

Lesson 14: Using operations on decimals to solve problems

Goals

- Apply operations with decimals to solve problems about the dimensions of a sports field or court, and explain (orally, in writing, and using other representations) the solution method.
- Choose whether an exact answer or an estimate is appropriate for a given problem.
- Interpret a verbal description or diagram that represents the dimensions of a sports field or court.

Learning Targets

- I can use addition, subtraction, multiplication, and division on decimals to solve problems.

Lesson Narrative

In this lesson, students apply their knowledge of operations on decimals to two sporting contexts. They analyse the distance between hurdles in a 110-metre hurdle race. In this situation, students use the given context to determine which arithmetic operations are relevant and use them to solve the problems. Additionally, they draw or use a diagram to help them make sense of the measurements, as well as to communicate their reasoning about the measurements. The numbers used in the problems reflect measurements that can be accurately measured on site, so the decimals can all be calculated by hand.

Building On

- Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognise that $3 \times (18\,932 + 921)$ is three times as large as $18\,932 + 921$, without having to calculate the indicated sum or product.

Addressing

- Fluently divide multi-digit numbers using the standard algorithm.
- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- Group Presentations
 - Collect and Display
 - Co-Craft Questions
 - Three Reads
-

- Think Pair Share

Required Materials

Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles.

When compasses are required, they are listed as a separate Required Material.

Student Learning Goals

Let's solve some problems using decimals.

14.1 Close Estimates

Warm Up: 5 minutes

The purpose of this warm-up is to encourage students to think about the reasonableness of a quotient by looking closely at the values of the dividend and divisor. The digits in the answer choices reflect those in the actual answer. While some students may try to mentally evaluate each one precisely, encourage them to think about their answer in relation to the numbers in the expressions. For each expression, ask students if the actual answer would be greater than or less than their estimate and how they know.

Launch

Display one expression at a time, or ask students to work on one expression at a time and begin when cued. Give students 1 minute of quiet think time per question and ask them to give a signal when they have an answer and can explain their strategy. Follow each question with a brief whole-class discussion.

Discuss each problem one at a time with this structure:

- Ask students to indicate which option they chose.
- If everyone agrees on one answer, ask a few students to share their reasoning. Record and display the explanations for all to see. If there is disagreement on an answer, ask students with opposing answers to explain their reasoning to come to an agreement on an answer.

Student Task Statement

For each expression, choose the best estimate of its value.

1. $76.2 \div 15$
 - 0.5
 - 5
-

-
- 50
2. $56.34 \div 48$
- 1
- 10
- 100
3. $124.3 \div 20$
- 6
- 60
- 600

Student Response

1. 5, because 76.2 is close to 75 and $15 \times 5 = 75$.
2. 1, because 48 and 56 are close to one another, so the answer must be close to 1.
3. 6, because 124.3 is close to 120, and 20 fits into 120 six times, so the quotient must be close to 6.

Activity Synthesis

Ask students to share their general strategies for these problems. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone solve the problem the same way but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to ___’s strategy?”
- “Do you agree or disagree? Why?”

14.2 Applying Division with Decimals

Optional: 15 minutes

In this activity, students practise calculating quotients of decimals by multiplying both the divisor and the dividend by an appropriate power of 10. Then, they extend their practice to calculate quotients of decimals in real-world contexts. Problem A reiterates earlier experiences with the “how much is in each group” and “how many groups” interpretations of division. Problem B recalls students’ prior work with ratios and determining rate of

speed. While students can use ratio techniques (e.g. a ratio table) to answer these questions, encourage them to use division of decimals.

Instructional Routines

- Group Presentations
- Collect and Display

Launch

Arrange students in groups of 3–5. Assign each group Problem A or B and have students circle the problem they are assigned. Give groups 5–7 minutes to work on their assigned problem. If time permits, consider giving students access to tools for creating a visual display. Have them create a simple visual display to showcase their solutions and prepare a short presentation in which they explain their reasoning and calculations.

Give students 2–3 minutes to review one another’s work followed by groups’ presentations of their displays.

Engagement: Develop Effort and Persistence. Provide prompts, reminders, guides, rubrics, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, provide a rubric or checklist to ensure the contents of the visual display are made explicit.

Supports accessibility for: Attention; Social-emotional skills Speaking: Collect and Display. Use this routine to capture the language students use as they calculate quotients of decimals in real-world contexts. Circulate and listen to students talk during small-group and whole-class discussion. Record the words, phrases, and writing students use to describe their strategies. Capture student language that reflects a variety of ways to show how they are making sense of each problem such as, “how many in each group”, “how many groups” or “multiply by the same factor.” Display for the whole class to see and to use as a reference throughout the lesson. This will help students develop meta-awareness of the language of division.

Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

Your teacher will assign to you either Problem A or Problem B. Work together as a group to answer the questions. Be prepared to create a visual display to show your reasoning with the class.

Problem A:

A piece of rope is 5.75 metres in length.

1. If it is cut into 20 equal pieces, how long will each piece be?
2. If it is cut into 0.05 metre pieces, how many pieces will there be?

Problem B:

A tortoise travels 0.945 miles in 3.5 hours.

1. If it moves at a constant speed, how many miles per hour is it travelling?
2. At this rate, how long will it take the tortoise to travel 4.86 miles?



Student Response

Problem A:

1. 0.2875 metres. Each piece of rope is $5.75 \div 20$ metres long. This is a “how many in each group” division problem with the groups being the pieces of rope and the “how much in each group” being the length of the rope.
2. 115. There are $5.75 \div 0.05$ pieces of rope. This is a “how many groups” division problem with the groups being the pieces of rope and the “how much in each group” being the length of the rope.

Problem B:

1. 0.27 miles per hour. The tortoise travelled 0.945 miles in 3.5 hours. To find the speed, divide the distance by the number of hours, 3.5. The speed is given by $0.945 \div 3.5$, which is the same value as $9.45 \div 35$.
2. 18 hours, $4.86 \div 0.27 = 18$

Activity Synthesis

Have each group post their display and read aloud each problem to the class. Then give all students 2–3 minutes to circulate and look at the displays. Ask them to think about how the work on each display is similar to or different from their own group’s work. Afterwards,

ask all groups to share their display. As each group presents, be sure they explain how they approached the problems and why these problems apply decimal division.

For Problem A ask:

- “Does the problem use the ‘how many groups’ or ‘how many in each group’ interpretation of division? How do you know?”

For Problem B ask:

- “What is the ratio relationship?” (Speed is the ratio of distance to time.) “If you know the rate per one hour, how can you determine the time it will take to travel any given distance?” (Divide the distance by the unit rate.)

After all groups present, ask:

- “Did groups that solved the same problem approach it in the same way? If not, how did strategies differ?”
- “What methods did students use for calculating quotients of decimals?”

14.3 Distance between Hurdles

20 minutes

Sports provide many good contexts for doing arithmetic. This is the first of two activities using a sporting context. Students use arithmetic with decimals to study the 110 metre hurdle race. The first question prompts students to draw a diagram to capture and make sense of all of the given information. The second prompts them to find the distance between hurdles.

As students work, find a variety of work samples (particularly ones that make use of a drawing) to share with the class during the discussion.

Instructional Routines

- Three Reads

Launch

Give a brief overview of hurdle races. Ask students if they have had the chance to watch track and field competitions. Have them describe what a hurdle race is. Display images such as the following or show a short video of a hurdle race.



Tell students that they will now use what they have learned about decimals to solve a couple of problems involving hurdles.

Arrange students in groups of 2. Give students about 5–6 minutes to draw a diagram for the first question and discuss their drawing with their partner. Then, give them another 5–6 minutes to complete the other two questions. Follow with a whole-class discussion.

Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students have calculated the correct distance between the first and last hurdles. Also, check to make sure students understand that there are only 9 spaces between 10 hurdles prior to calculating the distance between the hurdles.

Supports accessibility for: Memory; Organisation Reading, Listening, Conversing: Three Reads. Use this routine to support reading comprehension of this problem, without solving it for students. In the first read, students read the problem with the goal of understanding the situation (e.g., equally-spaced hurdles on a race track). After the second read, ask students to identify the quantities that can be used or measured (e.g., number of equally-spaced hurdles, length of the first and final hurdles, length of the race track, etc.). After the third read, ask students to discuss possible strategies, referencing the relevant quantities identified in the second read. This will help students make sense of the problem context before being asked to solve it.

Design Principle(s): Support sense-making; Maximise meta-awareness

Anticipated Misconceptions

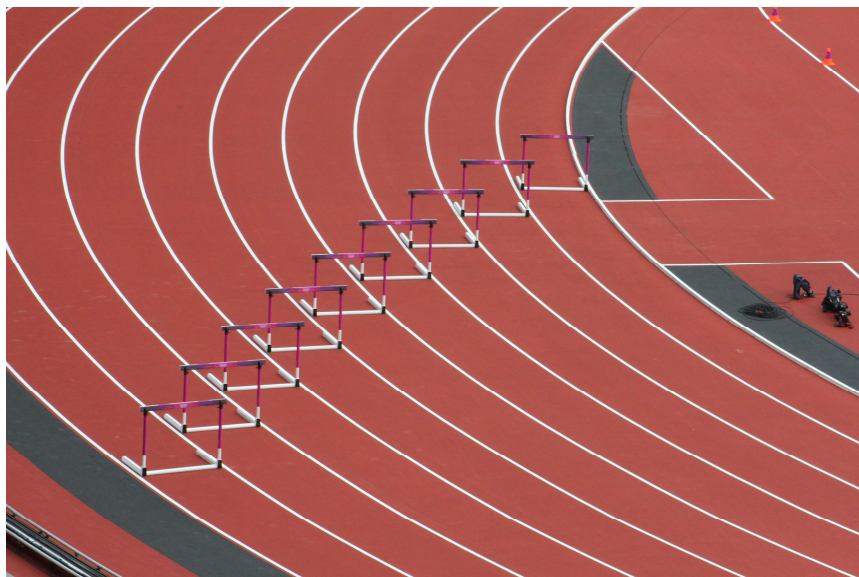
Students may not realise that there are only 9 spaces—not 10 spaces— between 10 hurdles, leading them to miscalculate the distance between hurdles. Have students study the number of spaces in their diagram, or ask them to think about how many spaces are between 2 hurdles, 3 hurdles, 4 hurdles, etc. and extend the pattern to 10 hurdles.

A calculation error in dividing may lead to a quotient with a non-terminating decimal. Look out for arithmetic errors when students calculate the distance between the first and last hurdles (82.26 metres) and when students perform division. If students end up with a non-

terminating decimal for their answer, ask them to revisit each step and see where an error might have occurred.

Student Task Statement

There are 10 equally-spaced hurdles on a race track. The first hurdle is 13.72 metres from the start line. The final hurdle is 14.02 metres from the finish line. The race track is 110 metres long.

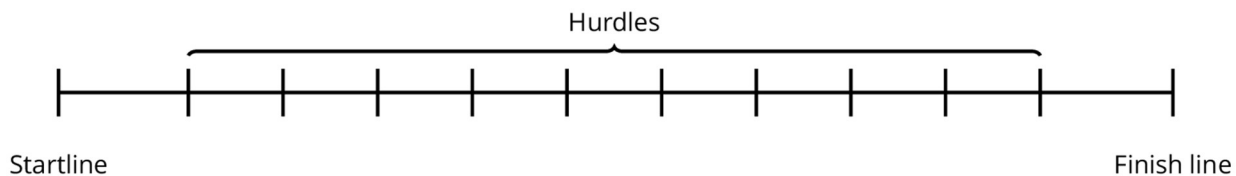


1. Draw a diagram that shows the hurdles on the race track. Label all known measurements.
2. How far are the hurdles from one another? Explain or show your reasoning.
3. A professional runner takes 3 strides between each pair of hurdles. The runner leaves the ground 2.2 metres *before* the hurdle and returns to the ground 1 metre *after* the hurdle.

About how long are each of the runner's strides between the hurdles? Show your reasoning.

Student Response

1. Diagrams vary. Sample diagram:



2. 9.14 metres. Sample reasoning: The distance between the first and last hurdles, in metres, is $110 - 13.72 - 14.02 = 82.26$. Since they are equally spaced, the ten equally

spaced hurdles divide the track into 9 equal parts between the first and last hurdles. The distance between them is $82.26 \div 9 = 9.14$ metres.

3. 1.98 metres (or about 2 metres). Sample reasoning: There are 9.14 metres between the hurdles. The runner comes down 1 metre beyond the previous hurdle and takes off 2.2 metres before the following hurdle so that means the 3 strides cover a distance of $9.14 - 1 - 2.2 = 5.94$ metres. Each of the three strides covers $5.94 \div 3 = 1.98$ metres, just under 2 metres.

Activity Synthesis

Select one or more previously identified students to share their diagrams for the first question. Ask other students if theirs are comparable to these, and if not, where differences exist. If not mentioned by students, be sure to highlight proper labelling of the parts of the diagram. Then, discuss the second and third questions. For the second question, discuss:

- How many 'gaps' are there between the hurdles? (9)
- What is the distance from the first hurdle to the last hurdle? (82.26 metres)
- What arithmetic operation is applied to the two numbers, 9 and 82.26? Why? (Division, because the 10 equally spaced hurdles divide 82.26 metres into 9 equal groups.)

For the third question, discuss:

- How far does the runner go in three strides? How do you know?
- Is 1.98 metres (the exact answer) an appropriate answer for the question? Why or why not? (Most likely not, because the runner is not going to control their strides and jumps to the nearest centimetre. About 2 metres would be a more appropriate answer.)

Consider asking a general question about hurdle races: Is it important for the runner that the hurdles be placed as closely as possible to the correct location? The answer is yes, because runners train to take a precise number of strides and to hone their jumps to be as regular as possible. Moving a hurdle a few centimetres is unlikely to create a problem, but moving a hurdle by a metre would ruin the runners' regular rhythm in the race.

14.4 Examining a Tennis Court

Optional: 20 minutes

In this activity, students study the dimensions of a tennis court and apply their understanding of decimals to solve problems in another sporting context. This activity is optional, because it is an opportunity for students to further practise solving problems with decimals.

Visually, it appears as if each half of the tennis court (divided by the net) is a square. Similarly, it appears as if the service line is about halfway between the net and the baseline. Calculations show that in both cases, however, neither half of the tennis court is a square, and that the service line is not half way between the baseline and the net.

Just as with the distances between hurdles in the previous activity, the dimensions of a tennis court are very precisely determined. It is also very important for professional tennis players to regularly play on courts that have consistent dimensions, as the smallest differences could affect whether the ball is in or out. The third question gets at how small the lines are, compared to the full service box.

There are some subtleties in this task related to measurement in the real world and the idealised version in the task. On a tennis court, the lines have width. For the first two questions of the task, the strips can be taken as dimensionless lines. In the final questions, students deal explicitly with these strips.

Instructional Routines

- Co-Craft Questions
- Think Pair Share

Launch

Give a brief introduction to tennis and tennis courts. Ask students if they play tennis or have ever watched a tennis match. Look at the picture of the tennis court and discuss the purpose of the boundary lines. Have students locate various sections of the court by pointing to rectangles, parallel segments, right angles, and the service box. (More detailed measurements for the parts of a tennis court can be found online.)

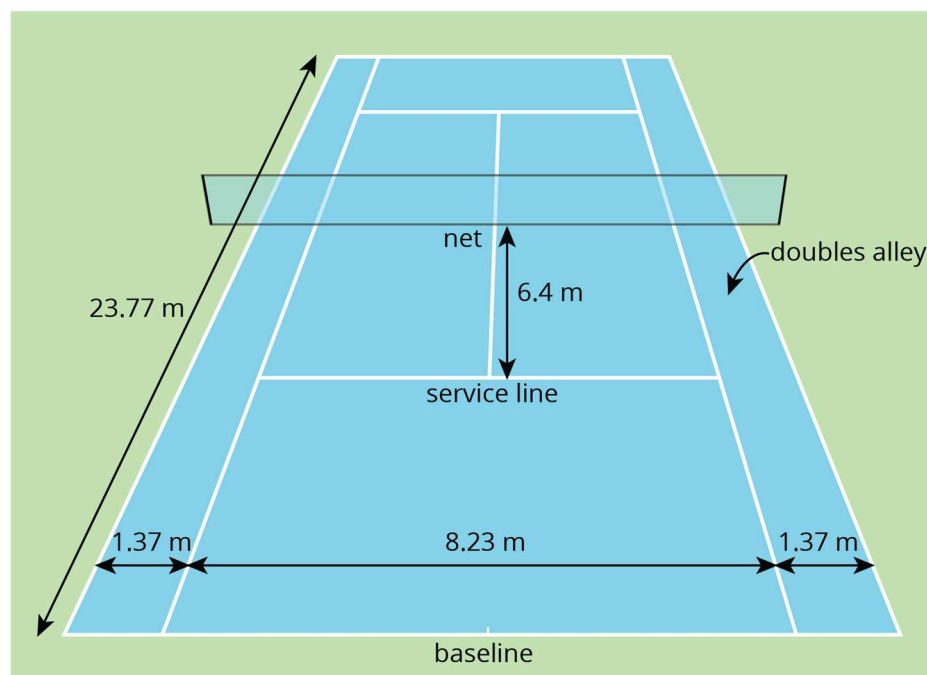
Keep students in groups of 2. Give students 2 minutes of quiet time to read and think about how they might approach the questions. Have them share their thinking with a partner for another 2 minutes. Then, give students 7–8 minutes to complete the questions. Follow with a whole-class discussion.

Writing, Speaking, Conversing: Co-Craft Questions. Use this routine to provide students with an opportunity to develop questioning skills and understand the context of the situation. Begin by showing the diagram of the tennis court. Next, ask students to write down possible mathematical questions that might be asked about the diagram. These questions could include information that might be missing, or even assumptions that students think are important (e.g., each half looks like a square or distance between the service line and baseline). Next, invite students to compare the questions they generated with a partner before they share with the whole class. Finally, reveal the actual questions that students are expected to work on. This helps students develop their skills to generate questions and begin to comprehend the context of the problem before solving it.

Design Principle(s): Maximise meta-awareness; Cultivate conversation

Student Task Statement

Here is a diagram of a tennis court.



The full tennis court, used for doubles, is a rectangle. All of the angles made by the line segments in the diagram are right angles.

1. The net partitions the tennis court into two halves. Is each half a square? Explain your reasoning.
2. Is the service line halfway between the net and the baseline? Explain your reasoning.
3. Lines painted on a tennis court are 5 cm wide. A painter made markings to show the length and width of the court, then painted the lines to the outside of the markings.
 - a. Did the painter's mistake increase or decrease the overall size of the tennis court? Explain how you know.
 - b. By how many square metres did the court's size change? Explain your reasoning.

Student Response

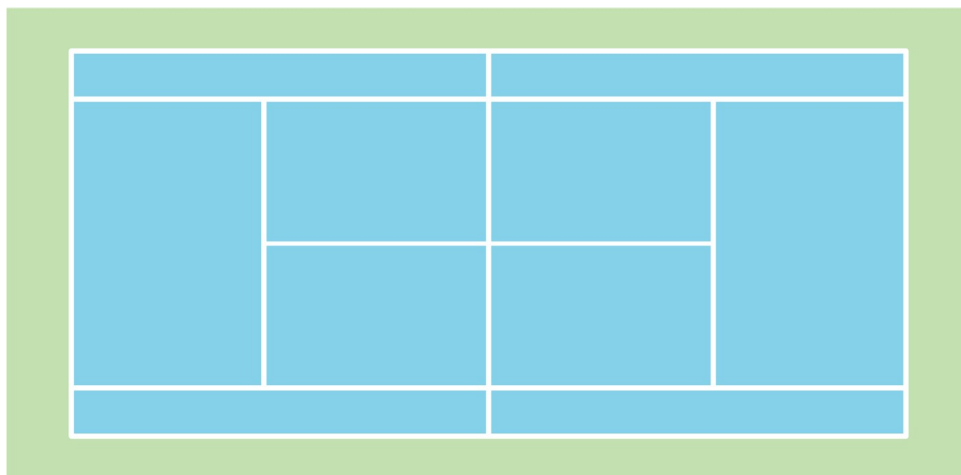
1. No, the tennis court is 23.77 metres long, and the net is in the middle. This means that each half of the court is $\frac{23.77}{2}$ metres long. This is 11.885 metres. Since the tennis court is only 10.97 metres wide, it is not a square.
2. No, the service line is 6.4 metres from the net. Since the length of the (half) court is 11.885 metres, this means that the service line is $11.885 - 6.4$ metres from the

baseline. This is 5.485 metres. So the service line is almost a metre closer to the baseline than it is to the net.

- a. The painter's mistake made the court larger. The painter's outline of the court *begins* where the outline of the court is supposed to *end*.
- b. The painter added two extra 23.77 m by 0.05 m strips and two 10.97 m by 0.05 m strips along the sides, and four 0.05 m by 0.05 m squares in the corners. All measurements are in metres. The two long strips make 2.377 square metres, while the two shorter strips add 1.097 square metres. The four small squares add another 0.01 square metres. This adds up to an extra 3.484 square metres.

Activity Synthesis

Here is a picture of the tennis court from directly above. In this picture we can see that the service box is not a square but it is difficult to determine, just by looking, whether or not each half of the court is a square:



On the other hand, it is possible to judge, from this picture, that the service line is closer to the baseline than it is to the net.

To reinforce these points, discuss:

- What do you look for to decide if you think a shape is a square? (4 equal sides, 4 right angles.)
- Can you tell by looking whether or not the service box is a square? (Yes, it looks significantly deeper than it is wide.)
- Can you tell by looking whether or not half of the full tennis court is a square? (Answers may vary, but it is close enough that it is not *clear* one way or another.)
- Would the painter's mistake change your answers to questions 1 or 2? (No, the sides of the quadrilaterals would all increase by 10 cm and they still would not be squares)

- Would you notice the painter's mistake if you were playing on the mis-painted court? (A professional player may notice, but an unseasoned player may not because 5 cm is very little compared to 11 metres and 24 metres.)

Lesson Synthesis

In this lesson, we used decimal operations to solve several problems involving measurements in sports. We noticed that diagrams can help us represent and communicate our mathematical thinking. We also saw that precision in our measurements and calculations matter.

- For which problems did you use a diagram? How did the diagram help you solve the problem?
- For which problems was it appropriate to round and estimate?
- For which problems was a precise answer necessary? For which ones was it not as critical to be precise?
- Find an example of how you applied each of the decimal operations to solve the problems in this lesson.

14.5 Middle School Hurdle Race

Cool Down: 5 minutes

Student Task Statement

Andre is running in an 80 metre hurdle race. There are 8 equally-spaced hurdles on the race track. The first hurdle is 12 metres from the start line and the last hurdle is 15.5 metres from the finish line.

1. Estimate how far the hurdles are from one another. Explain your reasoning.
2. Calculate how far the hurdles are from one another. Show your reasoning.

Student Response

1. Answers vary. Sample responses:
 - About 7 metres. The distance from the first hurdle to the last hurdle is about 50 metres, and there are 7 gaps between hurdles: $50 \div 7$ is about 7.
 - Between 6 and 7 metres. This race is about $\frac{7}{10}$ as long as the 110-metre hurdle race, so the distance between the hurdles is about $\frac{7}{10}$ as much. In the 110-metre hurdle race, the hurdles are a little over 9 metres apart, so in the 80-metre race they are between 6 and 7 metres apart.

2. 7.5 metres. The distance, in metres, from the first to the last hurdle is $80 - 12 - 15.5 = 52.5$. There are 7 equal gaps between hurdles so each of these gaps, in metres, is $52.5 \div 7 = 7.5$.

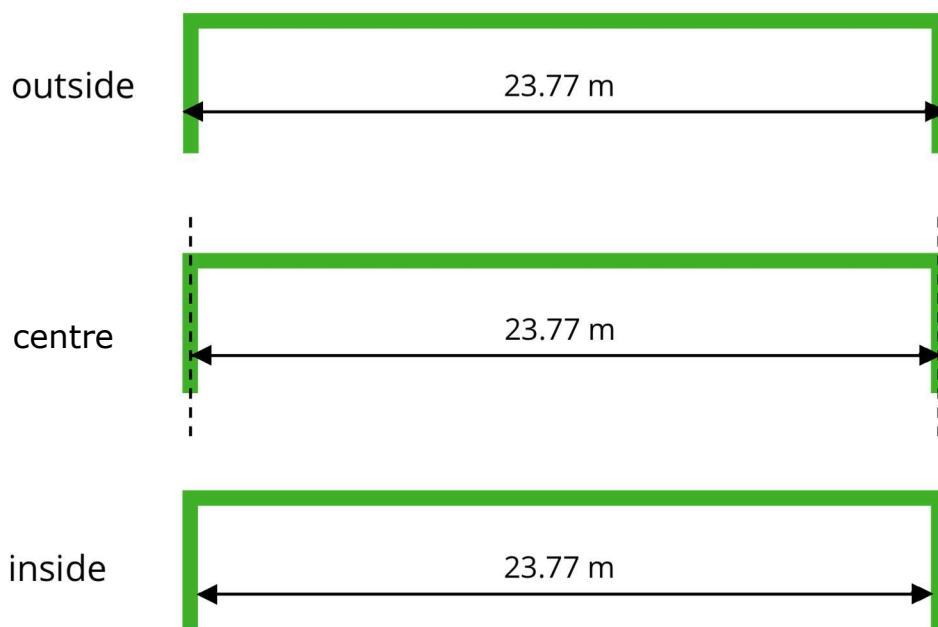
Student Lesson Summary

Diagrams can help us communicate and model mathematics. A clearly-labelled diagram helps us visualise what is happening in a problem and accurately communicate the information we need.

Sports offer great examples of how diagrams can help us solve problems. For example, to show the placement of the running hurdles in a diagram, we needed to know what the distances 13.72 and 14.02 metres tell us and the number of hurdles to draw. An accurate diagram not only helped us set up and solve the problem correctly, but also helped us see that there are only *nine* spaces between ten hurdles.

To communicate information clearly and solve problems correctly, it is also important to be precise in our measurements and calculations, especially when they involve decimals.

In tennis, for example, the length of the court is 23.77 metres. Because the boundary lines on a tennis court have a significant width, we would want to know whether this measurement is taken between the inside of the lines, the centre of the lines, or the outside of the lines. Diagrams can help us attend to this detail, as shown here.



The accuracy of this measurement matters to the tennis players who use the court, so it matters to those who paint the boundaries as well. The tennis players practise their shots to be on or within certain lines. If the tennis court on which they play is not precisely measured, their shots may not land as intended in relation to the boundaries. Court

painters usually need to be sure their measurements are accurate to within $\frac{1}{100}$ of a metre or one centimetre.

Lesson 14 Practice Problems

1. Problem 1 Statement

A roll of ribbon was 12 metres long. Diego cut 9 pieces of ribbon that were 0.4 metre each to tie some presents. He then used the remaining ribbon to make some wreaths. Each wreath required 0.6 metre. For each question, explain your reasoning.

- How many metres of ribbon were available for making wreaths?
- How many wreaths could Diego make with the available ribbon?

Solution

- 8.4 metres. Reasoning varies. Sample reasoning: Diego used $9 \times (0.4)$ or 3.6 metres for the presents, which leaves 8.4 metres in the roll, because $12 - 3.6 = 8.4$.
- 14 wreaths. ($8.4 \div 0.6 = 14$)

2. Problem 2 Statement

The Amazon rainforest covered 6.42 million square kilometres in 1994. In 2014, it covered only $\frac{50}{59}$ as much. Which is closest to the area of the Amazon forest in 2014? Explain how you know without calculating the exact area.

- 6.4 million km^2
- 5.4 million km^2
- 4.4 million km^2
- 3.4 million km^2
- 2.4 million km^2

Solution B

3. Problem 3 Statement

To get an A in her maths class, Jada needs to have at least 90% of the total number of points possible. The table shows Jada's results before the final test in the class.

	Jada's points	total points possible
Homework	141	150
Test 1	87	100

Test 2	81	100
Test 3	91	100

- Does Jada have 90% of the total possible points *before* the final test? Explain how you know.
- Jada thinks that if she gets at least 92 out of 100 on the final test, she will get an A in the class. Do you agree? Explain.

Solution

- No, before the final exam, Jada has 400 points out of 450. But 90% of 450 is $(0.9) \times 450 = 405$. So she is 5 points short of 90%.
- Answers vary. Sample responses:
 - No, Jada is 5 points short of 90% before the last test, so her score on the last test has to be at least 5 points *more* than 90% to make up for this.
 - Maybe. This would give Jada 492 points out of 550. The value of $493 \div 550$ is a little more than 89.6. If the teacher rounds up, Jada will get an A.

4. Problem 4 Statement

Find the following differences. Show your reasoning.

- $0.151 - 0.028$
- $0.106 - 0.0315$
- $3.572 - 2.6014$

Solution

- 0.123. Sample reasoning: 0.151 is 151 thousandths and 0.028 is 28 thousandths. $151 - 28 = 123$, so the difference is 123 thousandths.
- 0.0745. Sample reasoning: 0.106 can be written as 0.1060 and the subtraction can be done using vertical calculation (as shown).
- 0.9706. Sample reasoning: $3.572 - 2.6014$ can be thought of as $2.6014 + ? = 3.572$, and we can use vertical calculation to see what number, when added to 2.6014, makes 3.572 (the missing number is shown in boxes in the calculation).

$$\begin{array}{r}
 \text{b.} \quad \begin{array}{r}
 \overset{0}{\cancel{1}} \overset{10}{\cancel{0}} \overset{5}{\cancel{6}} \overset{10}{\cancel{0}} \\
 - 0.0315 \\
 \hline
 0.0745
 \end{array}
 \qquad
 \text{c.} \quad \begin{array}{r}
 \overset{1}{} \overset{1}{} \\
 2.6014 \\
 + \boxed{0} \boxed{9} \boxed{7} \boxed{0} \boxed{6} \\
 \hline
 3.5720
 \end{array}
 \end{array}$$

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