MEI Mathematics Education Innovation

Lesson 22: Combining like terms (Part 3)

Goals

- Explain (orally and in writing) how to write an equivalent expression with fewer terms.
- Generalise (orally) about what strategies are useful and what mistakes are common when writing equivalent expressions with fewer terms.
- Identify equivalent expressions, and justify (orally and in writing) that they are equivalent.

Learning Targets

- Given an expression, I can use various strategies to write an equivalent expression.
- When I look at an expression, I can notice if some parts have common factors and make the expression shorter by combining those parts.

Lesson Narrative

In this lesson, students have an opportunity to demonstrate fluency in combining like terms and look for and make use of structure to apply the distributive property in more sophisticated ways.

Building On

- Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

Addressing

- Apply properties of operations as strategies to add, subtract, factorise, and expand linear expressions with rational coefficients.

Building Towards

- Apply properties of operations as strategies to add, subtract, factorise, and expand linear expressions with rational coefficients.

Instructional Routines

- Discussion Supports
- Take Turns
- Think Pair Share

Student Learning Goals

Let's see how we can combine terms in an expression to write it with fewer terms.

22.1 Are They Equal?

Warm Up: 5 minutes

The purpose of this activity is to remind students of things they learned in the previous lesson using numerical examples. Look for students who evaluate each expression and students who use reasoning about operations and properties.

Launch

Remind students that working with subtraction can be tricky, and to think of some strategies they have learned in this unit. Encourage students to reason about the expressions without evaluating them.

Give students 2 minutes of quiet think time followed by whole-class discussion.

Anticipated Misconceptions

Students who selected $8 - 6 - 12 + 4$ or $8 - 12 + (6 + 4)$ might not understand that the subtraction sign outside the brackets applies to the 4 and that 4 is always to be subtracted in any equivalent expression.

Students who selected $8 - 12 + (6 + 4)$ might think the subtraction sign in front of 12 also applies to $(6 + 4)$ and that the two subtractions become addition.

Student Task Statement

Select **all** expressions that are equal to $8 - 12 - (6 + 4)$.

1. $8 - 6 - 12 + 4$
2. $8 - 12 - 6 - 4$
3. $8 - 12 + (6 + 4)$
4. $8 - 12 - 6 + 4$
5. $8 - 4 - 12 - 6$

Student Response

2, 5

Activity Synthesis

For each expression, poll the class for whether the expression is equal to the given expression, or not. For each expression, select a student to explain why it is equal to the given expression or not. If the first student reasoned by evaluating each expression, ask if anyone reasoned without evaluating each expression.

22.2 X's and Y's

15 minutes

In this activity students take turns with a partner and work to make sense of writing expressions in equivalent ways. This activity is a step up from the previous lesson because there are more negatives for students to deal with, and each expression contains more than one variable.

Instructional Routines

- Discussion Supports
- Take Turns

Launch

Arrange students in groups of 2. Tell students that for each expression in column A, one partner finds an equivalent expression in column B and explains why they think it is equivalent. The partner's job is to listen and make sure they agree. If they don't agree, the partners discuss until they come to an agreement. For the next expression in column A, the students swap roles. If necessary, demonstrate this protocol before students start working.

Representation: Internalise Comprehension. Differentiate the degree of difficulty or complexity by beginning with an example with more accessible values. For example, start with an expression with three terms such as "$6x - (2x + 8)$" and show different forms of equivalent expressions. Highlight connections between expressions by using the same colour on equivalent parts of the expression.
Supports accessibility for: Conceptual processing Listening, Speaking: Discussion Supports. Display sentence frames for students to use to describe the reasons for their matches. For example, "I matched expression ___ with expression ___ because . . . ." or "I used the ___ property to help me match expression ___ with expression ___." Provide a sentence frame for the partner to respond with, such as: "I agree/disagree with this match because . . . ." These sentence frames provide students with language structures that help them to produce explanations, and also to critique their partner's reasoning.
Design Principle(s): Maximise meta-awareness; Support sense-making

Anticipated Misconceptions

For the second and third rows, some students may not understand that the subtraction sign in front of the brackets applies to both terms inside that set of brackets. Some students may get the second row correct, but not realise how the third row relates to the fact that the product of two negative numbers is a positive number. For the last three rows, some students may not recognise the importance of the subtraction sign in front of $7y$. Prompt them to rewrite the expressions replacing subtraction with adding the inverse.

Students might write an expression with fewer terms but not recognise an equivalent form because the distributive property has been used to write a sum as a product. For example, $9x - 7y + 3x - 5y$ can be written as $9x + 3x - 7y - 5y$ or $12x - 12y$, which is equivalent to the expression $12(x-y)$ in column B. Encourage students to think about writing the column B expressions in a different form and to recall that the distributive property can be applied to either factorise or expand an expression.

Student Task Statement

Match each expression in column A with an equivalent expression from column B. Be prepared to explain your reasoning.

A

1. $(9x+5y)+(3x+7y)$
2. $(9x+5y)-(3x+7y)$
3. $(9x+5y)-(3x-7y)$
4. $9x-7y+3x+5y$
5. $9x-7y+3x-5y$
6. $9x-7y-3x-5y$

B

1. $12(x+y)$
2. $12(x-y)$
3. $6(x-2y)$
4. $9x+5y+3x-7y$
5. $9x+5y-3x+7y$
6. $9x-3x+5y-7y$

Student Response

The correct pairings:

A	B
$(9x+5y)+(3x+7y)$	$12(x+y)$
$(9x+5y)-(3x+7y)$	$9x-3x+5y-7y$
$(9x+5y)-(3x-7y)$	$9x+5y-3x+7y$
$9x-7y+3x+5y$	$9x+5y+3x-7y$
$9x-7y+3x-5y$	$12(x-y)$
$9x-7y-3x-5y$	$6(x-2y)$

Activity Synthesis

Much discussion takes place between partners. Invite students to share how they used properties to generate equivalent expressions and find matches.

- "Which term(s) does the subtraction sign apply to in each expression? How do you know?"
- "Were there any expressions from column A that you wrote with fewer terms but were unable to find a match for in column B? If yes, why do you think this happened?"
- "What were some ways you handled subtraction with brackets? Without brackets?"
- "Describe any difficulties you experienced and how you resolved them."

22.3 Seeing Structure and Factorising

10 minutes

This activity is an opportunity to notice and make use of structure in order to apply the distributive property in more sophisticated ways.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Display the expression $18 - 45 + 27$ and ask students to calculate as quickly as they can. Invite students to explain their strategies. If no student brings it up, ask if the three numbers have anything in common (they are all multiples of 9). One way to quickly compute would be to notice that $18 - 45 + 27$ can be written as $2 \times 9 - 5 \times 9 + 3 \times 9$ or $(2 - 5 + 3) \times 9$ which can be quickly calculated as 0. Tell students that noticing common factors in expressions can help us write them with fewer terms or more simply.

Keep students in the same groups. Give them 5 minutes of quiet work time and time to share their expressions with their partner, followed by a whole-class discussion.

Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2–3 minutes of work time. Look for students who identify common factors or rearrange terms to write the expressions with fewer terms.
Supports accessibility for: Memory; Organisation

Student Task Statement

Write each expression with fewer terms. Show or explain your reasoning.

1. $3 \times 15 + 4 \times 15 - 5 \times 15$

2. $3x + 4x - 5x$

3. $3(x-2) + 4(x-2) - 5(x-2)$

4. $3\left(\frac{5}{2}x + 6\frac{1}{2}\right) + 4\left(\frac{5}{2}x + 6\frac{1}{2}\right) - 5\left(\frac{5}{2}x + 6\frac{1}{2}\right)$

Student Response

1. 2×15 or 30. Explanations vary. Sample response: Move the common factor 15 out of each term and combine the terms in the other factor: $3 \times 15 + 4 \times 15 - 5 \times 15 = (3 + 4 - 5) \times 15 = 2 \times 15 = 30$.

2. $2x$. Explanations vary. Sample response: Move the common factor x out of each term and combine the terms in the other factor: $3x + 4x - 5x = (3 + 4 - 5)x = 2x$.

3. $2(x-2)$ or $2x - 4$. Explanations vary. Sample response: Move the common factor $(x - 2)$ out of each term and combine the terms in the other factor: $3(x-2) + 4(x-2) - 5(x-2) = (3 + 4 - 5)(x-2) = 2(x-2)$.

4. $2(\frac{5}{2}x + 6\frac{1}{2})$ or $5x + 13$. Explanations vary. Sample response: Move the common factor $(\frac{5}{2}x + 6\frac{1}{2})$ out of each term and combine the terms in the other factor: $3(\frac{5}{2}x + 6\frac{1}{2}) + 4(\frac{5}{2}x + 6\frac{1}{2}) - 5(\frac{5}{2}x + 6\frac{1}{2}) = (3 + 4 - 5)(\frac{5}{2}x + 6\frac{1}{2}) = 2(\frac{5}{2}x + 6\frac{1}{2}) = 5x + 13$.

Activity Synthesis

For each expression, invite a student to share their process for writing it with fewer terms. Highlight the use of the distributive property.

Speaking: Discussion Supports. Provide sentence frames to help students explain their strategies. For example, "I noticed that ______, so I ______." or "First, I ________ because ________." When students share their answers with a partner, prompt them to rehearse what they will say when they share with the full group. Rehearsing provides students with additional opportunities to clarify their thinking.
Design Principle(s): Optimise output (for explanation)

Lesson Synthesis

Ask students to reflect on their work in this unit. They can share their response to one or more of these prompts either in writing or verbally with a partner.

- "Describe something that you found confusing at first that you now understand well."
- "Think of a story problem that you would not have been able to solve before this unit that you can solve now."
- "What is a tool or strategy that you learned in this lesson that was particularly useful?"
- "Describe a common mistake that people make when using the ideas we studied in this unit and how they can avoid that mistake."

- “Which is your favourite, and why? The distributive property, rewriting subtraction as adding the opposite, or the commutative property.”

22.4 R's and T's

Cool Down: 5 minutes

Student Task Statement

Match each expression in column A with an equivalent expression from column B. Show or explain your reasoning.

Column A

1. $12r + t - 4r + 6t$
2. $(12r + 2t) - (4r + 10t)$
3. $12(r + t) - 6(r + t) + 4(r + t) - 2(r + t)$

Column B

- $8(r - t)$
- $8(r + t)$
- $8r + 7t$

Student Response

1. $8r + 7t$
2. $8(r - t)$
3. $8(r + t)$

Student Lesson Summary

Combining like terms is a useful strategy that we will see again and again in our future work with mathematical expressions. It is helpful to review the things we have learned about this important concept.

- Combining like terms is an application of the distributive property. For example:

$$2x + 9x$$
$$(2 + 9) \times x$$
$$11x$$

- It often also involves the commutative and associative properties to change the order or grouping of addition. For example:

$$2a + 3b + 4a + 5b$$
$$2a + 4a + 3b + 5b$$
$$(2a + 4a) + (3b + 5b)$$
$$6a + 8b$$

- We can't change order or grouping when subtracting; so in order to apply the commutative or associative properties to expressions with subtraction, we need to rewrite subtraction as addition. For example:

$$2a - 3b - 4a - 5b$$
$$2a + \text{-}3b + \text{-}4a + \text{-}5b$$
$$2a + \text{-}4a + \text{-}3b + \text{-}5b$$
$$\text{-}2a + \text{-}8b$$
$$\text{-}2a - 8b$$

- Since combining like terms uses properties of operations, it results in expressions that are equivalent.
- The like terms that are combined do not have to be a single number or variable; they may be longer expressions as well. Terms can be combined in any sum where there is a common factor in all the terms. For example, each term in the expression $5(x+3) - 0.5(x+3) + 2(x+3)$ has a factor of $(x+3)$. We can rewrite the expression with fewer terms by using the distributive property:

$$5(x+3) - 0.5(x+3) + 2(x+3)$$
$$(5 - 0.5 + 2)(x+3)$$
$$6.5(x+3)$$

Lesson 22 Practice Problems

1. **Problem 1 Statement**

   Jada says, "I can tell that $\frac{-2}{3}(x+5) + 4(x+5) - \frac{10}{3}(x+5)$ equals 0 just by looking at it." Is Jada correct? Explain how you know.

   Solution

   Yes. Explanations vary. Sample response: Take out the factor $x + 5$: $(x+5)(\frac{-2}{3} + 4 + \frac{-10}{3}) = (x+5)(\frac{-12}{3} + 4) = (x+5)(0) = 0$.

2. **Problem 2 Statement**

   In each row, decide whether the expression in column A is equivalent to the expression in column B. If they are not equivalent, show how to change one expression to make them equivalent.

   A

   a. $3x - 2x + 0.5x$

b. $3(x+4)-2(x+4)$

c. $6(x+4)-2(x+5)$

d. $3(x+4)-2(x+4)+0.5(x+4)$

e. $20\left(\frac{2}{5}x+\frac{3}{4}y-\frac{1}{2}\right)$

B

a. $1.5x$

b. $x+3$

c. $2(2x+7)$

d. 1.5

e. $\frac{1}{2}(16x+30y-20)$

Solution

a. Equivalent

b. Not equivalent. Answers vary. Sample responses: Change the column B entry to $x+4$, change the column A entry to $3(x+4)-2(x+4)-1$

c. Equivalent

d. Not equivalent. Answers vary. Sample response: Change the column B entry to $1.5(x+4)$.

e. Equivalent

3. **Problem 3 Statement**

For each situation, write an expression for the new balance using as few terms as possible.

a. A bank account has a balance of -£126.89. A customer makes two deposits, one $3\frac{1}{2}$ times the other, and then withdraws £25.

b. A bank account has a balance of £350. A customer makes two withdrawals, one £50 more than the other. Then he makes a deposit of £75.

Solution

a. $-126.89 + x + 3\frac{1}{2}x - 25 = £(4.5x - 151.89)$

b. $350 - x - (x + 50) + 75 = £(375 - 2x)$

4. **Problem 4 Statement**

Tyler is using the distributive property on the expression $9 - 4(5x - 6)$. Here is his work:

$9 - 4(5x - 6)$
$9 + (-4)(5x + -6)$
$9 + -20x + -6$
$3 - 20x$

Mai thinks Tyler's answer is incorrect. She says, "If expressions are equivalent then they are equal for any value of the variable. Why don't you try to substitute the same value for x in all the equations and see where they are not equal?"

a. Find the step where Tyler made an error.

b. Explain what he did wrong.

c. Correct Tyler's work.

Solution

Answers vary. Sample response:

a. Try 1:

- $9 - 4(5 \times 1 - 6) = 9 - 4(-1) = 9 + 4 = 13$
- $9 + (-4)(5 \times 1 - 6) = 9 + (-4)(-1) = 9 + 4 = 13$
- $9 + (-20)(1) + -6 = 9 + -20 + -6 = -17$
- $3 - 20(1) = 3 - 20 = -17$
  The value of the expression switched in the third step, so that's where the error is.

b. Tyler forgot to multiply the -6 term in the brackets by -4.

c. Starting at step 3:
$9 + -20x + 24$
$33 - 20x$

5. **Problem 5 Statement**

a. If $(11 + x)$ is positive, but $(4 + x)$ is negative, what is one number that x could be?

b. If $(-3 + y)$ is positive, but $(-9 + y)$ is negative, what is one number that y could be?

c. If $(-5 + z)$ is positive, but $(-6 + z)$ is negative, what is one number that z could be?

Solution

a. Answers vary: $-11 < x < -4$

b. Answers vary: $3 < y < 9$

c. Answers vary: $5 < z < 6$