

## Lesson 10: What are percentages?

### Goals

- Comprehend the word “percentage” (in written and spoken language) and the symbol “%” (in written language) to mean a rate per 100.
- Draw and label a double number line diagram to represent percentages of a pound and to find corresponding monetary values or percentages.

### Learning Targets

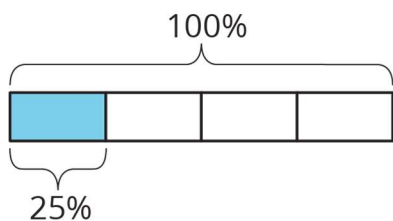
- I can create a double number line with percentages on one line and pound amounts on the other line.
- I can explain the meaning of percentages using pounds and pence as an example.

### Lesson Narrative

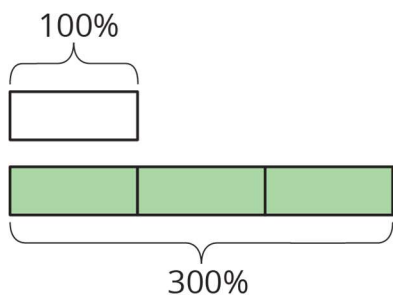
This lesson is the first of two that introduce students to **percentages** as a rate per 100 and the ways they are used to describe different types of situations.

Percentages are commonly used in two ways:

1. To describe a part of a whole. For example, “Jada drank 25% of the bottle of water.” In this case, the percentage expressing the amount consumed is not bigger than 100% because it refers to a part of a whole, as shown in the diagram below.



2. To describe the size of one quantity as a percentage of another quantity. For example, “Jada drank 300% as much water as Diego did.” In this case, there is no restriction on the size of the percentage, because the percentage is describing a multiplicative comparison between two quantities, as shown below.



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In the first usage there is a single quantity and we are describing a part of it; in the second usage we are comparing two quantities. Students may have prior exposure to percentages, but are likely to have only encountered the first usage and might not be able to make sense of percentages above 100% or those used in comparative contexts. This lesson exposes students to both applications of percentages.

Money is the main context for exploring percentages in this lesson and the warm up asks students to convert between pounds and pence providing an opportunity for the teacher to assess students' current abilities.

For the first several lessons exploring percentages, double number lines are the primary representation presented to students. This choice is intended to strongly communicate that we are working with percentage *rates*, and that students can and should use all of the reasoning they have developed to deal with equivalent ratios and rates when dealing with rates per 100. That said, if students prefer to reason using tables or by multiplying or dividing by unit rates, they should not be discouraged from doing so.

### Building On

- Solve word problems involving pounds and pence, using £ and p symbols appropriately. Example: If you have 2 10p coins and 3 pennies, how many pence do you have?
- Read, write, and compare decimals to thousandths.
- Understand ratio concepts and use ratio reasoning to solve problems.

### Addressing

- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means  $\frac{30}{100}$  times the quantity); solve problems involving finding the whole, given a part and the percentage.

### Building Towards

- Find a percentage of a quantity as a rate per 100 (e.g., 30% of a quantity means  $\frac{30}{100}$  times the quantity); solve problems involving finding the whole, given a part and the percentage.

### Instructional Routines

- Clarify, Critique, Correct
- Compare and Connect
- Think Pair Share

### Student Learning Goals

Let's learn about percentages.

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## 10.1 Pounds and Pence

### Warm Up: 5 minutes

This warm-up prompts students to reason in monetary terms, preparing them for subsequent tasks in the lesson. It also provides insight into students' understanding of pounds and pence as well as their ability to reason mentally.

### Launch

Display questions for all to see. Ask students to solve them mentally.

### Anticipated Misconceptions

In response to “how many pounds are in one penny,” students might say there are no pounds at all in one penny. Ask them what fraction of a pound one penny represents.

### Student Task Statement

Find each answer mentally.

1. A sticker costs 25 pence. How many pounds is that?
2. A pen costs 1.50 pounds. How many pence is that?
3. How many pence are in one pound?
4. How many pounds are in one penny?

### Student Response

1.  $\frac{1}{4}$  (or 0.25) of a pound. There are four lots of 25p in a pound.
2. 150 pence. There are 100 pence in 1 pound, so 1.50 pounds is multiplied by 100 to find the number of pence.
3. There are 100 pence in 1 pound.
4.  $\frac{1}{100}$  (or 0.01) of a pound. Since there are 100 pence in 1 pound, 1 pound is divided by 100.

### Activity Synthesis

After students solved all problems mentally, for each problem, ask 1–2 students to share their thinking. Pause between problems to give everyone time to reflect on the shared answers.

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## 10.2 Coins

### 15 minutes

In this activity, students learn the definition of a **percentage** as a rate per 100 and apply this definition in the context of money. They label various coin amounts as percentages of 100 pence or 1 pound.

Students are likely able to name the values of each coin and their individual percentages (in the first two questions) fairly quickly. Assigning a percentage to a group of coins adds complexity and should be the focus of the activity as students may use a variety of strategies. One possible strategy is to reason in terms of ratios. For example, a student may think that if a 10p coin is 10% of a pound, then 6 10p coins is 60% of a pound. This type of ratio thinking is a robust way for dealing with percentage problems and should be encouraged early.

As students work, notice the strategies being used to solve the two problems and identify those with effective approaches so they can share later.

### Instructional Routines

- Clarify, Critique, Correct

### Launch

Remind students that previously they have learned that a “unit rate” tells us the amount of one quantity for 1 of another quantity. Explain that in this task, they will explore “rates per 100.”

Solicit a couple of ideas on what “rates per 100” might mean. Students are likely to suggest a description along the lines of “the amount of something for 100 of something else.” Tell students that a rate per 100 is called a percentage and that they will explore percentages in the context of money. Point out the 50p and pound coins in the task, as some students may not be familiar with them.

Arrange students in groups of 2. Give students 3 minutes of quiet think time to begin work on the task. After that time, ask students to share their responses with a partner and complete the remaining questions together.

*Action and Expression: Develop Expression and Communication.* Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, “If \_\_\_\_ then \_\_\_\_ because...” or “How do you know...?”

*Supports accessibility for: Language; Organisation*

### Anticipated Misconceptions

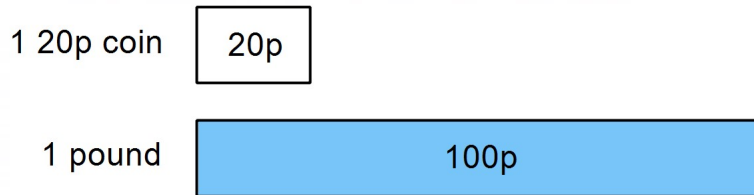
Students may notice a pattern particular to this activity—that the percentage value is the same as that for pence—and carry that assumption forward and apply it incorrectly to

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situations in which 100% does not correspond to 100. This conversation is addressed in the Activity Synthesis.

### Student Task Statement

The value of a 20p coin is 20% of the value of a pound because there are 20 pence for every 100 pence.



- Write the value of the coin that matches each expression.
  - 20% of a pound
  - 5% of a pound
  - 1% of a pound
  - 100% of a pound
  - 10% of a pound
  - 50% of a pound
- The value of 6 10p coins is what **percentage** of the value of a pound?
- The value of 6 20p coins is what percentage of the value of a pound?

### Student Response

- From left to right in the table: 1, 5, 10, 25, 50, 100
    - 20p
    - 5p
    - 1p
    - Pound
    - 10p
    - 50p
  - The value of 6 10p coins is 60 pence and a pound is 100 pence, so 6 10p coins are 60% of the value of 1 pound.
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3. The value of 6 20p coins is 120 pence and a pound is 100 pence, so 6 20p coins is 120% of the value of 1 pound.

### Are You Ready for More?

Find two different sets of coins that each make 120% of a pound, where no type of coin is in both sets.

### Student Response

Answers vary. Sample response: A pound and two 10p coins, five 20p coins and four 5p coins

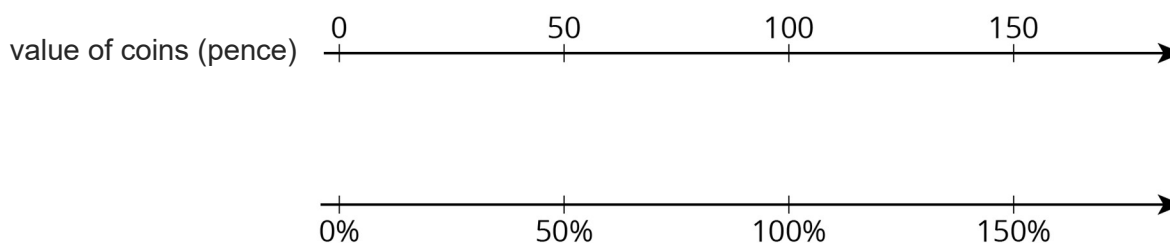
### Activity Synthesis

Focus the discussion on the ways students approached the last two questions and on precise use of language and notation. In the last two problems, expressing a percentage with only a number and without the % symbol should be considered an incomplete answer.

Select students with successful strategies to share their thinking with the class. Display a concise version of their reasoning for all to see. Invite others to express support, disagreement, or questions.

If no one reasoned about percentages in terms of ratios (e.g., If a 20p coin is 20% of a pound, 6 20p coins are 120% of a pound), illustrate it.

Many students may reason by noticing a pattern—that the number of pence in an amount matches its percentage of a pound (e.g., 60 pence is 60%)—rather by thinking in terms of ratio or scaling. Since the pattern only holds up in the context of percentages of 100 of a quantity, students will need to be prompted to look more closely at the meaning of “rate per 100.” Conclude the discussion by displaying the following double number line with 100 at the 100%:



Point out that we were finding **percentages** of 100, so in the double number line, we line up 100% and 100 because 100% of 100 is 100.

*Reading, Writing, Speaking: Clarify, Critique, Correct.* Before students share their explanations for the final question, present an incorrect answer and explanation. For example, “The value of 6 20p coins is 50% of the value of a pound because the value of 6

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20p coins is 120 pence, which is 20 pence greater than 100 pence. This means that the value of 6 20p coins is 20% of the value of a pound.” Ask students to identify the error, critique the reasoning, and write a correct explanation. As students discuss in pairs, listen for students who identify and clarify the ambiguous language in the statement. For example, the author probably meant to say that 6 20p coins is 20% greater than the value of a pound, or that 6 20p coins is 120% of the value of a pound. This will help students understand how to use percentages to describe the size of one quantity as a percentage of another quantity.

*Design Principle(s): Optimise output (for explanation); Maximise meta-awareness*

## 10.3 Coins on a Number Line

### 10 minutes

Previously, students found percentages of 100 pence. In this activity, they reason about percentages of 1 pound.

One important question to think about here is how students know or decide how the numbers on the double number line diagram should be aligned. Students build on their extensive work on equivalent ratios and double number lines to make sense of percentages and “per 100” reasoning.

#### Instructional Routines

- Compare and Connect
- Think Pair Share

#### Launch

Recap that in the previous activity students found percentages of 100 pence. Tell students they will now find percentages of 1 pound. Draw their attention to the fact that, on the double number line, the 1 pound and 100% are lined up vertically to reflect this.

Keep students in the same groups. Give students 2–3 minutes of quiet think time, and then ask them to share their responses with their partner. Display and read aloud the following questions. Ask partners to use them to guide their discussion.

- How did each of you arrive at your answers for the first two questions?
- Where do your answers fall on the double number line diagram? How do you know?
- Are your answers the same for the third question? If they are not, can they both be correct? If they are, can you think of another answer that would also be correct?

#### Anticipated Misconceptions

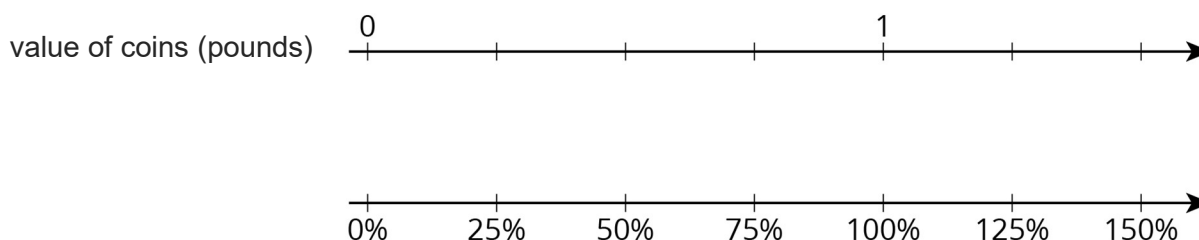
Based on previous work with labelling number lines less than 1, students may label the tick marks with fractions instead of the decimal value of the coins. This may not be helpful for answering the first two questions, but provides an opportunity to discuss alternative ways

to label the number line given the context of the problem. Consider prompting them to write fractional values as pence or to rewrite the pence as pound values.

### Student Task Statement

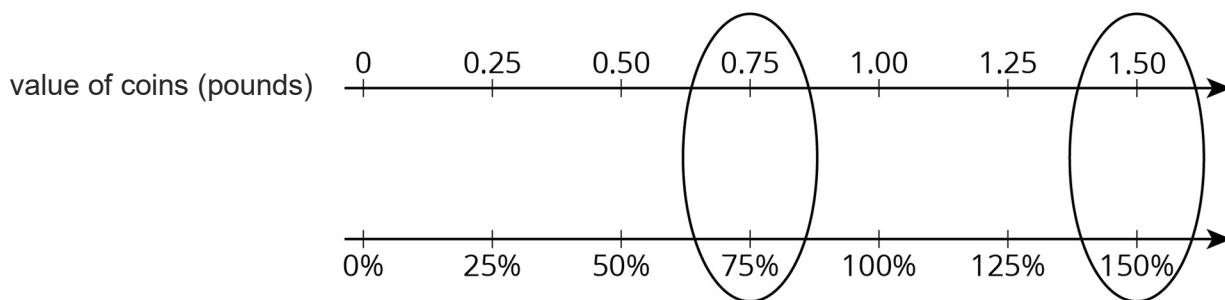


A £1 coin is worth 100% of the value of a pound. Here is a double number line that shows this.



1. The coins in Jada's pocket are worth 75% of a pound. How much are they worth (in pounds)?
2. The coins in Diego's pocket are worth 150% of a pound. How much are they worth (in pounds)?
3. Elena has 3 20p coins and 5 10p coins. What percentage of a pound does she have?

### Student Response



1. £0.75
2. £1.50
3. 110%



## Activity Synthesis

Select students who used the provided double number line to share their reasoning. This is an opportunity to refresh students' number line reasoning. Some students may see the four equally spaced tick marks from 0 to 1 and conclude that each is worth 0.25, or  $\frac{1}{4}$ . Others may fill in the 0.50 first, as it is half of 1, then the 0.25 for half of 0.50, and then use additive thinking to fill in the other tick mark values along the top.

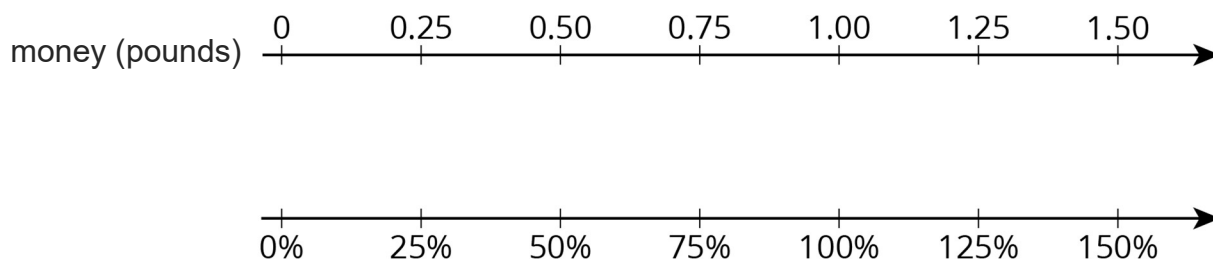
Some students may reason in terms of equivalent ratios and say, for example, that since 100 divided by 4 is 25, then  $1 \div 4 = 0.25$  must be 25% of 1. They would then assign the 0.25 value to the first tick mark and use additive thinking to conclude that 0.75 is 75% of a pound. Ask students who used such an approach to present last to emphasise that the familiar ratio thinking applies to percentage problems as well, even though the % symbol may be unfamiliar. If no students took this approach, illustrate it to make this point.

*Speaking, Listening: Compare and Connect.* As students prepare a visual display of how they made sense of the problem, look for students who labelled the tick marks on the double number line with fractions or pence instead of pound values. This may result in answers such as  $\frac{3}{4}$ , or 75 pence is 75% of a pound rather than 0.75 is 75% of a pound. Although 75 pence is 75% of a pound, the number line should be labelled with the decimal value of the coins in pounds. As students investigate each other's work, ask students to share what worked or did not work well in the way they labelled the double number line. Is there a particular advantage to using decimals instead of fractions to label the double number line? Emphasise that although there are several ways to label the double number line given the context of the problem, certain methods are more helpful for answering the question. This will foster students' meta-awareness and support constructive conversations as they compare the various ways to label a double number line given a context.

*Design Principles(s): Cultivate conversation; Maximise meta-awareness*

## Lesson Synthesis

Remind students that a **percentage** is a "rate per 100." We saw that the value of a 20p coin is 20% of the value of a pound, because a 20p coin is worth 20 pence and a pound is worth 100 pence. Reiterate that we found percentages of the value of a pound using a double number line as shown here:



Here, 100% corresponds to 1 pound, and this is reflected in the fact that the 1.00 and 100% are aligned in the double number line.

## 10.4 Eight 10p coins

### Cool Down: 5 minutes

The purpose of this activity is to see how students make sense of the percentage as a rate per 100.

### Anticipated Misconceptions

In the first question, students may write that 8 10p coins is 8% of the value of a pound because they account for the number of coins but not account their value. In the second question, students may put 130 pence as the answer, not differentiating between combination of coins and the value of the coins.

### Student Task Statement

1. Fill in the blank: The value of 8 10p coins is \_\_\_\_% of the value of a pound.
2. Name a combination of coins that is 130% of the value of a pound.

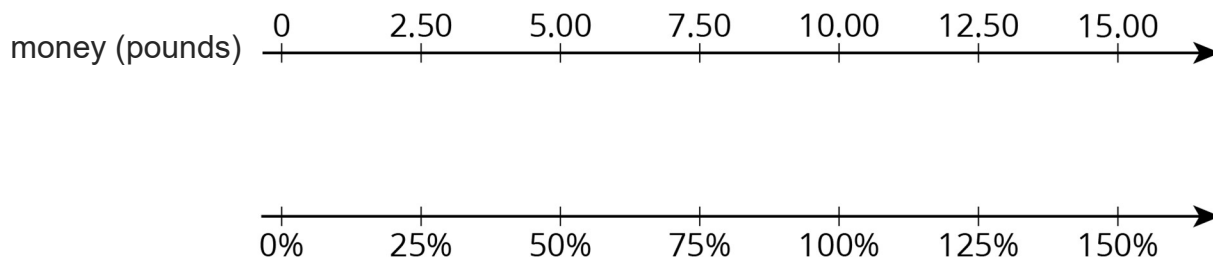
### Student Response

1. 80%. 8 10p coins are 80 pence, and a pound is 100 pence, so 8 10p coins are 80% the value of 1 pound.
2. Answers vary. Sample responses:
  - a. 1 pound and 3 10p coins
  - b. 6 20p coins and 1 10p coin
  - c. 13 10p coins
  - d. 26 5p coins

### Student Lesson Summary

A **percentage** is a *rate per 100*.

We can find percentages of £10 using a double number line where 10 and 100% are aligned, as shown here:



Looking at the double number line, we can see that £5.00 is 50% of £10.00 and that £12.50 is 125% of £10.00.

## Glossary

- percentage

## Lesson 10 Practice Problems

### Problem 1 Statement

What percentage of a pound is the value of each coin combination?

- 4 10p coins
- 1 5p coin and 3 pennies
- 5 20p coins and 1 10p coin

### Solution

- 40%
- 8%
- 110%

### Problem 2 Statement

- List three different combinations of coins, each with a value of 30% of a pound.
- List two different combinations of coins, each with a value of 140% of a pound.

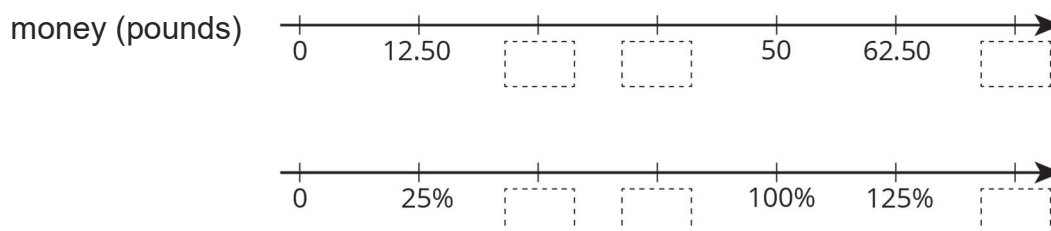
### Solution

Answers vary. Sample response:

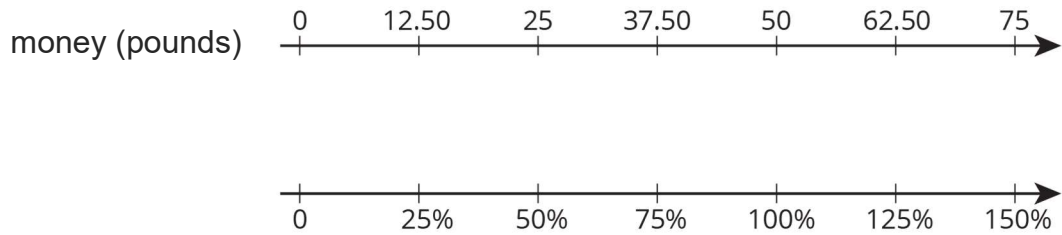
- 30 pennies, 6 5p coins, or 3 10p coins
- 140 pennies, 14 10p coins, or 7 20p coins

### Problem 3 Statement

Complete the double number line show percentages of £50.



**Solution**



**Problem 4 Statement**

Elena bought 8 tokens for £4.40. At this rate:

- a. How many tokens could she buy with £6.05?
- b. How much do 19 tokens cost?

**Solution**

- a. 11 tokens
- b. £10.45

**Problem 5 Statement**

A snail travels 10 cm in 4 minutes. At this rate:

- a. How long will it take the snail to travel 24 cm?
- b. How far does the snail travel in 6 minutes?

**Solution**

- a. 9.6 minutes (or equivalent)
- b. 15 cm

**Problem 6 Statement**

- a. 3 tacos cost £18. Complete the table to show the cost of 4, 5, and 6 tacos at the same rate.

number of tacos	cost in pounds	rate in pounds per taco
3	18	
4		
5		
6		

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b. If you buy  $t$  tacos for  $c$  pounds, what is the unit rate?

**Solution**

a.

number of tacos	cost in pounds	rate in pounds per taco
3	18	6
4	24	6
5	30	6
6	36	6

b.  $\frac{c}{t}$  pounds per taco or  $\frac{t}{c}$  tacos per pound.



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