Grades 9-12 (A)

Topic: Combinatorics

Duration: 30 min

Tools: one Logifaces Set / class

Individual / Pair work

Keywords: Counting principles, Logic

618 - Variety with Formulas

MATHS / COMBINATORICS

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

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DESCRIPTION

Logifaces blocks have 3 possible heights: 1, 2 and 3 standard units. In exercise 617 - Can you Match them? We counted all the different Logifaces blocks that are possible with these restraints.

Now students consider the number of possible different Logifaces blocks that have the following possible heights (but that are still triangular based prisms or truncated prisms):

LEVEL 1 1, 2, 3 and 4 standard units

LEVEL 2 1, 2, 3, …, 10 standard units

LEVEL 3 1, 2, 3, …, n standard units

Hint: as a Level 0 exercise first consider the original case, with 1, 2 or 3 standard units as heights

SOLUTIONS / EXAMPLES

LEVEL 0 n=3:

Number of different Logifaces blocks possible for 3 different heights, n=3

All heights are the same: $n = 3$

Two different heights: $3 \times 2 = 6$ (*first choose the two identical heights, then the third different one)*

Three different heights: $1 \times 2 = 2$ (*first choose the three different heights then the order)*

In total: $3 + 6 + 2 = 11$.

LEVEL 1 n=4:

Number of different Logifaces blocks possible for 4 different heights, n=4

All heights are the same: $n = 4$

Two different heights: $4 \times 3 = 12$ (*first choose the two identical heights, then the third different one)*

Three different heights: $C\ {}^{3}_{4} \times 2 = 4 \times 2 = 8$ (*first choose the three different heights then the order)*

In total: $4 + 12 + 8 = 24$.

The notation $C\ {}^{k}_{n}$ stands for the number of possibilities of choosing k elements from a fixed set of n elements. In particular, for $k = 3$, $C\ {}^{3}_{n} = \frac{n(n-1)(n-2)}{3\times2\times1}$.

LEVEL 2 n=10:

Number of different Logifaces blocks possible for 10 different heights, n=10

All heights are the same: $n = 10$

Two different heights: $n \times (n - 1) = 10 \times 9 = 90$ (*first choose the two identical heights, then the third different one)*

Three different heights: $C\ {}^{3}_{n} \times 2 = C\ {}^{3}_{10} \times 2 = 120 \times 2 = 240$ (*first choose the three different heights then the order)*

In total: $10 + 90 + 240 = 340$ or with formula: $n + n \times (n - 1) + C\ {}^{3}_{n} \times 2$

LEVEL 3 general n:

Therefore in general for n different heights the formula is $n + n \times (n - 1) + C\ {}^{3}_{n} \times 2$.

NOTES ON COUNTING

- Students do not need to know or use counting principles for the original question.

- They can systematically count the pieces based on multiple aspects, e.g. number of different heights (outlined above), sum of the heights (3 to 9), or the lowest height (1, 2 or 3)

PRIOR KNOWLEDGE

Counting principles

RECOMMENDATIONS / COMMENTS

It is recommended to first solve the exercise 617 - Can you Match them? where the task is to count the different possibilities.