Función Beta.

Definimos para cualquier $\alpha, \beta \in \mathbb{R}^{+}$ la función Beta como:

$$\mathrm{B}(\alpha,\beta)=\int_0^1 x^{\alpha-1}\cdot(1-x)^{\beta-1}\cdot dx$$

Además, se cumple:

- $\mathrm{B}(1,1)=\int_0^1 dx.=1$.
- $\mathrm{B}(\alpha,1)=\int_0^1 x^{\alpha-1}\cdot dx.=\frac{1}{\alpha}, si\, \alpha>1$.
- $\mathrm{B}(1,\beta)=\int_0^1 (1-x)^{\beta-1}\cdot dx.=\frac{1}{\beta}, si\, \beta>1$.

Como:

$$\Gamma(\alpha)\cdot\Gamma(\beta)=\int_0^\infty u^{\alpha-1}\cdot e^{-u}\cdot du\cdot\int_0^\infty v^{\beta-1}\cdot e^{-v}\cdot dv=\int_0^\infty\int_0^\infty u^{\alpha-1}\cdot v^{\beta-1}\cdot e^{-(u+v)}\cdot du\cdot dv=$$

$$Haciendo\, el\, cambio: \left\{\begin{array}{ll} x=\dfrac{u}{u+v} & \Rightarrow u=x\cdot y \\ y=u+v & \Rightarrow y\, Si\, u\in(0,\infty) \end{array}\right\};$$

$$Además \left\{\begin{array}{ll} Si\, u\in(0,\infty) & \Rightarrow x\in(0,1) \\ Si\, v\in(0,\infty) & \Rightarrow y\in(0,\infty) \end{array}\right\}$$

- $$Y\ como \begin{vmatrix} \dfrac{du}{dx} & \dfrac{du}{dy} \\ \dfrac{dv}{dx} & \dfrac{dv}{dy} \end{vmatrix} = \begin{vmatrix} y & x \\ -y & 1-x \end{vmatrix} = y;$$

$Se\, cumple$:

$$\Gamma(\alpha)\cdot\Gamma(\beta)=\int_0^1\int_0^\infty x^{\alpha-1}\cdot(1-x)^{\beta-1}\cdot y^{\alpha+\beta-1}\cdot e^{-y}\cdot dy\cdot dx=\Gamma(\alpha+\beta)\cdot\int_0^1\int_0^\infty x^{\alpha-1}\cdot(1-x)^{\beta-1}\cdot dx=$$

$$=\Gamma(\alpha+\beta)\cdot\beta(\alpha,\beta)$$

$$\Rightarrow \beta(\alpha,\beta)=\frac{\Gamma(\alpha)\cdot\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

Si $x\in(0,1)$, denominamos Beta incompleta a la función:

$$\mathrm{B}(\alpha,\beta,x)=\int_0^x t^{\alpha-1}\cdot(1-t)^{\beta-1}\cdot dt$$