

## Lesson 10: Using data displays to find associations

### Goals

- Create a two-way table and a compound bar graph that represent relative frequencies, and interpret (orally) the frequencies in context.
- Determine (in writing) whether categorical/qualitative data has a positive, negative, or no association using a relative frequency table or compound bar graph, and justify (orally) the reasoning. Association is used here as correlation has not yet been formally defined.

### Learning Targets

- I can create relative frequency tables, bar graphs, and compound bar graphs from frequency tables to find associations among variables.

### Lesson Narrative

In this lesson, students use two-way tables, bar graphs, and compound bar graphs to decide whether there is evidence of an association in categorical/qualitative data or not.

### Addressing

- Understand that patterns of association can also be seen in bivariate categorical/qualitative data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarising data on two categorical/qualitative variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

### Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

### Required Materials

#### Coloured pencils

#### Straightedges

A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

### Required Preparation

Use the data from the previous lesson's cool-down to build a two-way table of students' responses. Provide access to materials for students to create their own compound bar graphs including coloured pencils and straightedges.

## Student Learning Goals

Let's use data displays to find associations.

### 10.1 Sports and Musical Instruments

#### Warm Up: 5 minutes

The purpose of this warm-up is for students to answer questions about relative frequency of items after finding missing information in a two-way table.

Monitor for students who find the percentages for the final two questions using different strategies to share during the whole-class discussion.

#### Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

#### Student Task Statement

For a survey, students in a class answered these questions:

- Do you play a sport?
  - Do you play a musical instrument?
1. Here is a two-way table that gives some results from the survey. Complete the table, assuming that all students answered both questions.

	plays instrument	does not play instrument	total
plays sport	5		16
does not play sport			
total		15	25

2. To the nearest whole percentage, what percentage of students who play a sport *don't* play a musical instrument?
3. To the nearest whole percentage, what percentage of students who *don't* play a sport also *don't* play a musical instrument?

#### Student Response

1.

	plays instrument	does not play instrument	total
plays sport	5	11 since $16 - 5 = 11$ .	16

does not play sport	5 since $9 - 4 = 5$ .	4 since $15 - 11 = 4$ .	9 since $25 - 16 = 9$ .
total	10 since $25 - 15 = 10$ .	15	25

2. 69% because  $\frac{11}{16} = 0.6875$ .

3. 44% because  $\frac{4}{9} = 0.444\dots$

### Activity Synthesis

Ask students to share the missing information they found for the table. Record and display their responses for all to see.

Select students previously identified to explain how they found the percentages for the final two questions and what that percentage represents.

- Students who find a percentage using the values given (for example 31% since  $\frac{5}{16} \approx 0.31$ ), then subtract from 100% (for example 69% since  $100 - 31 = 69$ ) to answer the question.
- Students who find the actual values first by subtracting (for example  $16 - 5 = 11$ ) then calculate the percentage (for example 69% because  $\frac{11}{16} = 0.6875$ ).

Ask the rest of the class if they agree or disagree with the strategies and give time for any questions they have.

## 10.2 Sports and Music Association

### 20 minutes

Now that students are more familiar with two-way tables showing relative frequency, they are ready to create their own compound bar graphs. In this activity, students create two compound bar graphs based on the same two-way table by considering percentages of the rows and columns separately. After creating the compound bar graphs, they are analysed to determine if there is an association present in the data.

#### Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

#### Launch

Arrange students in groups of 2. After a brief introduction, give 5–10 minutes of quiet work time. Ask students to compare their answers with their partner and try to resolve any differences. Finish with a whole-class discussion.

Display the two-way table from the previous lesson's cool-down activity containing the data collected about the class's playing sports and musical instruments. If the data is unavailable, the data from this lesson's warm-up can be used.

Tell students they should work with their partners to each work on one of the graphs. One student should work on problems 1 and 2 while their partner should work on 3 and 4. After they have completed their graphs, they should work together to understand their partners graphs and complete the last problem together.

*Action and Expression: Internalise Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organisation and problem solving. For example, present one question at a time. Some students may benefit from a checklist on how to create a compound bar graph.

*Supports accessibility for: Organisation; Attention*

### Anticipated Misconceptions

Students may draw the compound bar graph incorrectly. Most likely, they will accidentally graph frequency instead of relative frequency. They may also graph relative frequencies, but without stacking them. Both compound bars should go from 0 to 100.

### Student Task Statement

Your teacher will give you a two-way table with information about the number of people in your class who play sports or musical instruments.

1. Complete this table to make a two-way table for the data from earlier. The table will show relative frequencies *by row*.

	plays instrument	does not play instrument	row total
plays sport			100%
does not play sport			100%

2. Make a compound bar graph for the table. Use one bar of the graph for each row of the table.



3. Complete the table to make a two-way table for the data from earlier. The table will show relative frequencies *by column*.

	plays instrument	does not play instrument
plays sport		
does not play sport		
column total	100%	100%

4. Using the values in the table, make a compound bar graph. Use one bar of the graph for each column of the table.



5. Based on the two-way tables and compound bar graphs, do you think there is an association between playing a sport and playing a musical instrument? Explain how you know.

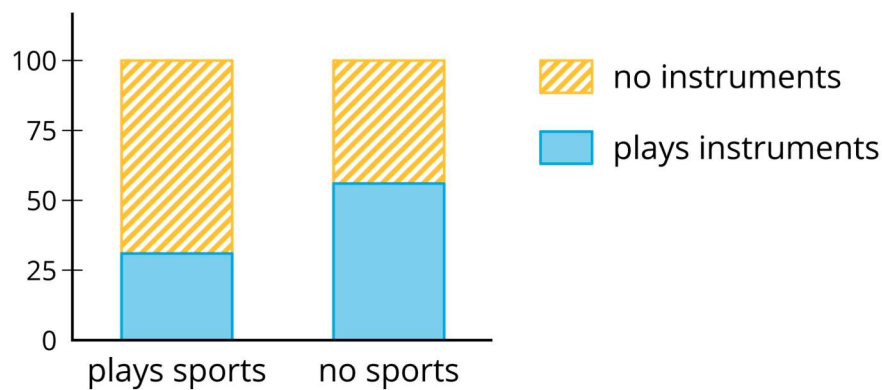
**Student Response**

Answers vary based on class data from previous cool-down. For the warm-up data:

1.

	plays instrument	does not play instrument	row total
plays sport	31%, since $6 \div 16 = 0.3125$	69%, since $11 \div 16 = 0.6875$	100%
does not play sport	56%, since $5 \div 9 = 0.5$	44%, since $4 \div 9 = 0.4$	100%

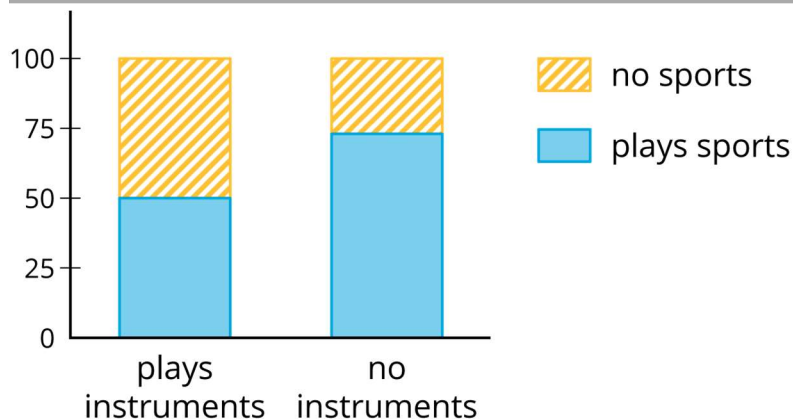
2.



3.

	plays instrument	does not play instrument
plays sport	50%, since $5 \div 10 = 0.5$ .	73%, since $11 \div 15 = 0.73$ .
does not play sport	50%, since $5 \div 10 = 0.5$ .	27%, since $4 \div 15 = 0.26$ .
column total	100%	100%

4.



5. I think there is a negative association between playing a sport and playing an instrument. Students who play a sport are less likely than their other classmates to play an instrument, and students who play an instrument are less likely than their other classmates to play a sport.

### Activity Synthesis

To clarify how to create and interpret compound bar graphs, ask:

- “What different information can be seen by the two compound bar graphs?”
- “Why are the numbers in the top left box in the two tables different? What do they mean?” (In the first table it represents the percentage who also play musical instruments out of all the people who play sports. In the second table it represents the percentage of people who also play sports out of all the people who play musical instruments.)
- “Is there an association between the two variables? Explain or show your reasoning.” (The answer will depend on class data, but the reasoning should include an analysis of the relative frequencies within categories. There is an association if the percentages within one category are very different from the percentages in another category.)

If there is an association, ask what the compound bar graphs would look like if there was no association. If there is not an association, ask what the compound bar graphs would look like if there was one.

*Writing, Speaking: Stronger and Clearer Each Time.* Use this routine to give students a structured opportunity to revise and refine their response to the last question. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., “Why do you think there is a (positive/negative) association?”, “How do the relative frequencies help to answer this question?”, “How could you say that another way?”, etc.). Students can borrow ideas and language from each partner to strengthen the final product. They can return to the first partner and revise and refine their initial response.

*Design Principle(s): Optimise output (for explanation)*

## 10.3 Coloured Erasers

### 15 minutes

This activity provides students less structure for their work in creating compound bar graphs to determine an association. In addition, the data in this activity is split into more than two options. Students work individually to create a compound bar graph based on either columns or rows and then share their information with a partner who has created the other compound bar graph. Together, partners discuss the compound bar graphs to determine if there is an association between the variables. In particular, students should notice that there is evidence of an association is the relative frequencies within a category are very different from the relative frequencies in another category.

As students work, identify groups that use the different compound bar graphs to explain why there is an association between the colour of the eraser and flaws.

### Instructional Routines

- Think Pair Share

### Launch

Keep students in groups of 2. Give 5 minutes quiet work time followed by 5 minutes of partner discussion and then a whole-class discussion.

Provide students access to coloured pencils. Either assign or have partners choose which will make a graph for each row and which will make a graph for each column.

*Representation: Access for Perception.* Read the directions aloud. Students who both listen to and read the information will benefit from extra processing time. Check for understanding by inviting students to rephrase directions in their own words.

*Supports accessibility for: Language*

### Student Task Statement

An eraser factory has five machines. One machine makes the eraser shapes. Then each shape goes through the red machine, blue machine, yellow machine, or green machine to have a side coloured.

The manager notices that an uncoloured side of some erasers is flawed at the end of the process and wants to know which machine needs to be fixed: the shape machine or some of the colour machines. The manager collected data on the number of flawed and unflawed erasers of each colour.

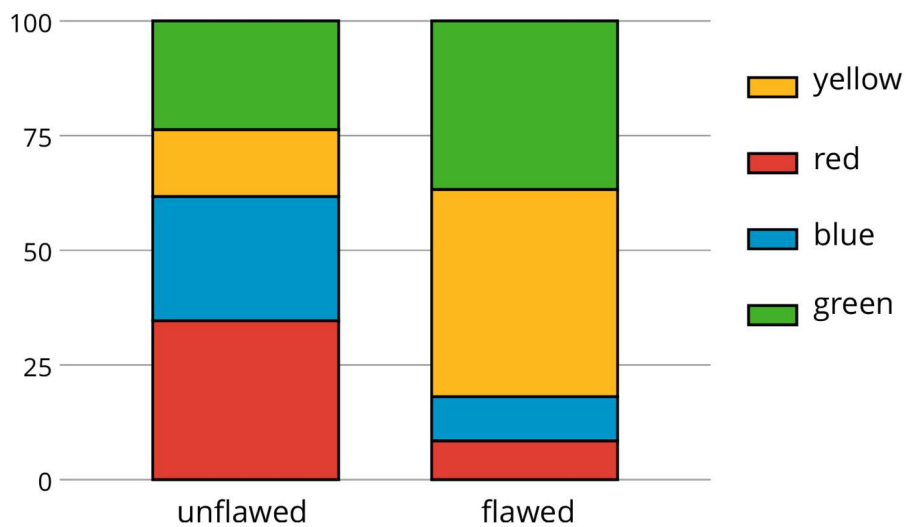
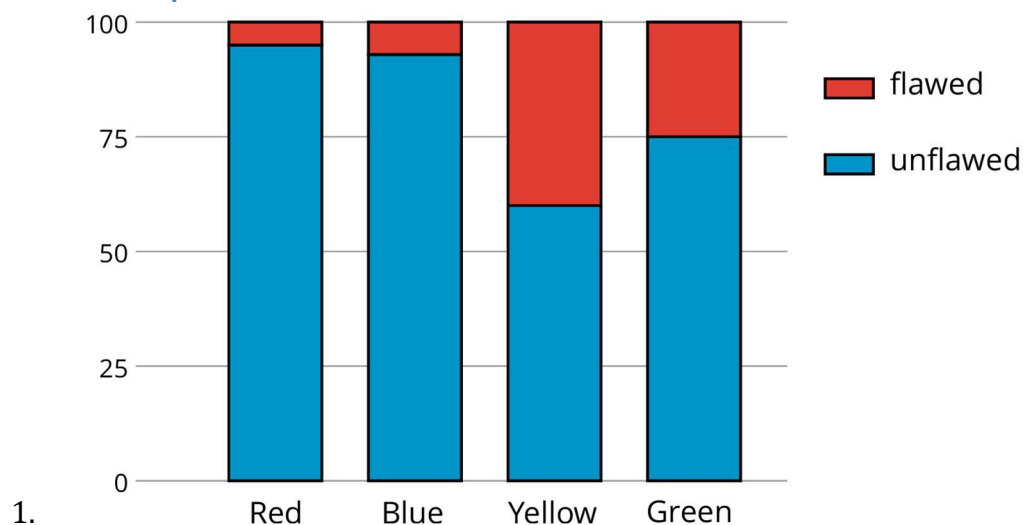
	unflawed	flawed	total
red	285	15	300
blue	223	17	240
yellow	120	80	200



green	195	65	260
total	823	177	1000

1. Work with a partner. Each of you should make one compound bar graph for the data in the table. One compound bar graph should have a bar for each *row* of the table. The other compound bar graph should have one bar for each *column* of the table.
2. Are the flawed erasers associated with certain colours? If so, which colours? Explain your reasoning.

**Student Response**



- The flawed erasers are positively associated with the yellow and green erasers. A much higher percentage of the total yellow and green erasers are flawed than red and blue are flawed. The yellow and green erasers also make up a larger percent of the total flawed erasers.

### Are You Ready for More?

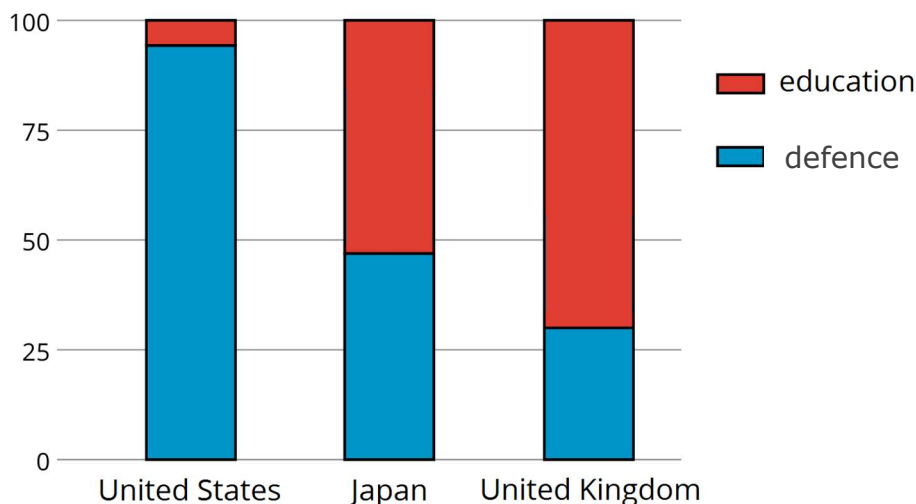
Based on the federal budgets for 2009, the table shows where some of the federal money was expected to go. The values are in billions of U.S. Dollars.

	United States	Japan	United Kingdom
defence	718.4	42.8	49.2
education	44.9	47.5	113.9

- Why would a compound bar graph be more useful than the table of data to see any associations between the country and where the money is spent?
- Create a compound bar graph that represents the data from the table.
- Is there an association between the country's budget and their spending in these areas? Explain your reasoning.

### Student Response

- Answers vary. Sample response: The total amount of money spent by the United States is so much greater than the other countries that it would help to see the percentages rather than actual dollars spent.



- There is an association. The United States spends a much larger percentage of its budget on defence than on education relative to the other countries. Japan spends almost equal amounts on defence and education while the United Kingdom spends a lot more on education than on defence.

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## Activity Synthesis

The purpose of this discussion is to identify strategies for creating compound bar graphs and for analysing them to determine if there is an association among variables.

Ask, “What strategies did you use to create the compound bar graphs?” (First, we created a new table of the relative frequencies. Then we approximated the heights of the segments based on the percentages from the table.)

Select previously identified groups to share their explanation for noticing an association.

1. Groups that use the compound bar graph based on rows.
2. Groups that use the compound bar graph based on columns.

After both explanations are shared, ask students, “Do you think that noticing the association was easier with one of the graphs?” (Likely the compound bar graph based on rows is easier since there are only 2 segments and it is easier to see that the yellow and green erasers are more flawed.)

Finally, ask students, “If there was not an association between colour and flaws, what might the compound bar graph based on the rows look like? What might the compound bar graph based on the columns look like?” (The compound bar graph based on the rows would have each compound bar look about the same. That is, the line dividing the two segments would be at about the same height in each bar. The compound bar graph based on the columns would have segments that are all approximately equal. That is, each segment should represent about 25% of the entire bar.)

## Lesson Synthesis

Remind students that we have been looking for associations in categorical/qualitative data, and that there is evidence of an association if the relative frequencies of some characteristic are very different from each other in the different groups. Ask:

- “Is it easier to see evidence of an association in a frequency table or a *relative* frequency table?” (It depends on the data. If the two groups are approximately the same size, it doesn't matter very much, but when they are different sizes, it is usually easier to compare using relative frequencies.)
- “How can we see evidence of an association in a two-way table of either kind?” (By numerically comparing the proportions between the two groups.)
- “How can we see evidence of an association in a bar graph or compound bar graph?” (By visually comparing the proportions between the two groups.)

## 10.4 Class Preferences

**Cool Down: 5 minutes**

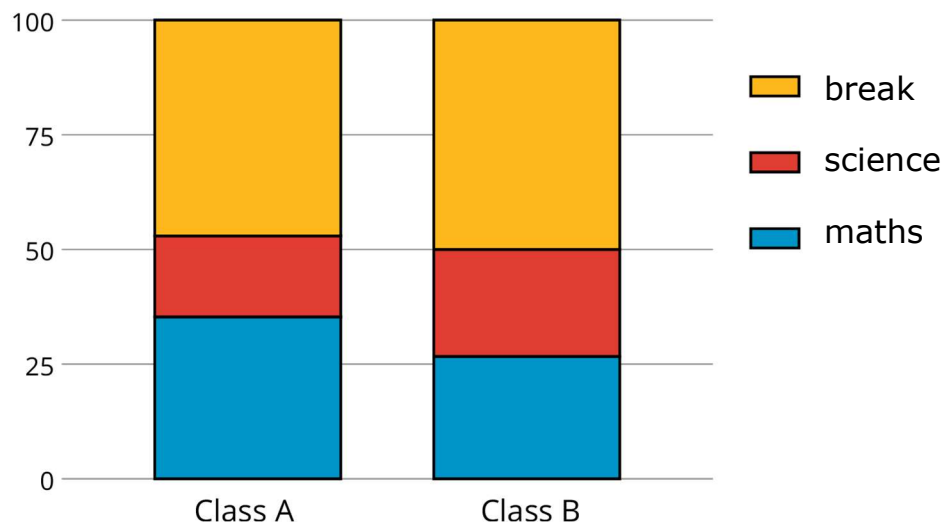
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Students are given a two-way table in which one variable has more than two outcomes and compound bar graph to determine if there is an association between the two variables.

### Student Task Statement

Here are a two-way table and compound bar graph for data from students in two classes. Do they show evidence of differences between the two classes?

	prefers maths	prefers science	prefers break
class A	6	3	8
class B	8	7	15

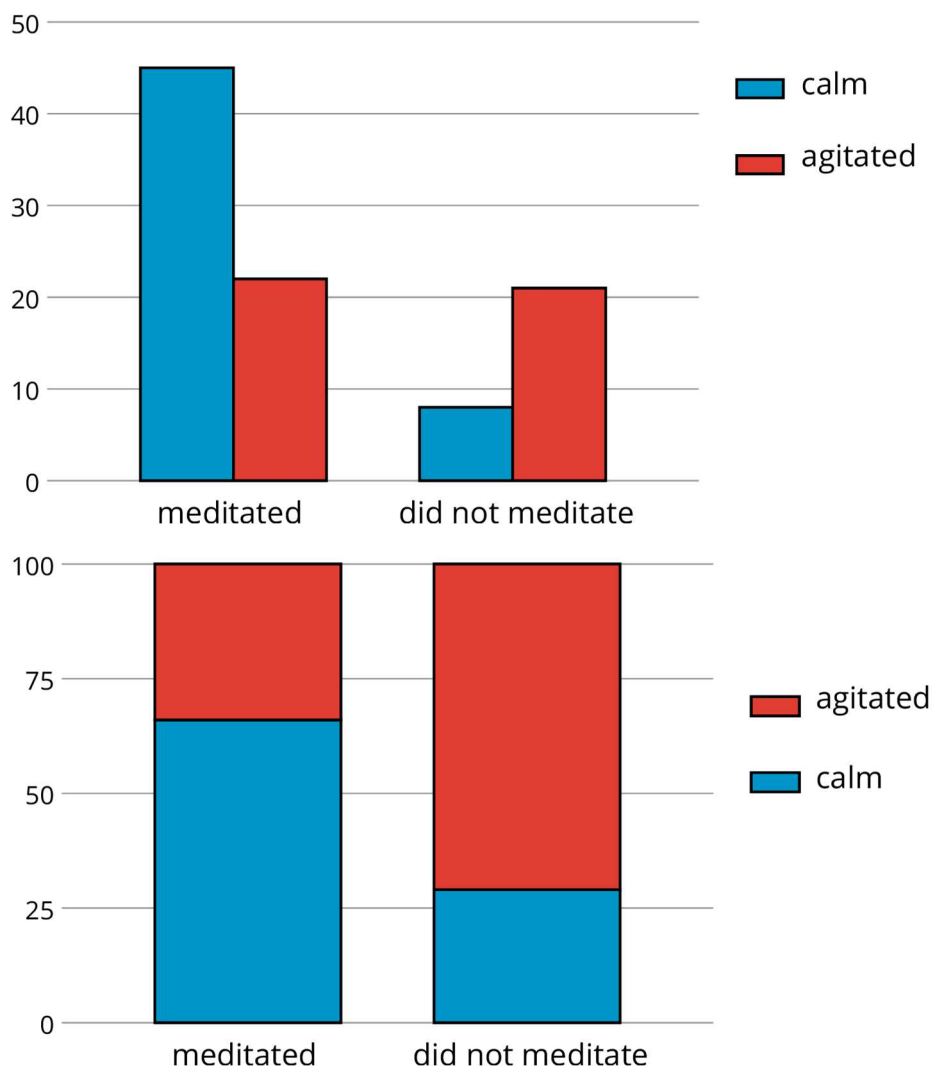


### Student Response

There is no evidence of different preferences associated with each class because the segments in the bars are about the same size.

### Student Lesson Summary

In an earlier lesson, we looked at data on meditation and state of mind in athletes.



Is there an association between meditation and state of mind?

The bar graph shows that more athletes were calm than agitated among the group that meditated, and more athletes were agitated than calm among the group that did not. We can see the proportions of calm meditators and calm non-meditators from the compound bar graph, which shows that about 66% of athletes who meditated were calm, whereas only about 27% of those who did not meditate were calm.

This does not necessarily mean that meditation causes calm; it could be the other way around, that calm athletes are more inclined to meditate. But it does suggest that there is an association between meditating and calmness.

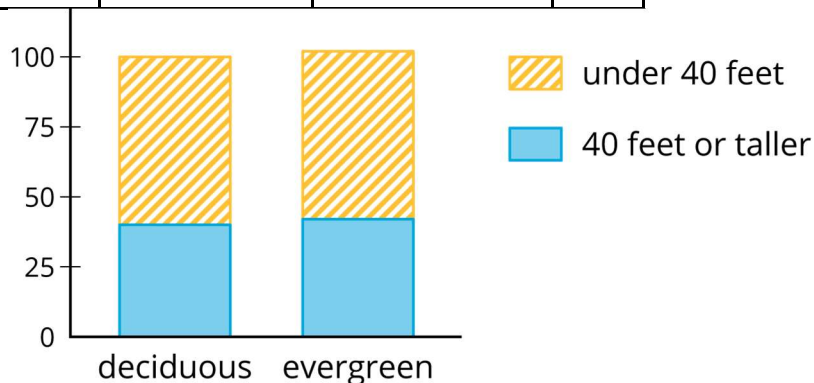
## Lesson 10 Practice Problems

### Problem 1 Statement

An ecologist is studying a forest with a mixture of tree types. Since the average tree height in the area is 40 feet, he measures the height of the tree against that. He also

records the type of tree. The results are shown in the table and compound bar graph. Is there evidence of an association between tree height and tree type? Explain your reasoning.

	under 40 feet	40 feet or taller	total
deciduous	45	30	75
evergreen	14	10	24
total	59	40	99



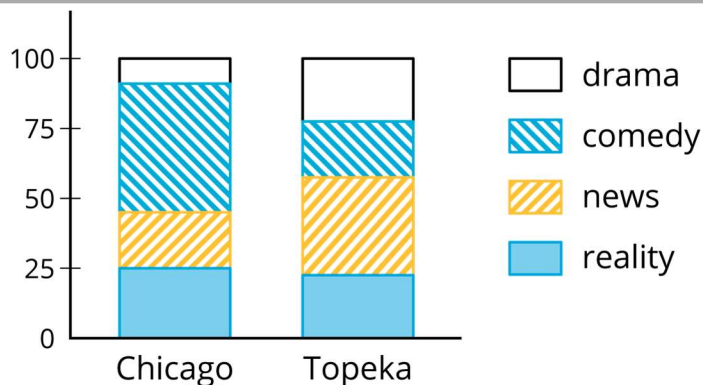
**Solution**

No, 60% of the deciduous trees are under 40 feet, and 40% are at least 40 feet. Similarly, 58% of evergreens are under 40 feet, and 42% are at least 40 feet. From the data recorded, there is not a clear association.

**Problem 2 Statement**

Workers at an advertising agency are interested in people’s TV viewing habits. They take a survey of people in two cities to try to find patterns in the types of shows they watch. The results are recorded in a table and shown in a compound bar graph. Is there evidence of different viewing habits? If so, explain.

	reality	news	comedy	drama
Chicago	50	40	90	20
Topeka	45	70	40	45



**Solution**

Yes, there are differences. Topekans watch news and dramas much more than Chicagoans. Chicagoans watch comedies much more. They watch about the same amount of reality TV.

**Problem 3 Statement**

A scientist is interested in whether certain species of butterflies like certain types of local flowers. The scientist captures butterflies in two zones with different flower types and records the number caught. Do these data show an association between butterfly type and zone? Explain your reasoning.

	zone 1	zone 2
eastern tiger swallowtail	16	34
monarch	24	46

**Solution**

No, there is not an association. 32% of eastern tiger swallowtails and 34% of monarchs were found in zone 1, so there is not a large difference in type of butterfly.



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