Lesson 6: Write expressions where letters stand for numbers

Goals

- Explain (orally) how to create and solve an equation that represents a situation with an unknown amount.
- Write an expression with a variable to generalise the relationship between quantities in a situation.

Learning Targets

- I can use an expression that represents a situation to find an amount in a story.
- I can write an expression with a variable to represent a calculation where I do not know one of the numbers.

Lesson Narrative

This lesson is a shift from previous work in this unit. Up until now, we were focused on writing and solving equations. Starting in this lesson, we begin to focus on writing expressions to represent situations. Students write expressions that record operations with numbers and with letters standing in for numbers. Students can choose to represent expressions with bar models if they wish.

Alignments

Addressing

- Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as $5 - y$.
- Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.
- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Instructional Routines

- Algebra Talk
- Anticipate, Monitor, Select, Sequence, Connect
- Collect and Display

- Discussion Supports
- Think Pair Share

Student Learning Goals

Let's use expressions with variables to describe situations.

6.1 Algebra Talk: When x is 6

Warm Up: 5 minutes

The purpose of this algebra talk is to elicit strategies and understandings students have for evaluating an expression for a given value of its variable. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to evaluate expressions.

Instructional Routines

- Algebra Talk
- Discussion Supports

Launch

Give students a minute to see if they recall that x^2 means $x \times x$ (which they learned about in an earlier unit in this course), but if necessary, remind them what this notation means.

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

If x is 6, what is:

$x + 4$

$7 - x$

x^2

$\frac{1}{3}x$

Student Response

- 10

- 1
- 36
- 2

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to _____’s strategy?”
- “Do you agree or disagree? Why?”

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

6.2 Lemonade Sales and Heights

15 minutes

Throughout this unit students have been matching equations to bar models, matching equations to situations, and solving equations. This lesson shifts the focus to writing the expressions that describe situations with an unknown quantity. Students use operations to calculate quantities and notice repeated patterns in those calculations. They replace a part of the calculation with a letter to represent any possible value and create an expression that represents the situation. Students learn that they can use these expressions to answer questions about specific values. Monitor for students who use different strategies to answer the second part of each question. For each, select at least one student who uses a less-efficient method like trial-and-error and one student who writes and solves an equation. Note if any student represents a situation with a bar model—this can be presented to support reasoning about an unknown quantity using a variable.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Collect and Display

Launch

Allow students 10 minutes quiet work time followed by a whole-class discussion.

Representation: Internalise Comprehension. Differentiate the degree of difficulty or complexity by beginning with more accessible values. Extend the given table, and begin by exploring values for money collected based on 1, 2, 3, 5, and 10 cups of lemonade sold. Draw students' attention to what changes and what stays the same each time they calculate the money collected.
Supports accessibility for: Conceptual processing Conversing, Representing, Writing: Collect and Display. During small-group discussion, listen for and collect the vocabulary and phrases students use to describe how to find the values of the table and how the expression represents the situation (e.g., "the number of cups is twice the number of pounds"). Make connections between how similar ideas are communicated and represented in different ways (e.g., "How do you see 'twice' in the bar models and expressions?"). Remind students to borrow language from the display as needed. This will help students to use academic mathematical language during paired and group discussions when writing expressions representing situations with an unknown quantity.
Design Principle(s): Maximise meta-awareness

Student Task Statement

1. Lin set up a lemonade stand. She sells the lemonade for £0.50 per cup.

   a. Complete the table to show how much money she would collect if she sold each number of cups.

lemonade sold (number of cups)	12	183	c
money collected (pounds, £)			

   b. How many cups did she sell if she collected £127.50? Be prepared to explain your reasoning.

2. Elena is 59 inches tall. Some other people are taller than Elena.

   a. Complete the table to show the height of each person.

person	Andre	Lin	Noah
how much taller than Elena (inches)	4	$6\frac{1}{2}$	d
person's height (inches)			

   b. If Noah is $64\frac{3}{4}$ inches tall, how much taller is he than Elena?

Student Response

a.	lemonade sold (number of cups)	12	183	c
	money collected (pounds, £)	6	91.50	$0.5c$

b. 255 cups. (The number of cups is twice the number of pounds.)

a.	person	Andre	Lin	Noah
	how much taller they are than Elena (inches)	4	$6\frac{1}{2}$	d
	person's height (inches)	63	$65\frac{1}{2}$	$59+d$

b. $5\frac{3}{4}$ inches. ($64\frac{3}{4}$ is $5\frac{3}{4}$ more than 59.)

Activity Synthesis

The goal of the discussion is to ensure students see that they can write a mathematical expression to represent a calculation, even if they do not know what one of the numbers is in the calculation. Select students to present who solved the second part of each problem with different strategies. If no student took an approach with equations, demonstrate that $0.5c = 127.50$ can represent the situation and remind students they can find c with $\frac{127.5}{0.5}$. Similarly for the second problem, demonstrate that $59 + d = 64\frac{3}{4}$ can represent the situation and d can be found with $64\frac{3}{4} - 59$. Ask students to explain how the equations make use of the expressions they wrote in the tables and why they can write the equations in these ways.

6.3 Building Expressions

15 minutes

The purpose of this activity is to help students write expressions given a situation, then to solve equations involving the same situation. For the first three questions, the work is still scaffolded by providing numbers to use to calculate before prompting students to write an expression that uses a variable.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 5–10 minutes of quiet work time and time to share with a partner, followed by a whole-class discussion.

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, invite students to draw a picture or bar model to help as an intermediate step before writing an equation.
Supports accessibility for: Social-emotional skills; Conceptual processing

Anticipated Misconceptions

There are multiple quantities in each problem. Students may lose track of what the variable represents.

Students may not see the value in setting up an equation if they can solve it mentally. Later problems are less likely to be solved mentally, so encourage students to write and solve an equation each time.

Student Task Statement

1. Clare is 5 years older than her cousin.

    a. How old would Clare be if her cousin is:

    10 years old?

    2 years old?

    x years old?

    b. Clare is 12 years old. How old is Clare's cousin?

2. Diego has 3 times as many comic books as Han.

    a. How many comic books does Diego have if Han has:

    6 comic books?

    n books?

    b. Diego has 27 comic books. How many comic books does Han have?

3. Two fifths of the vegetables in Priya's garden are tomatoes.

    a. How many tomatoes are there if Priya's garden has:

    20 vegetables?

    x vegetables?

    b. Priya's garden has 6 tomatoes. How many total vegetables are there?

4. A school paid £31.25 for each calculator.

    a. If the school bought x calculators, how much did they pay?

b. The school spent £500 on calculators. How many did the school buy?

Student Response

a. 15 years old, 7 years old, $x + 5$ years old

b. 7 years old, since $x + 5 = 12$ is true when x is 7.

a. 18 books, $3n$ books

b. 9 books, since $3n = 27$ is true when n is 9.

a. 8 tomatoes, $\frac{2}{5}x$ tomatoes

b. 15 vegetables, since $\frac{2}{5}x = 6$ is true when x is 15.

a. $31.25x$

b. 16 calculators, since $31.25x = 500$ is true when x is 16.

Are You Ready for More?

Kiran, Mai, Jada, and Tyler went to their school carnival. They all won chips that they could exchange for prizes. Kiran won $\frac{2}{3}$ as many chips as Jada. Mai won 4 times as many chips as Kiran. Tyler won half as many chips as Mai.

1. Write an expression for the number of chips Tyler won. You should only use one variable: J, which stands for the number of chips Jada won.
2. If Jada won 42 chips, how many chips did Tyler, Kiran, and Mai each win?

Student Response

1. $\frac{4}{3}J$
2. Tyler has 56 chips. Kiran has 28 chips. Mai has 112 chips.

Activity Synthesis

We can often express one quantity in terms of another unknown quantity because we know a relationship between them. Sometimes we also know a value of this expression, and we can use that understanding to write an equation and solve for the unknown quantity. Consider asking some of the following questions to guide the discussion:

- "What facts describing each situation helped you to write the expression?"
- "How did you use the expression to write an equation? Why were you able to set the quantities on each side of the equation equal to each other? Can you give an example of this?" (They represent the same quantity in the story so they have to be equal; for

example, the number of tomatoes in Priya's garden is both 6 and $\frac{2}{5}$ of the number of vegetables, x, in her garden.)

- "What strategies did you use to solve each equation?"
- "How did you check that your solution was correct?" (The best way to check when there is a context is to go back to the original situation and see if the solution makes the statements true. Checking in the equation has the problem that you won't catch if the equation you wrote does not correctly represent the situation.)

Listening, Conversing: Discussion Supports. Support whole-class discussion by displaying and inviting students to use these sentence frames: "To solve each equation I _____ because _____. " or "To check my work is correct, I can _____ because _____." As students share, encourage other students revoice or press for more explanation by asking, "So what I heard you say is ____" or "Can you tell me more about _____?"
Design Principle(s): Cultivate conversation; Optimise output

Lesson Synthesis

Keep students in the same groups. One partner makes up a story, similar to the situations they saw in the lesson, where they describe a relationship between two quantities. The second student assigns a value to one quantity and the other is unknown. As an example, tell students that you have half as many books as your friend, and you have 130 books. Your friend's number of books is the unknown quantity, let's call it b. You can then write the expression $\frac{1}{2}b$ to represent your number of books, and the equation $\frac{1}{2}b = 130$ to describe the situation. b can then be found with $\frac{130}{\frac{1}{2}}$. Writing their own stories helps students reason about the meaning and structure of expressions and equations and the situations they represent.

6.4 Crazy Eights

Cool Down: 5 minutes

Student Task Statement

A plant measured x inches tall last week and 8 inches tall this week.

1. Circle the expression that represents the number of inches the plant grew this week. Explain how you know.
   - $x - 8$
   - $8 - x$
2. For the expression *not* chosen, describe a situation that the expression might represent.

Student Response

1. $8 - x$. Sample explanation: Since the plant grew taller this week, 8 is greater than x. The difference of 8 and x is the amount that the plant grew.

2. Answers vary. Sample response: Elena has more roses than Lin. Lin has 8 roses and Elena has x roses. $x - 8$ represents how many more roses Elena has than Lin.

Student Lesson Summary

Suppose you share a birthday with a neighbour, but she is 3 years older than you. When you were 1, she was 4. When you were 9, she was 12. When you are 42, she will be 45.

If we let a represent your age at any time, your neighbor's age can be expressed $a + 3$.

your age	1	9	42	a
neighbour's age	4	12	45	$a + 3$

We often use a letter such as x or a as a placeholder for a number in expressions. These are called *variables* (just like the letters we used in equations, previously). Variables make it possible to write expressions that represent a calculation even when we don't know all the numbers in the calculation.

How old will you be when your neighbour is 32? Since your neighbour's age is calculated with the expression $a + 3$, we can write the equation $a + 3 = 32$. When your neighbor is 32 you will be 29, because $a + 3 = 32$ is true when a is 29.

Lesson 6 Practice Problems

1. **Problem 1 Statement**

   Instructions for a craft project say that the length of a piece of red ribbon should be 7 inches less than the length of a piece of blue ribbon.

   a. How long is the red ribbon if the length of the blue ribbon is:

      10 inches?

      27 inches?

      x inches?

   b. How long is the blue ribbon if the red ribbon is 12 inches?

   Solution

   a. 3 inches $(10 - 7 = 3)$, 20 inches $(27 - 7 = 20)$, $x - 7$ inches

   b. 19 inches $(12 + 7 = 19)$

2. **Problem 2 Statement**

Tyler has 3 times as many books as Mai.

a. How many books does Mai have if Tyler has:

15 books?

21 books?

x books?

b. Tyler has 18 books. How many books does Mai have?

Solution

a. 5 books $(15 \div 3 = 5)$, 7 books $(21 \div 3 = 7)$, $\frac{x}{3}$ books

b. 6 books $(18 \div 3 = 6)$

3. **Problem 3 Statement**

A bottle holds 24 ounces of water. It has x ounces of water in it.

a. What does $24 - x$ represent in this situation?

b. Write a question about this situation that has $24 - x$ for the answer.

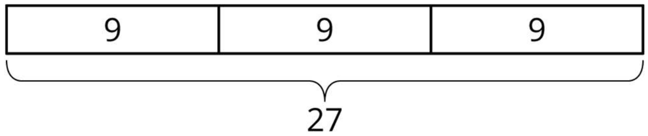
Solution

a. The amount of water that has been removed from the bottle.

b. Answers vary. Sample response: How many ounces of water did Jada drink from the full bottle if there are x ounces left?

4. **Problem 4 Statement**

Write an equation represented by this bar model using each of these operations.



a. addition

b. subtraction

c. multiplication

d. division

Solution

Answers vary. Sample responses:

a. $9 + 9 + 9 = 27$

b. $27 - 9 = 9 + 9$

c. $3 \times 9 = 27$

d. $27 \div 3 = 9$

5. **Problem 5 Statement**

Select **all** the equations that describe each situation and then find the solution.

a. Han's house is 450 meters from school. Lin's house is 135 meters closer to school. How far is Lin's house from school?

- $z = 450 + 135$
- $z = 450 - 135$
- $z - 135 = 450$
- $z + 135 = 450$

b. Tyler's playlist has 36 songs. Noah's playlist has one quarter as many songs as Tyler's playlist. How many songs are on Noah's playlist?

- $w = 4 \times 36$
- $w = 36 \div 4$
- $4w = 36$
- $\frac{w}{4} = 36$

Solution

a. $z = 450 - 135, z + 135 = 450; z = 315$

b. $w = 36 \div 4, 4w = 36; w = 9$

(From Algebra 1.4)

6. **Problem 6 Statement**

You had £50. You spent 10% of the money on clothes, 20% on games, and the rest on books. How much money was spent on books?

Solution

£35 Reasoning varies. Sample reasoning: £5 was spent on books, because $50 \times 0.1 = 5$. £10 was spend on games, because $50 \times 0.2 = 10$. £15 is the combined amount spent on books and games. That leaves £35, because $50 - 15 = 35$.

7. **Problem 7 Statement**

A trash bin has a capacity of 50 gallons. What percentage of its capacity is each amount? Show your reasoning.

a. 5 gallons

b. 30 gallons

c. 45 gallons

d. 100 gallons

Solution

a. 5 gallons is 10% of 50 gallons, because $5 \div 50 = 0.1$.

b. 30 gallons is 60% of 50 gallons, because $30 \div 50 = 0.6$.

c. 45 gallons is 90% of 50 gallons, because $45 \div 50 = 0.9$.

d. 100 gallons is 200% of 50 gallons, because $100 \div 50 = 2$.