Grades 9-12 (A)

Duration: 40 min

Tools: one Logifaces block / student

Pair / Group work

Keywords: Probability, Favourable outcome, Total outcome

623 - Cutting along Edges

MATHS / PROBABILITY

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

2019-1-HU01-KA201-0612722019-1

DESCRIPTION

WARM-UP QUESTION

Imagine that we have a Logifaces block made of paper. We want to cut along a few edges to get a net of the block. How many edges should we cut along?

MAIN QUESTION

Choose 5 edges of a Logifaces block at random and cut along these edges. What is the probability that we get a net of the block?

SOLUTIONS / EXAMPLES

Any block can be chosen, because only the relative location of the faces is important. All the blocks have 9 types of nets (see exercise 510 - Nets of Prism).

WARM-UP QUESTION

To get a net, we need to cut along exactly 5 edges. Then there are four edges that are not cut.
One way to see that exactly 5 edges are needed to be cut is to check that this holds for each net of the blocks (see exercise 510 - Nets of Prism for the list of all nets).
We show another reasoning, because these observations help answer the main question.

- More than 5 edges cannot be cut, because at least 4 edges are needed not to be cut. To see this, one can think reversed: one edge is needed to glue two faces together to form a polygon. Adding one more face needs one more edge to glue along. Altogether 4 edges are needed to glue together 5 faces to form a potential net of the block.

- Less than 5 edges cannot be cut, because at most 4 edges are needed not to be cut. To see this, we continue the reverse thinking: suppose that 5 faces are glued together along 4 edges to form a polygon. To add one more edge would mean to glue together 2 more edges of the polygon. This cannot be performed in such a way that it remains a 2 dimensional shape, hence it cannot be a net of the block.

In fact, each net is a polygon consisting of the 5 faces in such a way that they are glued together to form a connected polygon. Based on this, if we get a connected polygon after performing the five cuts, it must be a net.

MAIN QUESTION

Total outcome: 5 different edges can be chosen from the 9 edges of the block in $C_{9}^{5} = \frac{9\times8\times7\times6\times5}{5\times4\times3\times2\times1} = 126$ ways.

We calculate the number of favourable outcomes by counting the unfavourable outcomes first. The unfavourable outcomes are the cases when the paper falls into two separate parts. This happens when the cut edges form at least one cycle. The 5 cut edges can only form one cycle, and in each case, the paper falls into two parts consisting of different numbers of faces. We count the cases by the part consisting of fewer faces. In that way, the following list consists of every unfavourable outcome exactly once.

First, if all edges of a face are chosen, then this face is separated from the other faces:

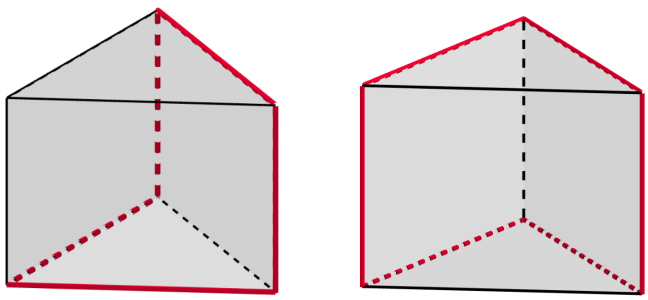
- A triangular face has 3 edges, and the remaining two edges are chosen from the other 6 edges. Since there are two triangular faces, this gives $2 \times C_{6}^{2} = 30$ cases.

- A quadrilateral face has 4 edges, and the remaining one edge is chosen from the other 5 edges. Since there are three quadrilateral faces, this gives $3 \times 5 = 15$ cases.

Second, when the part consisting of fewer faces consists of 2 adjacent faces:

- When this part consists of a triangular and a quadrilateral face, then every choice of a triangular and a quadrilateral face gives a proper case, see the diagram. This gives $2 \times 3 = 6$ cases.

- This part cannot consist of two adjacent quadrilateral faces, 6 cut edges would be needed to separate them from the other three faces, see the diagram.



The number of unfavourable outcomes is $30 + 15 + 6 = 51$.
Hence there are $126 - 51 = 75$ cases when cutting five edges at random results in a net, so the probability of this event is $\frac{75}{120} = 0.625$.

PRIOR KNOWLEDGE

The traditional model of probability, Net of a polyhedron

RECOMMENDATIONS / COMMENTS

Before the exercise, it is worth familiarising students with the nets of the blocks (see exercise 510 - Nets of Prism).
This exercise is suitable for differentiation and group work. The groups can be encouraged to first make a plan and then divide the calculation tasks between the members.