Lesson 13: Definition of standard form

Goals

- Identify (in writing) numbers written in standard form, and describe (orally) the features of an expression in standard form.

Learning Targets

- I can tell whether or not a number is written in standard form.

Lesson Narrative

In the previous few lessons, students have built familiarity with arithmetic involving powers of 10 to solve problems with very large and very small quantities. This lesson formalises what they have learned by introducing the definition of **standard form**. A number is said to be in standard form if it is written as a product of two factors: the first factor is a number greater than or equal to 1, but less than 10; and the second factor is an integer power of 10. This definition does not include negative numbers for simplicity. Students must attend to precision as they decide whether or not numbers are in standard form and convert to standard form.

Building On

- Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of each digit when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Addressing

- Perform operations with numbers expressed in standard form, including problems where both decimal and standard form are used. Use standard form and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimetres per year for seafloor spreading). Interpret standard form that has been generated by technology.

Building Towards

- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.

Instructional Routines

- Clarify, Critique, Correct
- Discussion Supports
- Number Talk

Required Materials

Pre-printed slips, cut from copies of the blackline master

Standard Form Matching - Set A 5×10^{5}	Standard Form Matching - Set A 5×10^{6}	Standard Form Matching - Set A 5×10^{-5}	Standard Form Matching - Set A 5×10^{-6}
Standard Form Matching - Set A 0.5×10^{6}	Standard Form Matching - Set A 50×10^{5}	Standard Form Matching - Set A 0.5×10^{-4}	Standard Form Matching - Set A 50×10^{-7}
Standard Form Matching - Set A 500 000	Standard Form Matching - Set A 5 000 000	Standard Form Matching - Set A 0.00005	Standard Form Matching - Set A 0.000005
Standard Form Matching - Set A 500 000	Standard Form Matching - Set A 5 000 000	Standard Form Matching - Set A 0.00005	Standard Form Matching - Set A 0.000005

Standard Form Matching - Set B 4.3×10^{7}	Standard Form Matching - Set B 4.3×10^{4}	Standard Form Matching - Set B 0.43×10^{-1}	Standard Form Matching - Set B 4.3×10^{-7}
Standard Form Matching - Set B 4.3×10^{5}	Standard Form Matching - Set B 4.3×10^{2}	Standard Form Matching - Set B 4.3×10^{-5}	Standard Form Matching - Set B 4.3×10^{-4}
Standard Form Matching - Set B 43×10^{4}	Standard Form Matching - Set B 0.43×10^{3}	Standard Form Matching - Set B 0.43×10^{-4}	Standard Form Matching - Set B 43×10^{-3}
Standard Form Matching - Set B 43 000	Standard Form Matching - Set B 43 000 000	Standard Form Matching - Set B 0.00000043	Standard Form Matching - Set B 0.00043
Standard Form Matching - Set B 430 000	Standard Form Matching - Set B 430	Standard Form Matching - Set B 0.000043	Standard Form Matching - Set B 0.043
Standard Form Matching - Set B 430 000	Standard Form Matching - Set B 430	Standard Form Matching - Set B 0.000043	Standard Form Matching - Set B 0.043

Standard Form Matching - Set C 6.3×10^4	Standard Form Matching - Set C 6.3×10^5	Standard Form Matching - Set C 6.3×10^{-4}	Standard Form Matching - Set C 6.3×10^{-5}
Standard Form Matching - Set C 6.3×10^3	Standard Form Matching - Set C 6.3×10^6	Standard Form Matching - Set C 6.3×10^{-3}	Standard Form Matching - Set C 6.3×10^{-6}
Standard Form Matching - Set C 0.63×10^4	Standard Form Matching - Set C 63×10^5	Standard Form Matching - Set C 0.63×10^{-5}	Standard Form Matching - Set C 63×10^{-4}
Standard Form Matching - Set C 63 000	Standard Form Matching - Set C 630 000	Standard Form Matching - Set C 0.00063	Standard Form Matching - Set C 0.000063
Standard Form Matching - Set C 6 300	Standard Form Matching - Set C 6 300 000	Standard Form Matching - Set C 0.0063	Standard Form Matching - Set C 0.0000063
Standard Form Matching - Set C 6 300	Standard Form Matching - Set C 6 300 000	Standard Form Matching - Set C 0.0063	Standard Form Matching - Set C 0.0000063

Required Preparation

The blackline master for Standard form Matching has three sets of cards. Set A is for the teacher to demonstrate the process, so only one copy of set A is needed. Cut out one set of cards (either set B or set C) for every 2 students. If possible, copy each complete set on a different colour of paper, so that a stray slip can quickly be put back.

Student Learning Goals

Let's use standard form to describe large and small numbers.

13.1 Number Talk: Multiplying by Powers of 10

Warm Up: 5 minutes

The purpose of this Number Talk is to elicit strategies and understandings students have for multiplying by a power of 10. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to work with numbers in standard form. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the talk. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Find the value of each expression mentally.

$123 \times 10\,000$

$(3.4) \times 1\,000$

$(0.6) \times 100$

$(7.3) \times (0.01)$

Student Response

Explanations vary. Sample responses:

- $123 \times 10^4 = 1\,230\,000$ because multiplying by 10^4 puts 4 more decimal places left of the decimal point.
- $(3.4) \times 1\,000 = 3\,400$ because multiplying by $1\,000$ puts 3 more decimal places left of the decimal point.
- $(0.6) \times 100 = 60$ because multiplying by 100 puts 2 more decimal places left of the decimal point.
- $(7.3) \times (0.01) = 0.073$ because multiplying by 0.01 puts 2 more decimal places right of the decimal point.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. After the last problem, ask students, "How could we rewrite each expression as a product of a number and a power of 10?" Record and display their responses next to each of the original expressions for all to see.

To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

13.2 The Structure of Standard form

15 minutes

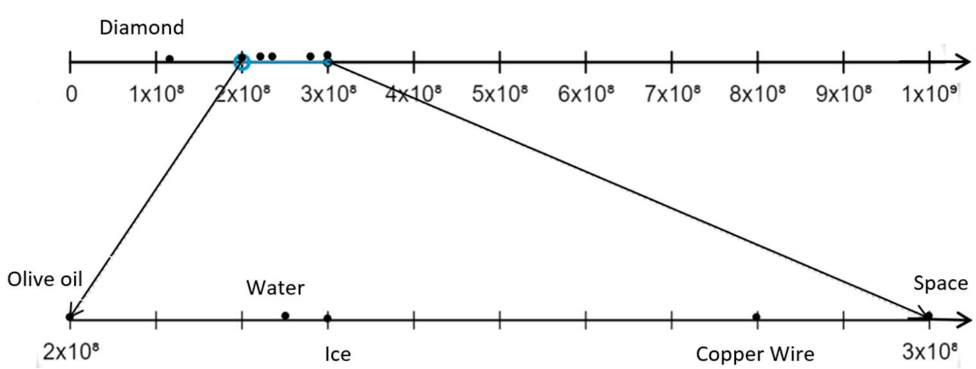
Students learn the definition of standard form and practise using it. Students attend to precision when determining whether or not a number is in standard form and converting numbers into standard form.

Instructional Routines

- Clarify, Critique, Correct

Launch

Tell students, "Earlier, we examined the speed of light through different materials. We zoomed into the number line to focus on the interval between 2.0×10^8 metres per second and 3.0×10^8 metres per second as shown in the figure." Display the following image notation for all to see.



Tell students, "We saw that the speed of light through ice was 2.3×10^8 metres per second. This way of writing the number is called *standard form*. Standard form is useful for understanding very large and very small numbers."

Display and explain the following definition of standard form for all to see.

A number is said to be in standard form when it is written as a product of two factors:

- The first factor is a number greater than or equal to 1, but less than 10, for example 1.2, 8, 6.35, or 2.008.
- The second factor is an integer power of 10, for example 10^8, 10^{-4}, or 10^{22}.

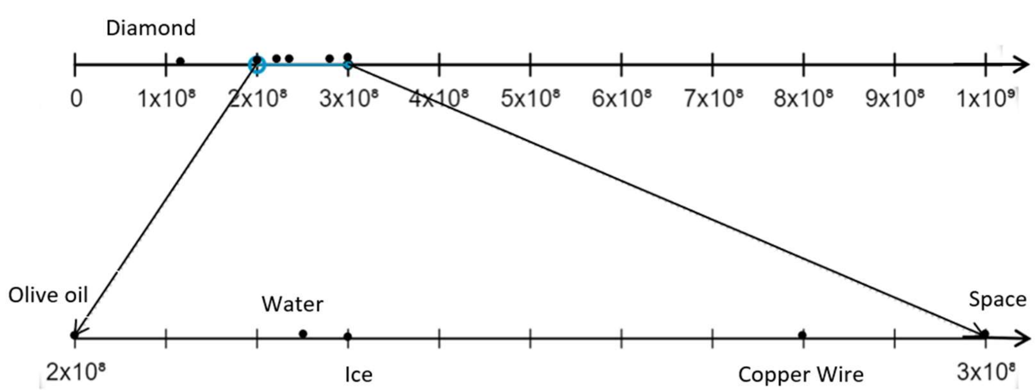
Carefully consider the first question and go through the list of numbers as a class, frequently referring to the definition to decide whether the number is written in standard form. When all numbers written in standard form have been circled,
consider demonstrating or discussing how a number that was not circled could be written in standard form. Then, ask students to complete the second question (representing the other numbers in standard form). Leave 3–4 minutes for a whole-class discussion.

Student Task Statement

The table shows the speed of light or electricity through different materials.

material	speed (metres per second)
space	300 000 000
water	2.25×10^8
copper (electricity)	280 000 000
diamond	124×10^6
ice	2.3×10^8
olive oil	0.2×10^9

Circle the speeds that are written in standard form. Write the others using standard form.



Student Response

The speeds of light through water and ice are given in standard form.

The following are speeds of light through the material (metres per second):

- Space: 3×10^8
- Copper: 2.8×10^8
- Diamond: 1.24×10^8
- Olive oil: 2×10^8

Activity Synthesis

Ask students to come up with at least two examples of numbers that are *not* in standard form. Select responses that highlight the fact that the first factor must be between 1 and 10 and other responses that highlight that one of the factors must be an integer power of 10. Make sure students recognise what does and does not count as standard form.

Also make sure students understand how to write an expression that may use a power of 10 but is not in standard form as one that is in standard form. Consider using the speed of light through diamond as an example. Ask a series of questions such as:

- "In 124×10^6, how must we write the first factor for the expression to be in standard form?" (A number between 1 and 10, so 1.24 in this case)
- "How can we rewrite 124 as an expression that has 1.24?" (Write it as 1.24×100 or 1.24×10^2)
- "What is the equivalent expression in standard form?" $(1.24 \times 10^2) \times 10^6$, which is 1.24×10^8)

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following term and maintain the display for reference throughout the unit: standard form. Invite students to suggest language or diagrams to include on the display that will support their understanding of this term.
Supports accessibility for: Memory; Language Writing: Clarify, Critique, Correct. Display a hypothetical student statement that represents a misunderstanding about how to write values in standard form, such as: "To write the speed of light through a diamond is 12.4×10^7." Ask pairs of students to critique the response by asking, "Do you agree with the author? Why or why not?" Invite students to write feedback to the author that identifies the reasoning error and how to improve the statement. Listen for students who include in their feedback a need for the first factor to be between 1 and 10. This helps students evaluate, and improve on, the written mathematical arguments of others.
Design Principle(s): Maximise meta-awareness; Support sense-making

13.3 Standard form Matching

15 minutes

In this activity, students match cards written in standard form with their decimal values. The game grants advantage to students who distinguish between numbers written in standard form from numbers that superficially resemble standard form (e.g. 0.43×10^5).

Instructional Routines

- Discussion Supports

Launch

The blackline master has three sets of cards: set A, set B, and set C. Set A is meant for demonstration purposes, so only a single copy of set A is necessary.

Arrange students in groups of 2. Consider giving students a minute of quiet time to read the directions. Then, use set A to demonstrate a round of the game for the class. Explain to students that a match can be made by pairing any two cards that have the same value, but it is favourable to be able to tell the difference between numbers in standard form and numbers that simply look like they are in standard form.

When students indicate that they understand how to play, distribute a set of cards (either set B or set C) to each group. Save a few minutes for a whole-class discussion.

Action and Expression: Internalise Executive Functions. Begin with a whole-class think aloud to demonstrate the steps of the game. Consider providing some groups with cards that contain more accessible values to begin with.
Supports accessibility for: Memory; Conceptual processing Representing, Conversing: Discussion Supports. Demonstrate the steps of how to play the game. To do this, select a student to play the game with you while the rest of the class observes. This will help clarify the expectations of the task, invite more student participation, and facilitate meta-awareness of the language involving standard form.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

Your teacher will give you and your partner a set of cards. Some of the cards show numbers in standard form, and other cards show numbers that are not in standard form.

The reason you shout "Science" is that standard form is also called scientific notation.

1. Shuffle the cards and lay them facedown.

2. Players take turns trying to match cards with the same value.

3. On your turn, choose two cards to turn faceup for everyone to see. Then:

    a. If the two cards have the same value *and* one of them is written in standard form, whoever says "Science!" first gets to keep the cards, and it becomes that player's turn. If it's already your turn when you call "Science!", that means you get to go again. If you say "Science!" when the cards do not match or one is not in standard form, then your opponent gets a point.

b. If both partners agree the two cards have the same value, then remove them from the board and keep them. You get a point for each card you keep.

c. If the two cards do not have the same value, then set them face down in the same position and end your turn.

4. If it is not your turn:

   a. If the two cards have the same value *and* one of them is written in standard form, then whoever says "Science!" first gets to keep the cards, and it becomes that player's turn. If you call "Science!" when the cards do not match or one is not in standard form, then your opponent gets a point.

   b. Make sure both of you agree the cards have the same value.
   If you disagree, work to reach an agreement.

5. Whoever has the most points at the end wins.

Student Response

No response required. Sample pairs of cards in standard form and decimal:

Set B: 4.3×10^{-7} and 0.00000043; 4.3×10^{4} and 43 000; 4.3×10^{7} and 43 000 000; 4.3×10^{-4} and 0.00043; 4.3×10^{5} and 430 000; 4.3×10^{-5} and 0.000043; 4.3×10^{2} and 430; 4.3×10^{-2} and 0.043;

Not in standard form: 43×10^{4}; 0.43×10^{3}; 0.43×10^{-4}; 43×10^{-3}

Set C: 6.3×10^{4} and 63 000; 6.3×10^{5} and 630 000; 6.3×10^{-4} and 0.00063; 6.3×10^{-5} and 0.000063; 6.3×10^{3} and 6 300; 6.3×10^{6} and 6 300 000; 6.3×10^{-3} and 0.0063; 6.3×10^{-6} and 0.0000063

Not in standard form: 63×10^{5}; 0.63×10^{4}; 0.63×10^{-5}; 63×10^{-4}

Are You Ready for More?

1. What is $9 \times 10^{-1} + 9 \times 10^{-2}$? Express your answer as:

   a. A decimal

   b. A fraction

2. What is $9 \times 10^{-1} + 9 \times 10^{-2} + 9 \times 10^{-3} + 9 \times 10^{-4}$? Express your answer as:

a. A decimal

b. A fraction

3. The answers to the two previous questions should have been close to 1. What power of 10 would you have to go up to if you wanted your answer to be so close to 1 that it was only $\frac{1}{1\,000\,000}$ off?

4. What power of 10 would you have to go up to if you wanted your answer to be so close to 1 that it was only $\frac{1}{1\,000\,000\,000}$ off? Can you keep adding numbers in this pattern to get as close to 1 as you want? Explain or show your reasoning.

5. Imagine a number line that goes from your current position (labelled 0) to the door of the room you are in (labelled 1). In order to get to the door, you will have to pass the points 0.9, 0.99, 0.999, etc. The Greek philosopher Zeno argued that you will never be able to go through the door, because you will first have to pass through an infinite number of points. What do you think? How would you reply to Zeno?

Student Response

1. 0.99, $\frac{99}{100}$

2. 0.9999, $\frac{9\,999}{10\,000}$

3. 10^{-6}

4. 10^{-9}

   Yes. In the previous example, adding $9 \times 10^{-1} + \cdots + 9 \times 10^{-9}$ gave us a number that was 10^{-9} away from 1. In general, adding $9 \times 10^{-1} + \cdots + 9 \times 10^{-n}$ will be 10^{-n} away from 1, and we can choose n to make this distance as small as we want.

5. Answers vary. The goal is for students to think about and discuss the problem rather than coming to a substantive conclusion. Sample response: the points 0.9, 0.99, 0.999 get much closer together the farther we go in the sequence, and so the time it takes to pass each one will shrink accordingly.

Activity Synthesis

The main idea is for students to practise using the definition of standard form and flexibly convert numbers to standard form. Consider selecting students to explain how they could tell whether two cards had the same value and whether they were written in standard form.

Lesson Synthesis

The purpose of the discussion is to make sure that students understand the definition of standard form. Consider displaying student responses for all to see.

- "What are some examples of expressions that are in standard form? How can you tell they are in standard form?"
- "What are some examples of expressions that are *not* in standard form? Try to come up with examples that would test whether someone knows what standard form is."
- "How would you write a very small number like 0.000021 in standard form?" (2.1×10^{-5})
- "How would you write a very large number like 21 000 000 in standard form?" (2.1×10^{7})
- "Why might standard form be useful?"
- "Can you think of information in the real world that might be easier to work with in standard form?"

If time allows, arrange students in groups of 2 and ask students to create a small decimal or large number for a partner to rewrite with standard form.

13.4 Standard form Check

Cool Down: 5 minutes

Students convert numbers to standard form.

Student Task Statement

State whether each of the following is in standard form. If not, write it in standard form.

1. 5.23×10^{8}
2. 48 200
3. 0.00099
4. 36×10^{5}
5. 8.7×10^{-12}
6. 0.78×10^{-3}

Student Response

1. Already in standard form
2. 4.82×10^{4}

3. 9.9×10^{-4}

4. 3.6×10^{6}

5. Already in standard form

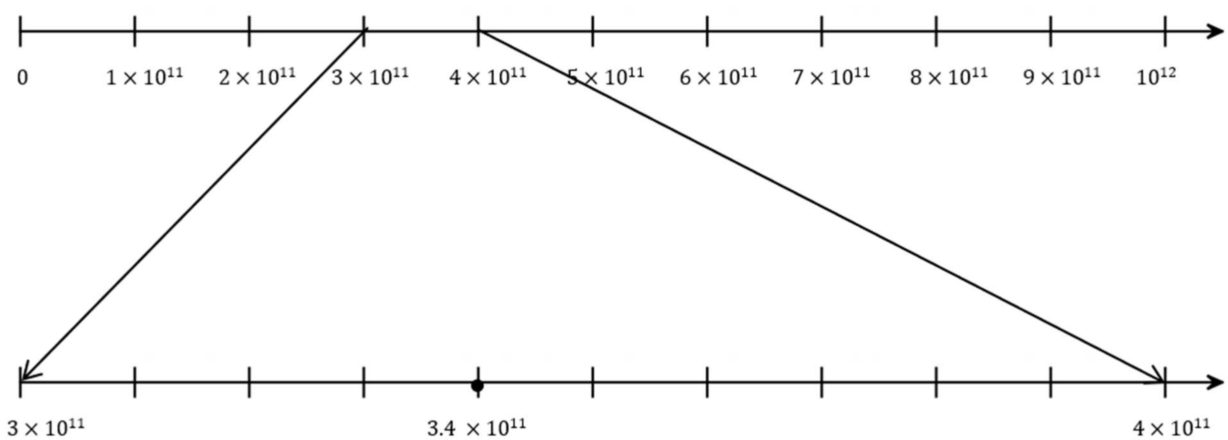
6. 7.8×10^{-4}

Student Lesson Summary

The total value of all the quarters made in 2014 is 400 million pounds. There are many ways to express this using powers of 10. We could write this as 400×10^6 pounds, 40×10^7 pounds, 0.4×10^9 pounds, or many other ways. One special way to write this quantity is called **standard form**. In standard form, 400 million pounds would be written as 4×10^8 pounds. Writing the number this way shows exactly where it lies between two consecutive powers of 10. The 10^8 shows us the number is between 10^8 and 10^9. The 4 shows us that the number is 4 tenths of the way to 10^9.

Some other examples of standard form are 1.2×10^{-8}, 9.99×10^{16}, and 7×10^{12}. The first factor is a number greater than or equal to 1, but less than 10. The second factor is an integer power of 10.

Thinking back to how we plotted these large (or small) numbers on a number line, standard form tells us which powers of 10 to place on the left and right of the number line. For example, if we want to plot 3.4×10^{11} on a number line, we know that the number is larger than 10^{11}, but smaller than 10^{12}. We can find this number by zooming in on the number line:



Glossary

- standard form

Lesson 13 Practice Problems

1. **Problem 1 Statement**

Write each number in standard form.

a. 14 700

b. 0.00083

c. 760 000 000

d. 0.038

e. 0.38

f. 3.8

g. 3 800 000 000 000

h. 0.0000000009

Solution

a. 1.47×10^4

b. 8.3×10^{-4}

c. 7.6×10^8

d. 3.8×10^{-2}

e. 3.8×10^{-1}

f. 3.8×10^0

g. 3.8×10^{12}

h. 9×10^{-10}

2. **Problem 2 Statement**

Perform the following calculations. Express your answers in standard form.

a. $(2 \times 10^5) + (6 \times 10^5)$

b. $(4.1 \times 10^7) \times 2$

c. $(1.5 \times 10^{11}) \times 3$

d. $(3 \times 10^3)^2$

e. $(9 \times 10^6) \times (3 \times 10^6)$

Solution

a. 8×10^5

b. 8.2×10^7

c. 4.5×10^{11}

d. 9×10^6

e. 2.7×10^{13}

3. **Problem 3 Statement**

Jada is making a scale model of the solar system. The distance from Earth to the Moon is about 2.389×10^5 miles. The distance from Earth to the Sun is about 9.296×10^7 miles. She decides to put Earth on one corner of her dresser and the Moon on another corner, about a foot away. Where should she put the sun?

- On a windowsill in the same room?
- In her kitchen, which is down the hallway?
- A city block away?

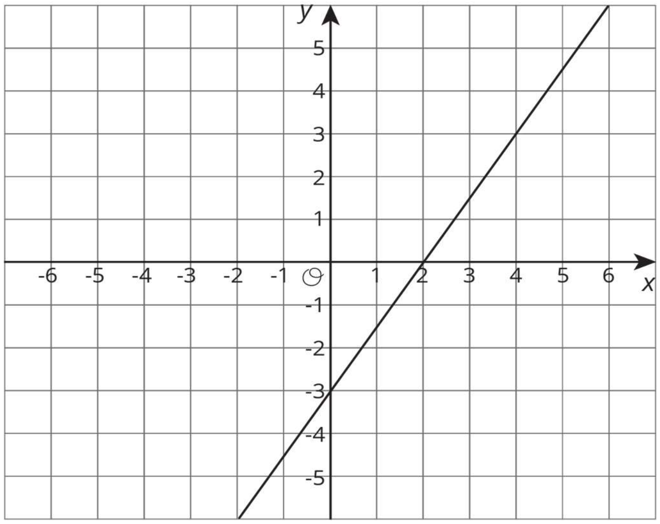
Explain your reasoning.

Solution

The model Sun should go down the block. Explanations vary. The distance from Earth to the Sun is about 4×10^2 or 400 times the distance from the Earth to the Moon. Since Jada's dresser is about a foot long, this means that her model Sun should be about 400 feet away from the dresser. Jada's house or apartment is probably not 400 feet long; a block away is about right.

4. **Problem 4 Statement**

Here is the graph for one equation in a system of equations.



a. Write a second equation for the system so it has infinitely many solutions.

b. Write a second equation whose graph goes through (0,2) so that the system has no solutions.

c. Write a second equation whose graph goes through (2,2) so that the system has one solution at (4,3).

Solution

a. $y = \frac{3}{2}x - 3$

b. $y = \frac{3}{2}x + 2$

c. $y = \frac{1}{2}x + 1$