frac{1}{2}$ the length of the corresponding side on $ABCD$. For example, if $B'C'$ on the scaled copy corresponds to BC, then C' should be down 1 grid square and right 2 grid squares from B'. The same should be true for all other sides: this guarantees that corresponding angles will have the same measure.