Grades 5-8 (SA), 9-12 (SA)

Topic: Combinatorics

Duration: 60-90 min

Tools: 1 block / 1-3 student (block 111, 222 or 333)

Individual / Pair / Group work

Keywords: Colouring edges, All cases, Spatial vision, Symmetry

612 - Colouring Edges 222

MATHS / COMBINATORICS

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

2019-1-HU01-KA201-0612722019-1

DESCRIPTION

Students colour the edges of the block 222 (or 111 or 333) with 2 colours (red and blue) and consider the number of possible colourings. (Two colourings are different, if they cannot be moved into each other.)

LEVEL 1 1 red edge or 1 blue edge.

LEVEL 2 2 red edges or 2 blue edges.

LEVEL 3 3 red edges or 3 blue edges.

LEVEL 4 All possibilities.

SOLUTIONS / EXAMPLES

SOLUTION There are 104 colourings of the edges of the block 222.

There are 1, 2, 8, 17, 24, 24, 17, 8, 2, 1 edge colourings with 0, 1, 2, ..., 9 red edges respectively, see the figures below. By symmetry, the number of colourings with k red edges is the same as with $9 - k$ red edges.

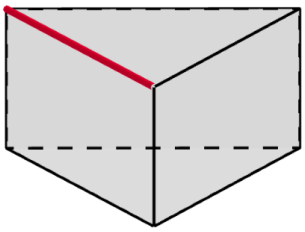
LEVELS 2 AND 3

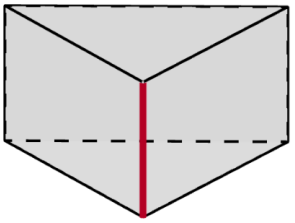
It is worth discussing that there are colourings that are the mirror images, but they can not be moved to each other. This is a similar phenomenon to the fact that the blocks 123 and 132 are not the same, see exercise 601 - Matchmaking.

LEVEL 2

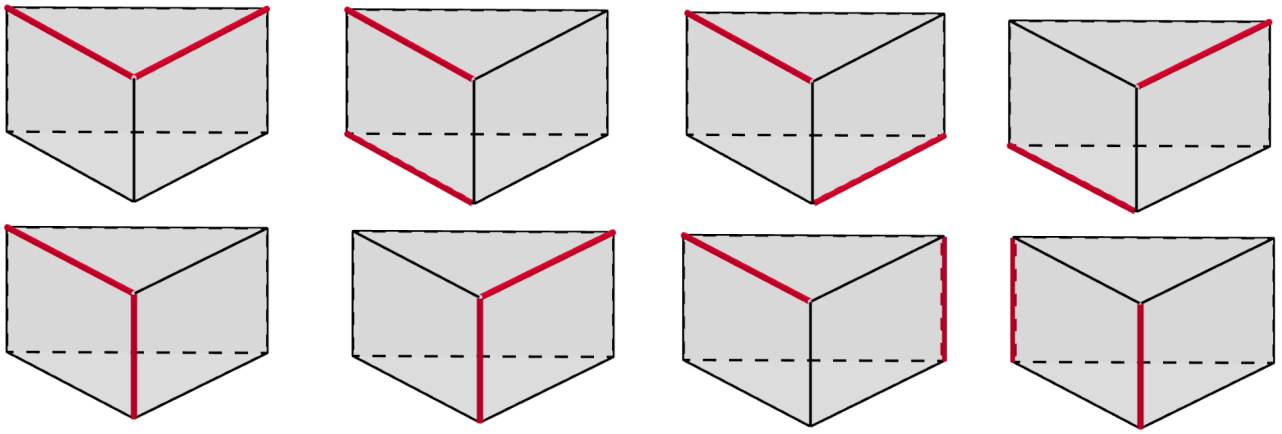
The mutual position of the edges can be discussed, see also exercise 512 - Mutual Position of Lines and Planes. The usage of the correct terminology helps discuss the solutions.

1 red edge

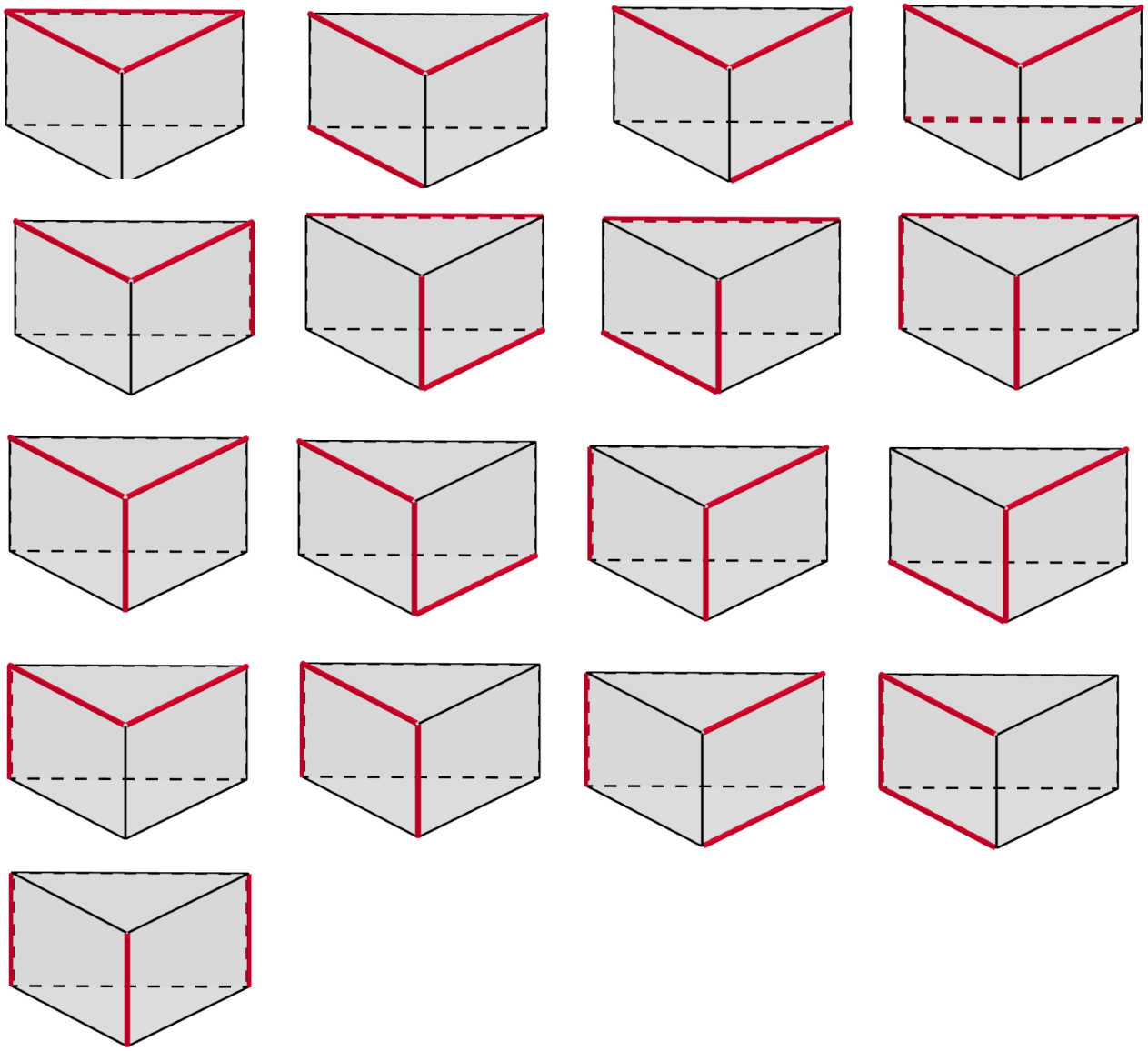




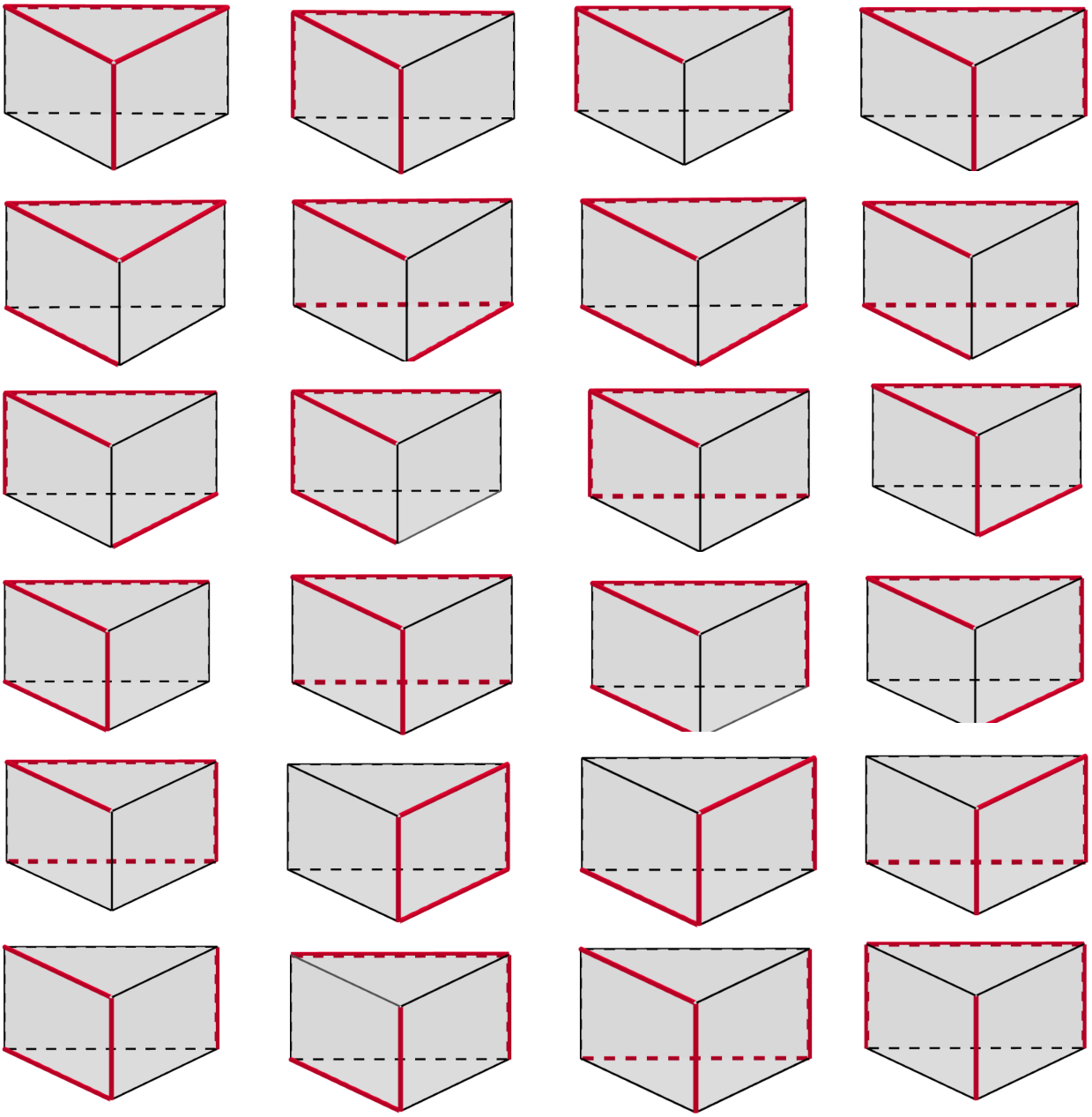
2 red edges



3 red edges



4 red edges



GUIDELINES FOR THE TEACHER

Discuss the symmetries that imply that there are only two colourings with one red edge.
Discuss the symmetries in the case of 2 red edges.
Discuss that by symmetry, there is the same number of colourings with n red edges as with n blue edges.

PRIOR KNOWLEDGE

Edge of polyhedron, Symmetry

RECOMMENDATIONS / COMMENTS

This exercise develops combinatorial and logical thinking, while also builds on spatial vision.

Levels 1-3 can be given to different students or groups, each level is more difficult than the previous one.

The previous level helps answer the next one, but each question can be asked independently of the others.

Level 4 is the combination of Levels 1-3 and a few more difficult cases. This is a very difficult exercise.