Lesson 9: Drawing triangles (Part 1)

Goals

- Draw triangles with two given angle measurements and one side length, and describe (orally) how many different triangles could be drawn with the given conditions.
- Use drawings to justify (in writing) whether two given angle measurements and one side length determine one unique triangle.

Learning Targets

- Given two angle sizes and one side length, I can draw different triangles with these measurements or show that these measurements determine one unique triangle or no triangle.

Lesson Narrative

In the previous lesson, students were given collections of triangles and noticed that they shared angle and side measurements, and that sometimes there was more than one type of triangle with the same measurements. In this lesson and the next, they build on that experience by drawing their own triangles with specified measurements: a given angle, two given angles, and two given angles and a given side length. The purpose of the two lessons is to give students experience using various tools to draw triangles with given conditions, and to help them see that sometimes the given conditions allow only one possible triangle, sometimes more than one, and that sometimes none.

Addressing

- Draw, construct, and describe geometrical figures and describe the relationships between them.
- Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measurements of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Instructional Routines

- Compare and Connect
- Discussion Supports
- Which One Doesn't Belong?

Required Materials

Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Student Learning Goals

Let's see how many different triangles we can draw with certain measurements.

9.1 Which One Doesn't Belong: Triangles

Warm Up: 5 minutes

This warm-up prompts students to compare four images. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the images in comparison to one another. To allow all students to access the activity, each image has one obvious reason it does not belong. Encourage students to move past the obvious reasons (e.g., shape A has 3 equal angles) and find reasons based on geometrical properties (e.g., shape A is the only shape whose sides seem to have equal length). During the discussion, listen for important ideas and terminology that will be helpful in upcoming work of the lesson.

Instructional Routines

- Which One Doesn't Belong?

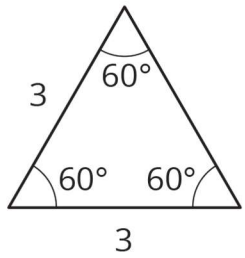
Launch

Arrange students in groups of 2–4. Display the image for all to see. Ask students to indicate when they have noticed one image that doesn't belong and can explain why. Give students 2 minutes of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask the group to offer at least one reason each image doesn't belong.

Student Task Statement

Which one doesn't belong?

A



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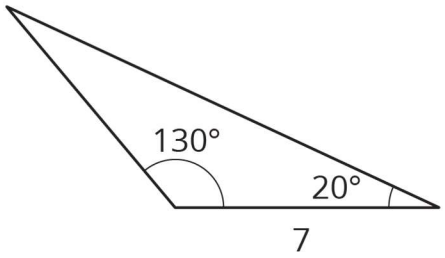

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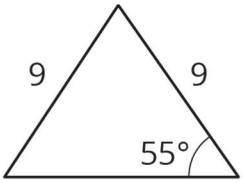

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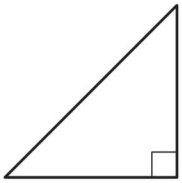


Student Response

Answers vary. Sample responses:

Shape 1 doesn't belong because:

- All the angles are equal to each other.
- All of the side lengths appear to be equal.

Shape 2 doesn't belong because:

- Only triangle with an obtuse angle.
- Only triangle with two angle measurements given.

Shape 3 doesn't belong because:

- Only triangle with two sides equal to each other (but not three)
- Only one angle measurement is given.

Shape 4 doesn't belong because:

- Only triangle with a right angle.
- No side lengths are given.

Activity Synthesis

Ask each group to share one reason why a particular image does not belong. Record and display the responses for all to see. After each response, poll the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct. During the discussion, ask students to explain the meaning of any terminology they use. Also, press students on unsubstantiated claims.

9.2 Does Your Triangle Match Theirs?

15 minutes (there is a digital version of this activity)

In this activity, students continue the work from the previous lesson by creating triangles from given conditions and seeing if it will match a given triangle. This activity transitions from students just noticing things about triangles already drawn to students drawing triangles themselves to test whether conditions result in unique triangles.

As student work on the task, monitor for students who draw different triangles than each other.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 2. Give students 3–5 minutes of quiet work time followed by time to check with their partner and discuss whether any of the triangles they drew are identical copies. Follow with whole-class discussion. Provide access to geometry toolkits, including rulers marked with centimetres and protractors.

Students using the digital version can create new triangles by dragging the vertices of the equilateral triangle in the applet. The measurements will be made for them, allowing them to focus on the new ideas.

Representation: Internalise Comprehension. Begin the activity with concrete or familiar contexts: Remind students to be measuring with centimetres and demonstrate how to use a protractor to draw the given angle.
Supports accessibility for: Conceptual processing; Memory

Anticipated Misconceptions

Students may have trouble recognising that Lin's triangle could have the pieces described in different orders. They are likely to immediately think of the side being between the two angles and not visualise other arrangements. Remind students of the task from the previous day and some of the triangles they saw there.

Student Task Statement

Three students have each drawn a triangle. For each description:

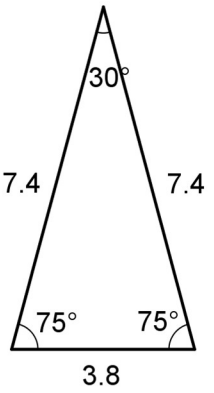
- Draw a triangle with the given measurements.
- Measure and label the other side lengths and angle sizes in your triangle.
- Decide whether the triangle you drew must be an identical copy of the triangle that the student drew. Explain your reasoning.

1. Jada's triangle has one angle measuring 75°.
2. Andre's triangle has one angle measuring 75° and one angle measuring 45°.
3. Lin's triangle has one angle measuring 75°, one angle measuring 45°, and one side measuring 5 cm.

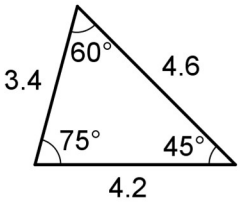
Student Response

Answers vary. Sample response:

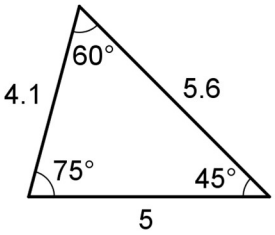
Jada's Triangle



Andre's Triangle



Lin's Triangle



None of the triangles are guaranteed to be identical copies.

- The description of Jada's triangle is very vague. You can choose lots of other angles and side lengths.
- The description of Andre's triangle makes it so you can't choose the third angle size (so all the drawings will be scaled copies), but you can still choose different sizes for the side lengths.
- The description of Lin's triangle might seem unique at first glance, but actually you could make any of the three sides be the 5 cm length, so you can still draw more than 1 triangle given these conditions.

Activity Synthesis

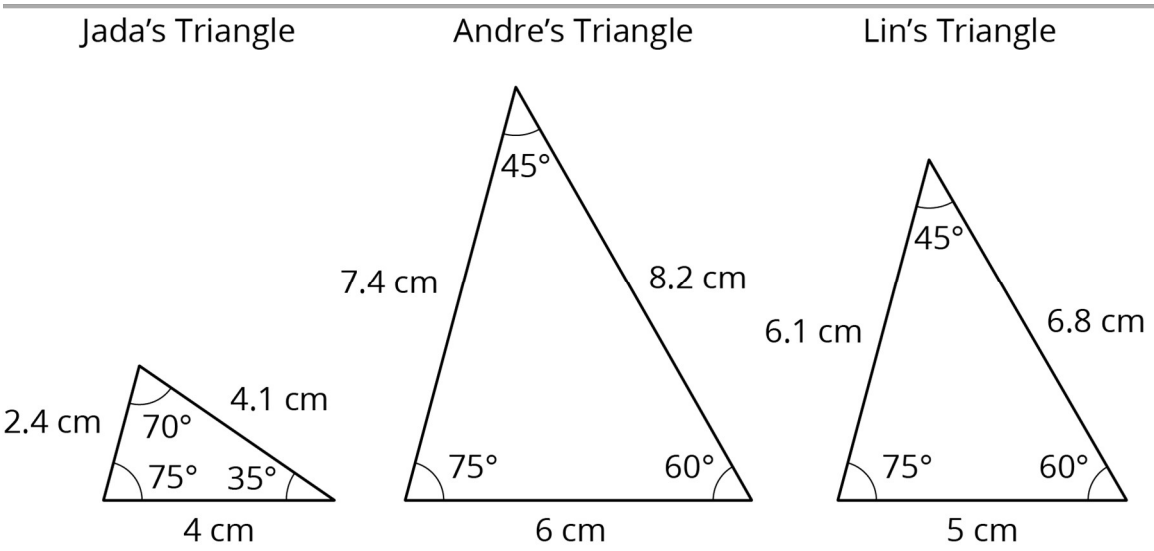
Select previously identified students to share their triangles.

To highlight the fact that there could be different triangles drawn, ask:

- "Did anybody draw a triangle that was identical to one drawn by their partner?"
- "Do we know enough about Jada's triangle to draw an identical copy of it? Andre's triangle? Lin's triangle?" (no)

If not mentioned by students, explain that it could be possible that we all drew identical copies for Lin's triangle (because it is most straightforward to draw the 5 cm side in between the 75° and 45° angles). However, that does not mean that we were given enough information about Lin's triangle to draw an identical copy of it. The problem did not say that we had to put the 5 cm side between those two angles.

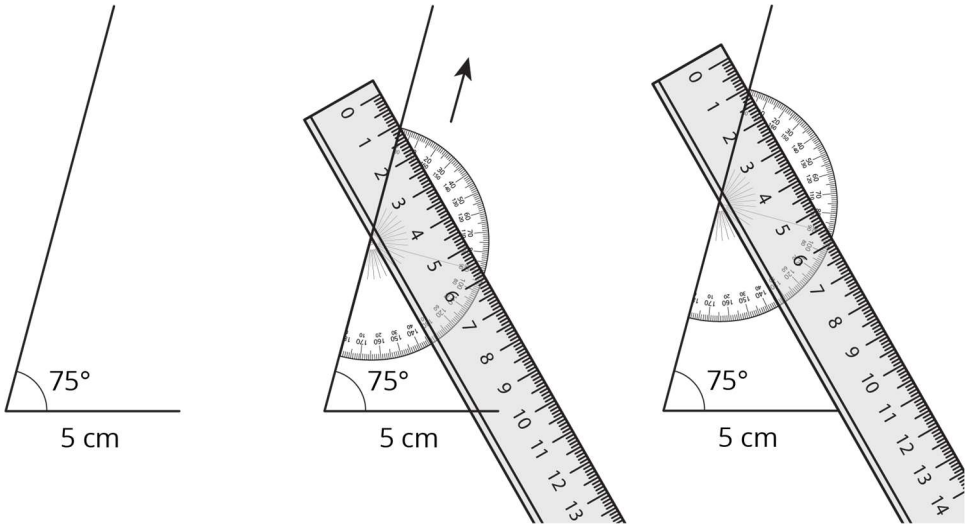
Display the image of Lin's triangle for all to see. Invite students to confirm that it matches the description of Lin's triangle. Ask whether any student drew an identical copy of Lin's triangle.



Introduce the word "unique." Explain to students that in all three cases, the information given is not enough to determine a unique triangle, not even for Lin's triangle, because there is more than one way we can draw a triangle with those given conditions. Ask students "what information would Lin have to give us to make the triangle unique (so we knew our drawing would be an identical copy of her triangle)?"

Before moving on to the next activity, it would be helpful to model how Lin drew her triangle:

1. Draw the 5 cm segment.
2. Draw the 75° angle on one end of the segment, with a very long ray.
3. Place a protractor along the ray.
4. Line up a ruler at the 45° measurement on the protractor.
5. Keeping the ruler and protractor together, slide them along the ray until the edge of the ruler intersects with the other end of the 5 cm segment.
6. Keeping the ruler in place on the paper, remove the protractor from underneath.
7. Draw a line along the ruler from the ray to the segment.



Speaking, Listening: Discussion Supports. Use this routine to help students explain whether any of the triangles they drew are identical copies to Jada's, Andre's or Lin's triangles. Provide sentence frames such as: "I noticed ___ so I …" ; "This triangle is/isn't identical because…." These help students use mathematical language related to triangles (e.g., angle, side) to reason about whether their triangle is identical to a given triangle.
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

9.3 How Many Can You Draw?

15 minutes (there is a digital version of this activity)

In this activity, students are asked to draw as many different triangles as they can with the given conditions. The purpose of this activity is to provide an opportunity for students to see the three main results for this unit: a situation in which only a unique triangle can be made, a situation in which it is impossible to create a triangle from the given conditions, and a situation in which multiple triangles can be created from the conditions.

Students are not expected to remember which conditions lead to which results, but should become more familiar with some methods for attempting to create different triangles. They will practise including various conditions into the triangles, including the conditions in different combinations, and recognising when the resulting triangles are identical copies or not.

Instructional Routines

- Compare and Connect

Launch

Keep students in same groups. Tell students they must try at least two different times to draw a triangle with the measurements given in each problem. Give students 5 minutes of quiet work time followed by time to discuss their different triangles with a partner. Follow with a whole-class discussion. Provide access to geometry toolkits.

Action and Expression: Internalise Executive Functions. Provide students with a printed graphic organiser to categorise the different triangles by condition.
Supports accessibility for: Language; Organisation

Anticipated Misconceptions

Some students may draw two different orientations of the same triangle for the third set of conditions, with the 4 cm side in between the 60° and 90° angles. Prompt them to use tracing paper to check whether their two triangles are really different (not identical copies).

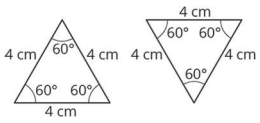
Some students may say the third set of measurements determines one unique triangle, because they assume the side length must go between the two given angle sizes. Remind them of the discussion about Lin's triangle in the previous activity.

Student Task Statement

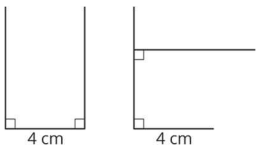
1. Draw as many different triangles as you can with each of these sets of measurements:
    a. Two angles measure 60°, and one side measures 4 cm.
    b. Two angles measure 90°, and one side measures 4 cm.
    c. One angle measures 60°, one angle measures 90°, and one side measures 4 cm.
2. Which of these sets of measurements determine one unique triangle? Explain or show your reasoning.

Student Response

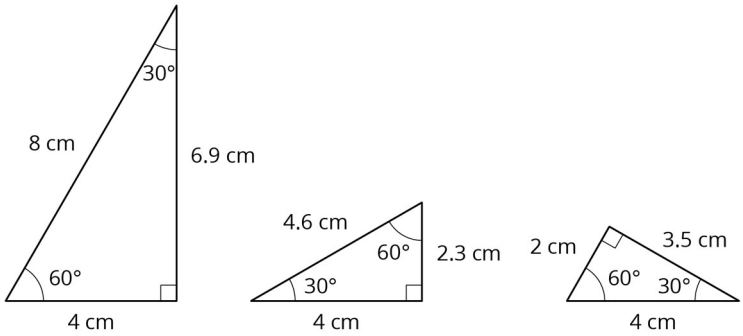
1. Answers vary. Sample responses:
    a. Two orientations of the same triangle.



    b. Two attempts to draw a triangle with two 90° angles and a 4 cm side. There is no possible triangle with these conditions.

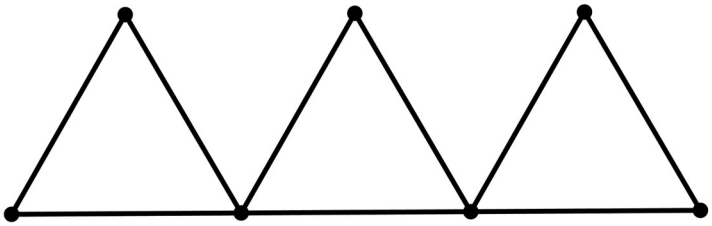


    c. Three different triangles can be made with the conditions.



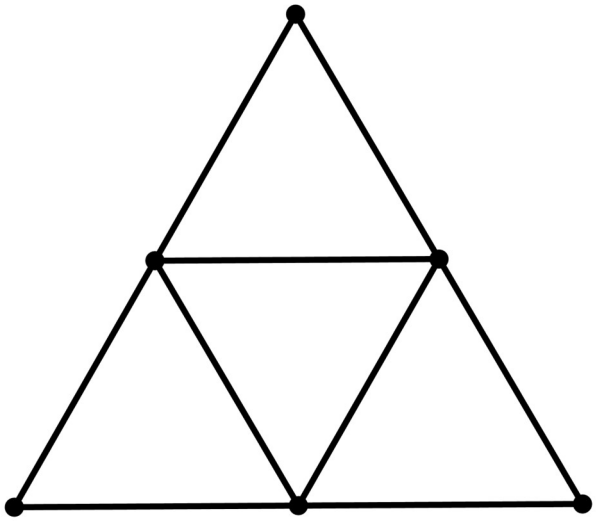
2. Only the first set of measurements determine a unique triangle.

Are You Ready for More?



In the diagram, 9 toothpicks are used to make three equilateral triangles. Figure out a way to move only 3 of the toothpicks so that the diagram has exactly 5 equilateral triangles.

Student Response



There are four small equilateral triangles and one large one.

Activity Synthesis

Ask students to indicate how many *different* triangles (triangles that are not identical copies) they could draw for each set of conditions. Select students to share their drawings

and reasoning about the uniqueness of each problem. Discuss methods students used to try to think about other triangles that might fit the conditions.

Consider asking some of the following questions:

- "Which conditions produced a unique triangle?" (the first set of conditions)
- "Were there conditions that produced more than one triangle?" (the third set of conditions)
- "Were there conditions you could not draw a triangle for?" (the second set of conditions)
- "Why could you not draw a triangle for the second set of conditions?" (because two sides are parallel and will never intersect)

If not mentioned by students, explain to students that for the third set of conditions it is possible that all students drew identical copies using the 4 cm length as the side between the 60° and 90° angles. Consider asking them to think of the previous activity and to try to draw the triangle the way Lin would.

Speaking: Compare and Connect. Use this routine to compare and contrast the different ways students reasoned about the uniqueness of the constructed triangles. Ask students to consider what is the same and what is different about the triangles produced for each condition. Draw students' attention to the association between the conditions given and the ability to construct unique, many, or no triangles. In this discussion, model the language used to make sense of the conditions that resulted in the three different scenarios. These exchanges strengthen students' mathematical language use and supports them to compare geometric shapes.
Design Principle(s): Maximise meta-awareness; Support sense-making

Lesson Synthesis

- Sometimes a set of conditions result in a unique triangle. What other results can come from a set of conditions? (It could be impossible or make multiple triangles.)
- If you are given a side length and two angles, what would you do to try to get started making different triangles? (Draw a line segment with the given length and put the two angles on each end. Then I would try leaving one angle on one end, but using Lin's method of using a protractor and sliding it along for the other angle to create a triangle. Finally, I would do something similar, but switch which angle is next to the given length.)

9.4 Checking Diego's Triangle

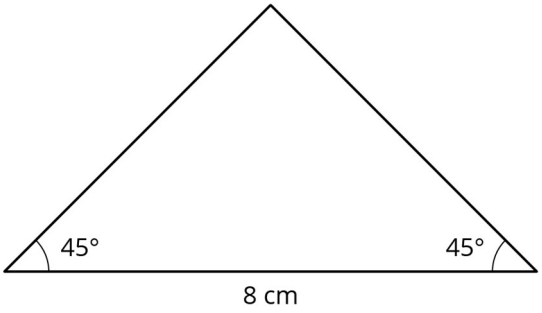
Cool Down: 5 minutes

Anticipated Misconceptions

Students may say that they do not agree with Diego's triangle, because the side length labelled 8 cm does not print at exactly 8 cm.

Student Task Statement

When asked to draw a triangle with two 45° angles and a side length of 8 cm, Diego drew this triangle.



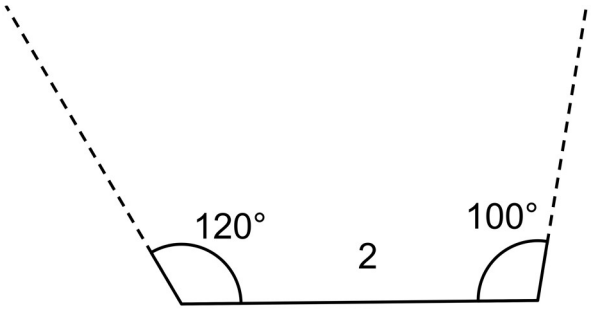
1. Do you agree with Diego's answer?
2. Is there a different triangle Diego could have drawn that would answer the question? Explain or show your reasoning.

Student Response

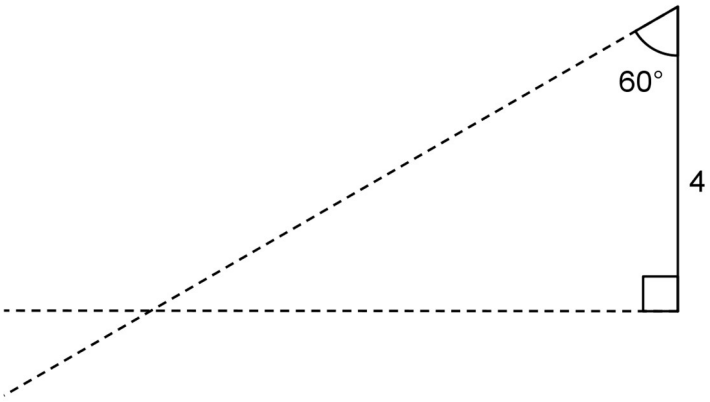
1. Yes, I agree that Diego's triangle has two 45° angles and a side length of 8 cm.
2. There is another possible triangle. Diego could keep one 45° angle next to the 8 cm side, but move the other one across from the 8 cm side.

Student Lesson Summary

Sometimes, we are given two different angle sizes and a side length, and it is impossible to draw a triangle. For example, there is no triangle with side length 2 and angle sizes 120° and 100°:



Sometimes, we are given two different angle sizes and a side length between them, and we *can* draw a unique triangle. For example, if we draw a triangle with a side length of 4 between angles 90° and 60°, there is only one way they can meet up and complete to a triangle:



Any triangle drawn with these three conditions will be identical to the one above, with the same side lengths and same angle sizes.

Lesson 9 Practice Problems

1. **Problem 1 Statement**

   Use a protractor to try to draw each triangle. Which of these three triangles is impossible to draw?

   a. A triangle where one angle measures 20° and another angle measures 45°

   b. A triangle where one angle measures 120° and another angle measures 50°

   c. A triangle where one angle measures 90° and another angle measures 100°

   Solution

   It is impossible to draw a triangle where one angle measures 90° and another angle measures 100°.

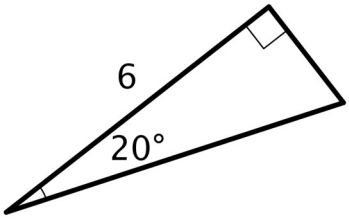
2. **Problem 2 Statement**

   A triangle has an angle measuring 90°, an angle measuring 20°, and a side that is 6 units long. The 6-unit side is in between the 90° and 20° angles.

   a. Sketch this triangle and label your sketch with the given measurements.

   b. How many unique triangles can you draw like this?

   Solution

a. 

b. There is only one triangle that fits this description, so long as the 6-unit side is between the two given angles.

3. **Problem 3 Statement**

   a. Find a value for x that makes $-x$ less than $2x$.

   b. Find a value for x that makes $-x$ greater than $2x$.

   Solution

   Answers vary. Sample response:

   1. 1, because -1 is less than 2×1.

   2. -3, because 3 is greater than 2×-3.

4. **Problem 4 Statement**

   One of the particles in atoms is called an electron. It has a charge of -1. Another particle in atoms is a proton. It has charge of +1.

   The overall charge of an atom is the sum of the charges of the electrons and the protons. Here is a list of common elements.

	charge from electrons	charge from protons	overall charge
carbon	-6	+6	0
aluminium	-10	+13	
phosphide	-18	+15	
iodide	-54	+53	
tin	-50	+50	

   Find the overall charge for the rest of the atoms on the list.

   Solution

Aluminium: $(-10) + (+13) = +3$

Phosphide: $(-18) + (+15) = -3$

Iodide: $(-54) + (+53) = -1$

Tin: $(-50) + (+50) = 0$

5. **Problem 5 Statement**

A factory produces 3 bottles of sparkling water for every 7 bottles of still water. If those are the only two products they produce, what percentage of their production is sparkling water? What percentage is still?

Solution

30% of the production is sparkling water. 70% of the production is still water.