Lesson 18: More relationships

Goals

- Coordinate (orally and in writing) graphs, tables, and equations that represent the same relationship.
- Create an equation and a graph to represent the relationship between two variables that are inversely proportional.
- Describe and interpret (orally and in writing) a graph that represents a nonlinear relationship between independent and dependent variables.

Learning Targets

- I can create tables and graphs that show different kinds of relationships between amounts.
- I can write equations that describe relationships with area and volume.

Lesson Narrative

This lesson is optional. It offers opportunities to look at multiple representations (equations, graphs, and tables) for some different contexts.

This final lesson on relationships between two quantities examines situations of constant area, constant volume, and a doubling relationship. Students have an opportunity to notice the similar structures of the situations in the Making a Banner and Cereal Boxes activities, as well as connecting the Multiplying Mosquitoes activity to prior work with exponents and the Genie's coins situation from earlier in the unit. They may use those observations and knowledge to more easily solve the problems in the activities.

Consider offering students a choice about which one they work on. Then, in the lesson synthesis, invite students to share their work with the class and compare and contrast the representations of the different contexts.

Alignments

Addressing

- Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyse the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Instructional Routines

- Three Reads
- Compare and Connect
- Discussion Supports
- Which One Doesn't Belong?

Student Learning Goals

Let's use graphs and equations to show relationships involving area, volume, and exponents.

18.1 Which One Doesn't Belong: Graphs

Warm Up: 5 minutes

The purpose of this warm-up is to prompt students to reason about what a set of organised points on a coordinate grid might mean. This activity invites students to explain their reasoning and hold mathematical conversations, and allows you to hear how they use terminology and talk about points in a coordinate plane. To allow all students to access the activity, there is not one correct answer so students are able to choose any figure as long as they can support their reasoning. As students share their responses, listen for important ideas and terminology that will be helpful in upcoming work.

Instructional Routines

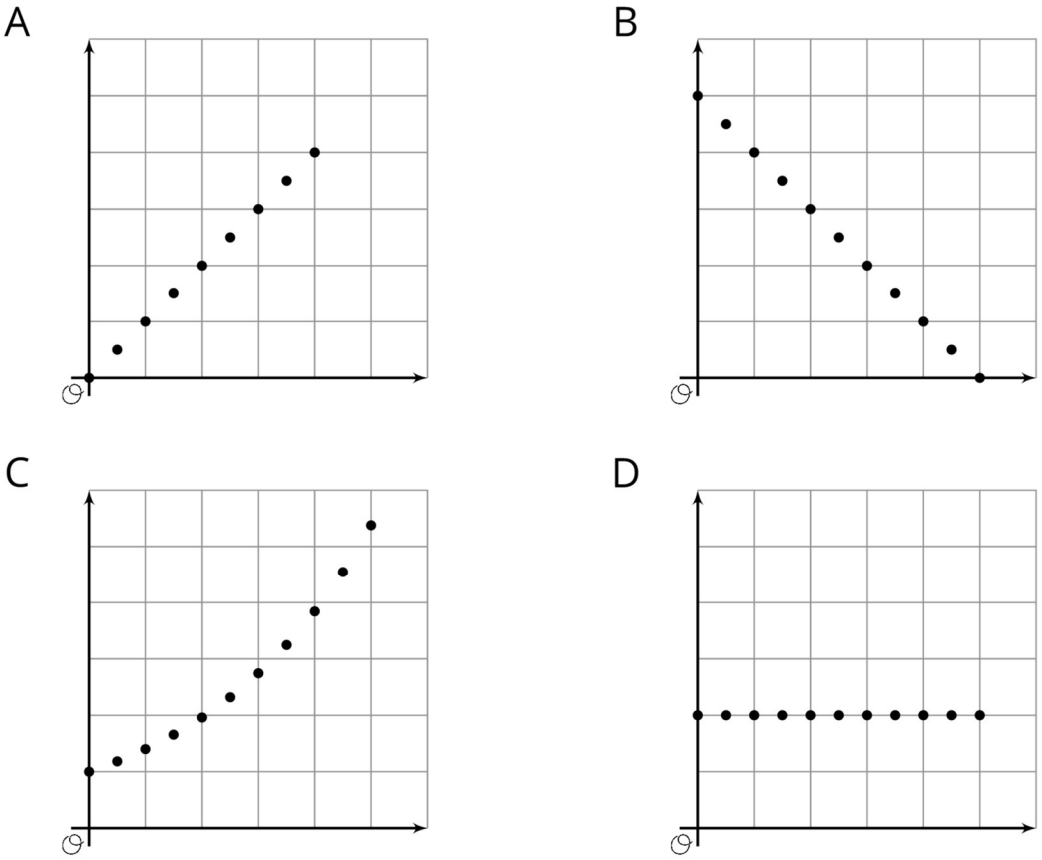
- Which One Doesn't Belong?

Launch

Arrange students in groups of 2-4. Display the image of the four figures for all to see. Ask students to indicate when they have noticed one figure that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask the group to offer at least one reason *each* figure doesn't belong.

Student Task Statement

Which one doesn't belong?



Student Response

Answers vary. Possible responses:

A: The only one that starts at the origin; the only one with the points (1,1), (2,2), (3,3) and (4,4). Students might note that this graph looks like the graphs of ratio relationships.

B: The only one that decreases as you move to the right

C: The only one where the points are not on a line. Points are not equally spaced; the higher points are farther apart.

D: The only one that is flat, not increasing or decreasing as you move to the right

Activity Synthesis

After students have conferred in groups, invite each group to share one reason why a particular figure might not belong. Record and display the responses for all to see. After each response, poll the rest of the class if they agree or disagree. Since there is no single correct answer to the question of which shape does not belong, attend to students'

explanations and ensure the reasons given are correct. During the discussion, prompt students with the following questions:

- "How would you describe the relationship between the two quantities represented by the two axes?"
- "Do you have any ideas about what quantities or relationships any of these graphs might represent?" (If students do have ideas, based on relationships they have explored or others they are thinking about, have them explain why the graph represents the relationship.)

18.2 Making a Banner

Optional: 15 minutes (there is a digital version of this activity)

In this activity, students consider the relationship between length and width for different rectangles with the same given area and are asked to compare strategies for finding various lengths and widths. They make sense of how the graph shows what happens to the width when the length changes and what the plotted points on the graph mean in the context of the problem.

Instructional Routines

- Three Reads

Launch

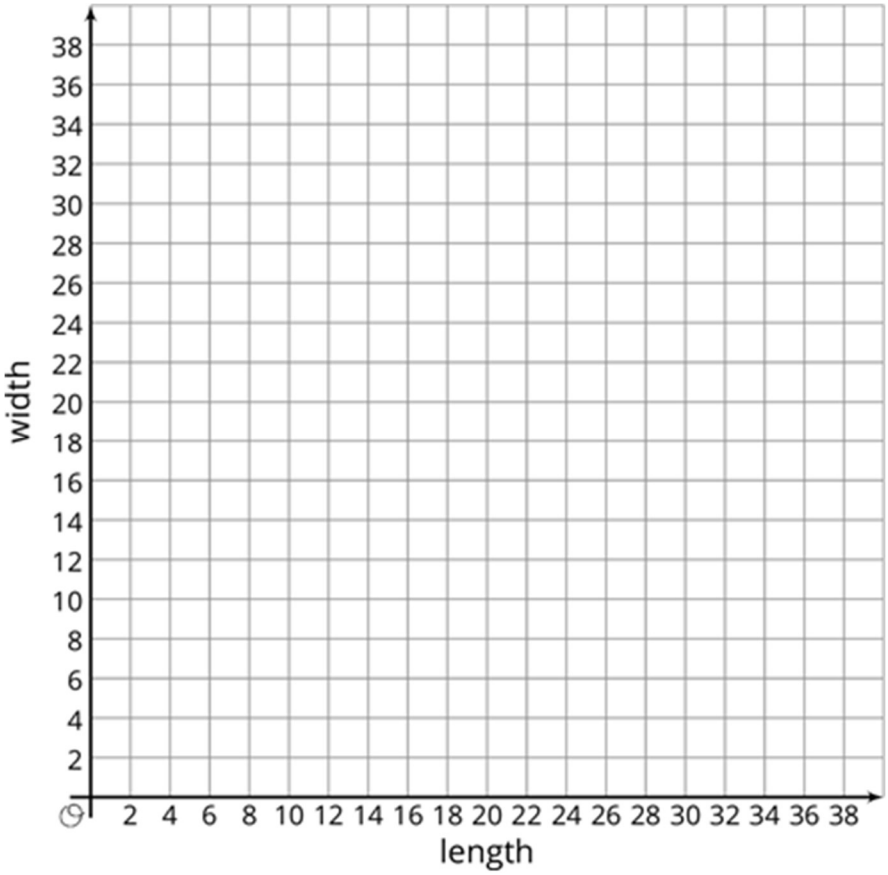
Give students 10 minutes of quiet work time, followed by a whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge. Remind students that they can draw rectangle diagrams to help them determine the missing values.
Supports accessibility for: Social-emotional skills; Conceptual processing Reading: Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., This problem is about creating a banner. Mai needs to buy material). Use the second read to identify the important quantities by asking students what can be counted or measured (e.g., The area of the banner needs to be 36 square units). In the third read, ask students to brainstorm possible strategies to answer the questions. This helps students connect the language in the word problem and the reasoning needed to solve the problem keeping the intended level of cognitive demand in the task.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

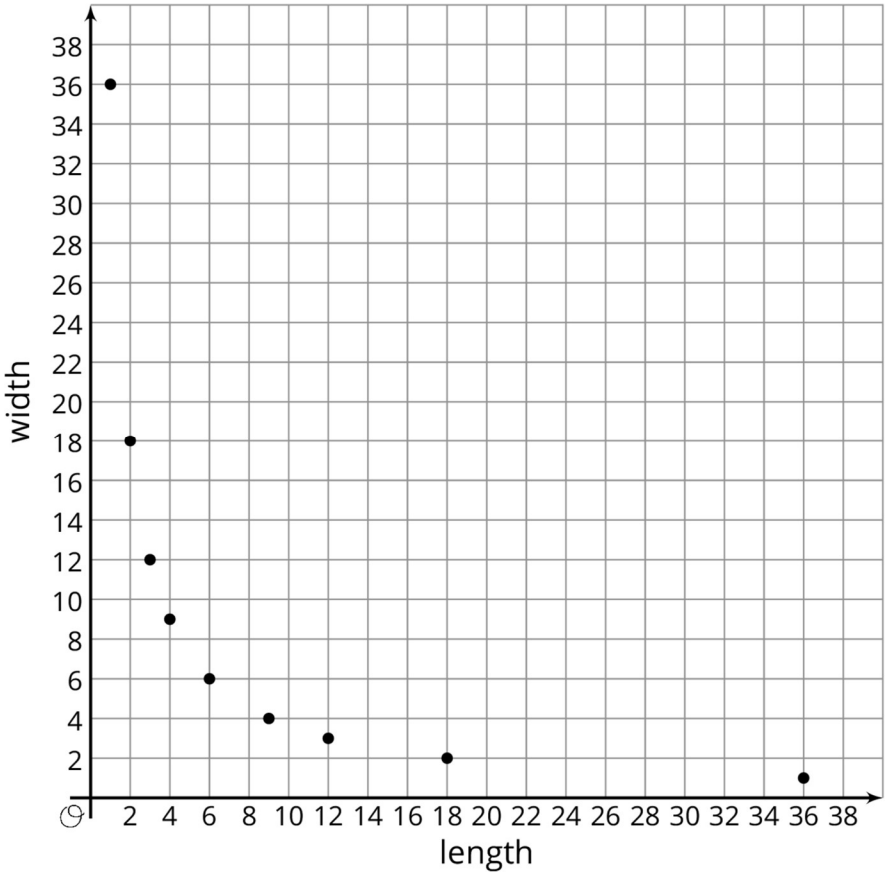
Mai is creating a rectangular banner to advertise the school play. The material for the banner is sold by the square foot. Mai has enough money to buy 36 square feet of material. She is trying to decide on the length and width of the banner.

1. If the length is 6 feet, what is the width?
2. If the length is 4 feet, what is the width?
3. If the length is 9 feet, what is the width?
4. To find different combinations of length and width that give an area of 36 square feet, Mai uses the equation $w = \frac{36}{\ell}$, where w is the width and ℓ is the length. Compare your strategy and Mai's method for finding the width. How were they the same or different?
5. Use several combinations of length and width to create a graph that shows the relationship between the side lengths of various rectangles with area 36 square feet.



6. Explain how the graph describes the relationship between length and width for different rectangles with area 36.
7. Suppose Mai used the equation $\ell = \frac{36}{w}$ to find the length for different values of the width. Would the graph be different if she graphed length on the vertical axis and width on the horizontal axis? Explain how you know.

Student Response

1. 6 feet
2. 9 feet
3. 4 feet
4. Answers vary. Sample response: I looked for a number to multiply the length by to get 36.
5. 

6. Answers vary. Sample response: As the length increases, the width decreases.
7. The graph would look the same. Explanation varies. Sample explanation: The values for length and width would just switch.

Activity Synthesis

The discussion should focus on the connection between the situation, the equation or strategy for finding combinations that make the area 36, and the graph that represents the relationship between length and width in the different rectangles.

Some guiding questions:

- "What does the point (2,18) (for example) in the graph mean? In general, what does each point represent?" (each point represents a rectangle with area 36. (2,18) represents a rectangle with length 2 and height 18)
- "Why does it make sense that the graph falls as you move to the right?" (The length and width are factors of a fixed product, so if one increases the other has to decrease)
- "Where do you see the area 36 in each of the situation, the equation or strategy for finding combinations, and the graph that shows those combinations?" (on the graph, the coordinates of each point multiply to 36)
- "Describe what the graph would look like if it were to extend more to the right. Name some points on the graph and describe their coordinates." (Points could be $(\frac{9}{10}, 40)$, $(\frac{18}{25}, 50)$, $(\frac{1}{2}, 72)$. w would be a fraction less than 1 as ℓ gets larger than 36 because w and ℓ have to multiply to 36. The points will keep getting closer to the x-axis as w gets smaller.

18.3 Cereal Boxes

Optional: 15 minutes (there is a digital version of this activity)

This activity presents a situation with a similar structure to the area situation in the Making a Banner activity. Students consider different combinations of base areas and heights that keep the volume of a rectangular box at 225 cubic inches. They complete a table for given values of area and height, write an equation relating the area and height, and graph the relationship.

If students have completed the Making a Banner activity, prompt them to think about similarities and differences they noticed between the activities. Invite students to share these thoughts during the discussion. Students have an opportunity to notice the similar structures of the two situations and use that knowledge to solve the problems in this activity.

Instructional Routines

- Compare and Connect

Launch

Give students 10 minutes of quiet work time and follow with a whole-class discussion.

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies. Allow students to use the applet for this activity to facilitate plotting the ordered pairs from the table on a graph.
Supports accessibility for: Visual-spatial processing; Conceptual processing; Organisation

Student Task Statement

A cereal manufacturer needs to design a cereal box that has a volume of 225 cubic inches and a height that is no more than 15 inches.

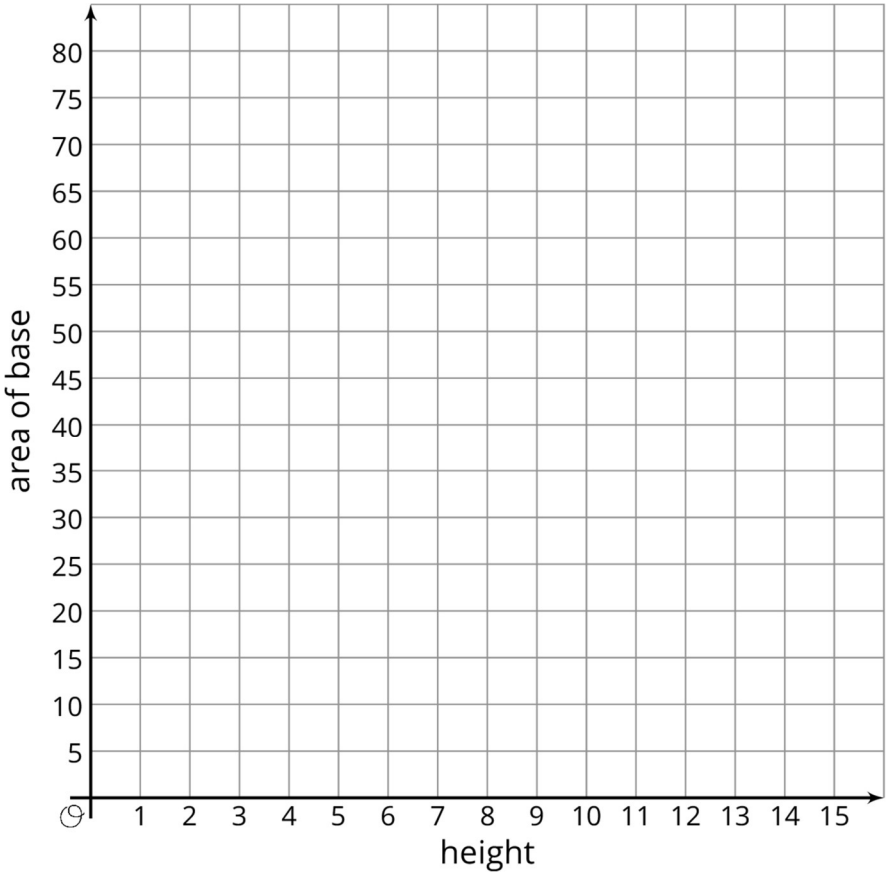
1. The designers know that the volume of a rectangular prism can be calculated by multiplying the area of its base and its height. Complete the table with pairs of values that will make the volume 225 in^3.

	height (in)		5	9	12		$7\frac{1}{2}$
	area of base (in^2)	75				15	

2. Describe how you found the missing values for the table.

3. Write an equation that shows how the area of the base, A, is affected by changes in the height, h, for different rectangular prisms with volume 225 in^3.

4. Plot the ordered pairs from the table on the graph to show the relationship between the area of the base and the height for different boxes box with volume 225 in^3.



Student Response

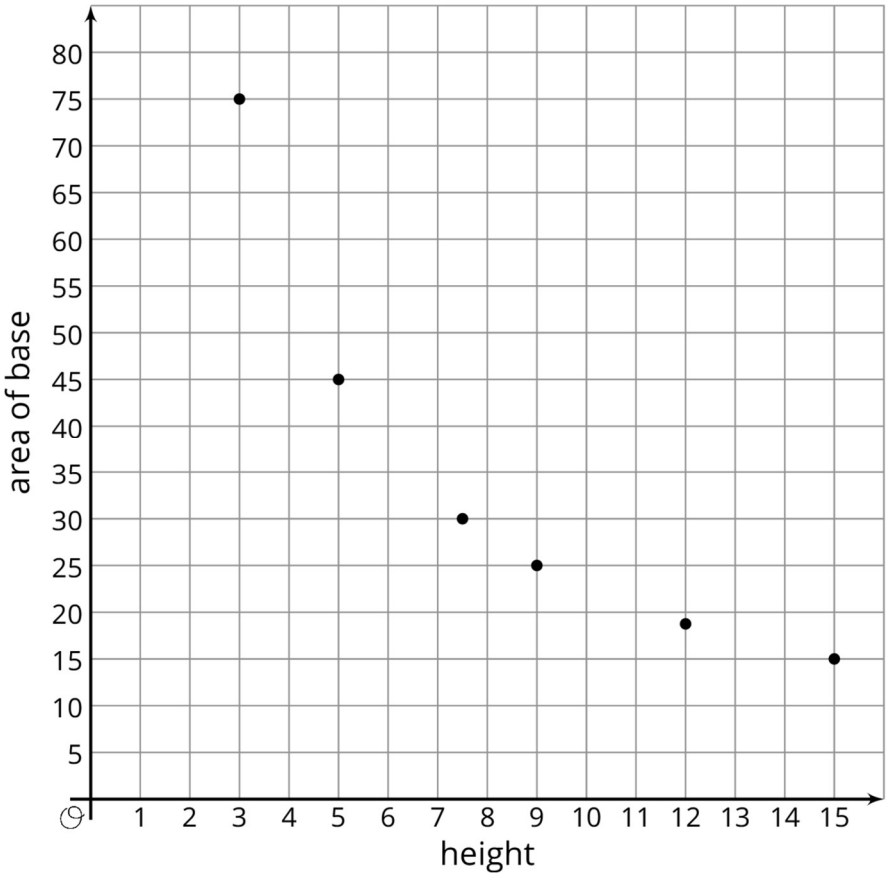
height (in)	3	5	9	12	15	$7\frac{1}{2}$
area of base (in^2)	75	45	25	18.75	15	30

1.

2. Answers vary. Sample response: Divide 225 by either the area or height.

3. Answers vary. Sample response: $A = \frac{225}{h}$ or equivalent

4.



Activity Synthesis

The purpose of the discussion is to connect this activity to the previous one and point out similarities and differences between them.

Consider asking the following questions:

- "How does this situation compare to the one in the Making a Banner activity?" (The structure of the equations was similar: the two quantities multiply to a constant.)

- "How did you decide on an equation that represents the relationship?"
- "How would the graph be different if height was on the vertical axis and area on the horizontal axis?" (It would look the same since the two numbers multiply to a constant.)

Representing, Speaking, Listening: Compare and Connect. Invite students to prepare a visual display of their table, graph, and equation that represent the relationship between the area of the base and height of the cereal box. As students analyse each others' work, ask them to share what is especially clear in a particular representation. Listen for and amplify the language students use to describe how the area of the base decreases as the height of the cereal boxes increases. Invite students to identify where they see the volume of 225 cubic inches represented in the table, graph, and equation. Listen for and amplify the language students use to describe how the coordinates of each point on the graph multiply to 225. This will foster students' meta-awareness about language and support constructive conversations.
Design Principles(s): Cultivate conversation; Maximise meta-awareness

18.4 Multiplying Mosquitoes

Optional: 10 minutes (there is a digital version of this activity)

In this activity, students consider a doubling relationship where the exponent is a variable. Monitor for students who connect this activity to the lessons on exponents, or who recognise that the quantities in this relationship are changing with respect to each other in a different manner than previous examples they have seen. Have these students share during the discussion.

Instructional Routines

- Discussion Supports

Launch

Give students 5-7 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

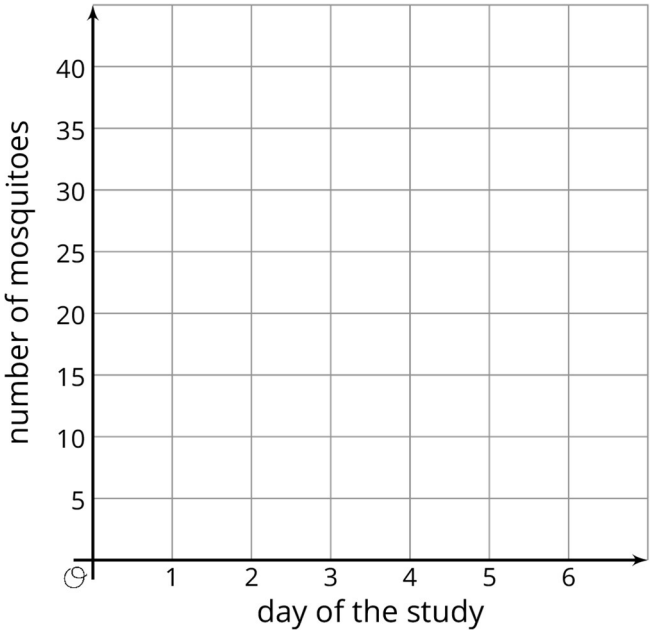
A researcher who is studying mosquito populations collects the following data:

day in the study (d)	number of mosquitoes (n)
1	2
2	4
3	8
4	16

5	32

1. The researcher said that, for these five days, the number of mosquitoes, n, can be found with the equation $n = 2^d$ where d is the day in the study. Explain why this equation matches the data.

2. Use the ordered pairs in the table to graph the relationship between number of mosquitoes and day in the study for these five days.

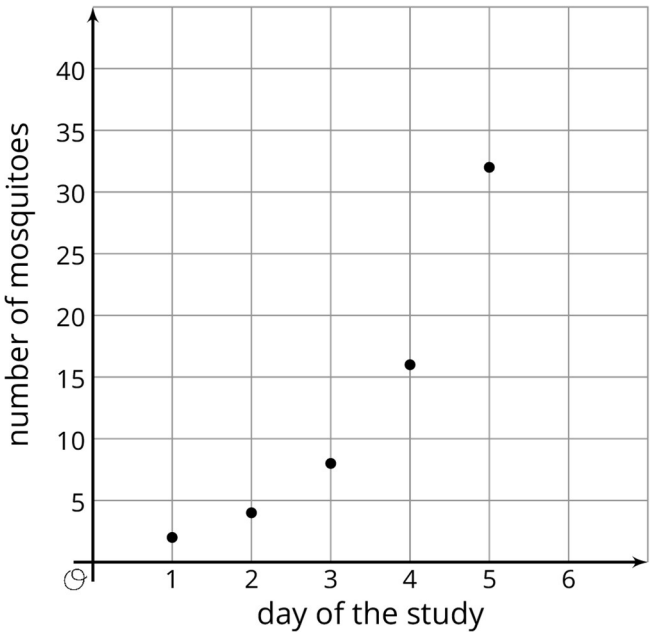


3. Describe the graph. Compare how the data, equation, and graph illustrate the relationship between the day in the study and the number of mosquitoes.

4. If the pattern continues, how many mosquitoes will there be on day 6?

Student Response

1. Answers vary. Sample response: $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, and $2^5 = 32$, so the equation matches the data.

2. 

3. Answers vary. Sample response: The graph is increasing and rises very quickly after the first few days. We see bigger and bigger jumps from day to day in the data. In the equation, the exponent means that the numbers are doubling each day.

4. $2^6 = 64$, 64 mosquitoes

Activity Synthesis

The purpose of this discussion is to help students make connections between the table, graph, and equation that describe this situation. Ask students to share who connected this activity to the lessons on exponents, or who recognise that the quantities in this relationship are changing with respect to each other in a different manner than previous examples they have seen.

Consider asking some of these questions:

- "Which of the data, equation, or graph gives you more of a feel for how the mosquito population changes with the day in the study?"
- "Think back to the lesson on exponents with the dot pattern and the Genie activity. How do those situations compare with this mosquito situation? How would tables of data, graphs, and equations compare?"
- "What are some similarities and differences you noticed between all the relationships you've seen in the last few lessons?"

Speaking, Listening: Discussion Supports. To support whole-class discussion, display sentence frames such as, "The dot pattern is similar to/ different than the mosquito situation because _____," and "The genie situation is similar to/ different than the

mosquito situation because _____." This routine will support rich and inclusive discussion about the similarities and differences between all of the relationships students have seen in the last few lessons.
Design Principle(s): Optimise output (for comparison); Cultivate conversation

Lesson Synthesis

In this lesson we looked at three situations: rectangles with the same area, rectangular prisms with the same volume, and one quantity that doubles repeatedly each time another quantity is increased by 1. In each situation we examined the relationship between two quantities: length and width of the rectangle, area of the base and height, number of mosquitos and number of days.

Invite students to share and discuss their work with the class. To facilitate discussion, ask students to restate each other's explanations. Draw attention to the meaning of the components of each representation, and how the representations of the different situations are alike and different. For example, ask questions like,

- "What is the meaning of the coordinates of that specific point?"
- "How are the coordinates of a specific point connected to an equation that represents the situation?"
- "In the first activity, for a larger length, there is a *smaller* width. But in the last activity, for a larger number of days, there is a *larger* number of mosquitos. How come?"

18.5 Interpret the Point

Cool Down: 5 minutes

Student Task Statement

The equation $\frac{1}{4}P = s$ relates the perimeter P of any square and its side length s. A graph of the equation includes the point (12,3).

1. What does the point (12,3) represent in this situation?
2. What point would represent a square with perimeter $\frac{20}{21}$?

Student Response

1. It represents a square with side length 3 and perimeter 12.
2. $(\frac{20}{21}, \frac{5}{21})$

Student Lesson Summary

Equations can represent relationships between geometric quantities. For instance:

- If s is the side length of a square, then the area A is related to s by $A = s^2$.
- Sometimes the relationships are more specific. For example, the perimeter P of a rectangle with length l and width w is $P = 2l + 2w$. If we consider only rectangles with a length of 10, then the relationship between the perimeter and the width is $P = 20 + 2w$.

Here is another example of an equation with exponent expressing the relationship between quantities:

- A super ball is dropped from 10 feet. On each successive bounce, it only goes $\frac{1}{2}$ as high as on the previous bounce.

  This means that on the first bounce, the ball will bounce 5 feet high, and then on the second bounce it will only go $2\frac{1}{2}$ feet high, and so on. We can represent this situation with an equation to find how high the super ball will bounce after any number of bounces.

  To find how high the super ball bounces on the n^{th} bounce, we have to multiply 10 feet (the initial height) by $\frac{1}{2}$ and multiply by $\frac{1}{2}$ again for each bounce thereafter; we need to do this n times. So the height, h, of the ball on the n^{th} bounce will be $h = 10\left(\frac{1}{2}\right)^n$. In this equation, the dependent variable, h, is affected by changes in the independent variable, n.

Equations and graphs can give us insight into different kinds of relationships between quantities and help us answer questions and solve problems.

Lesson 18 Practice Problems

1. **Problem 1 Statement**

   Elena is designing a logo in the shape of a parallelogram. She wants the logo to have an area of 12 square inches. She draws bases of different lengths and tries to compute the height for each.

   a. Write an equation Elena can use to find the height, h, for each value of the base, b.

   b. Use your equation to find the height of a parallelogram with base 1.5 inches.

   Solution

   a. $h = \frac{12}{b}$

   b. 8 inches

2. **Problem 2 Statement**

Han is planning to ride his bike 24 miles.

a. How long will it take if he rides at a rate of:

3 miles per hour?

4 miles per hour?

6 miles per hour?

b. Write an equation that Han can use to find t, the time it will take to ride 24 miles, if his rate in miles per hour is represented by r.

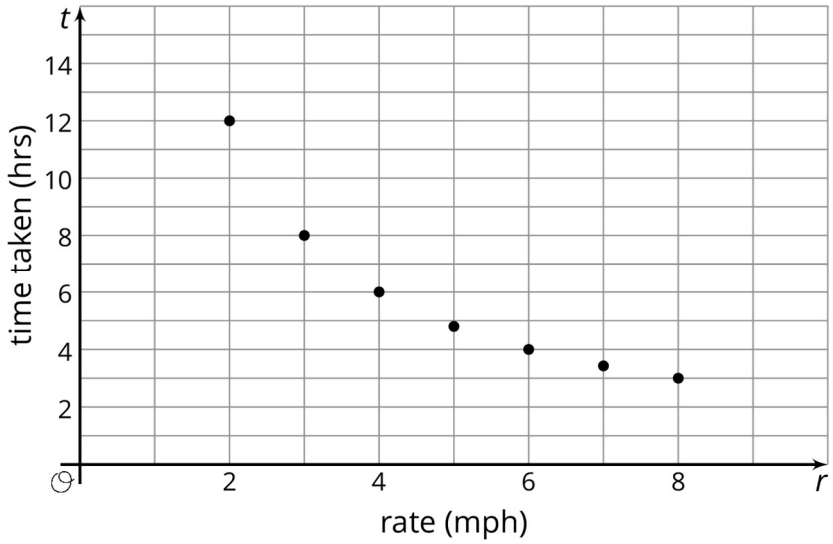
c. On graph paper, draw a graph that shows t in terms of r for a 24-mile ride.

Solution

a. 8 hours, 6 hours, 4 hours

b. $t = 24 \div r$ or $t = \frac{24}{r}$.

c.



3. **Problem 3 Statement**

The graph of the equation $V = 10s^3$ contains the points (2,80) and (4,640).

a. Create a story that is represented by this graph.

b. What do the points mean in the context of your story?

Solution

Answers vary. Sample response: Lin and Jada each build a tower of 10 cubes. Lin's cubes have edge length 2 and Jada's have edge length 4. They use the equation $V = 10s^3$ to compute the volume of their towers.

4. **Problem 4 Statement**

You find a brass bottle that looks really old. When you rub some dirt off of the bottle, a genie appears! The genie offers you a reward. You must choose one:

£50,000 or a magical £1 coin.

The coin will turn into two coins on the first day. The two coins will turn into four coins on the second day. The four coins will double to 8 coins on the third day. The genie explains the doubling will continue for 28 days.

a. Write an equation that shows the number of coins, n, in terms of the day, d.

b. Create a table that shows the number of coins for each day for the first 15 days.

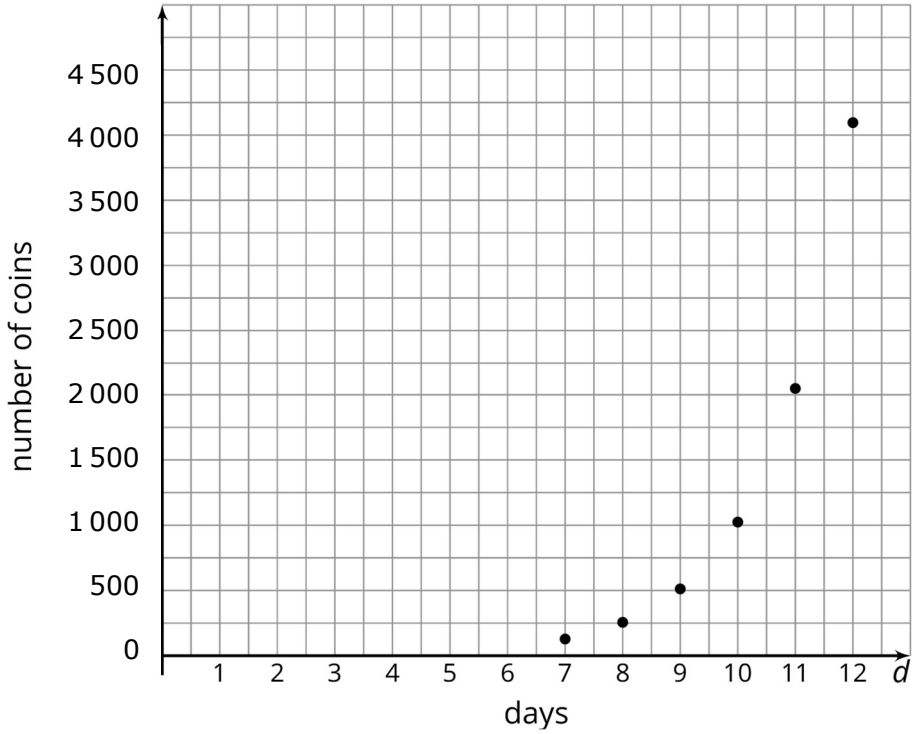
c. Create a graph for days 7 through 12 that shows how the number of coins grows with each day.

Solution

a. $n = 2^d$

day (d)	number of coins (n)
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1 024
11	2 048
12	4 096
13	8 192
14	16 384
15	32 768

b. The numbers are too large to show exact numbers on the vertical axis. It's best to mark increments of 500 or 1 000 and approximate the positions.



5. **Problem 5 Statement**

At a market, 3.1 pounds of peaches cost £7.72. How much did the peaches cost per pound? Explain or show your reasoning. Round your answer to the nearest penny.

Solution

£2.49 per pound. Reasoning varies. Sample reasoning: Divide 7.72 by 3.1 to get the price of the peaches per pound in pounds. If dividend and divisor are both multiplied by 100, the value of the quotient does not change. Then calculate 772 ÷ 310 by long division. The quotient is a little more than 2.490.

6. **Problem 6 Statement**

Andre set up a lemonade stand last weekend. It cost him £0.15 to make each cup of lemonade, and he sold each cup for £0.35.

a. If Andre collects £9.80, how many cups did he sell?

b. How much money did it cost Andre to make this amount of lemonade?

c. How much money did Andre make in profit?

Solution

a. 28 $(9.80 \div 0.35 = 28)$

b. £4.20 $(28 \times (0.15) = 4.20)$

c. £5.60 $(9.80 - 4.20 = 5.60)$