

Grades 5-8 (A), 9-12 (A)

Duration: 20 min

Tools: one Logifaces Set / class

Pair / Class work

Keywords: Volume, Truncated prism, Proof

518 - Proof of the Volume Formula



MATHS / 3D GEOMETRY



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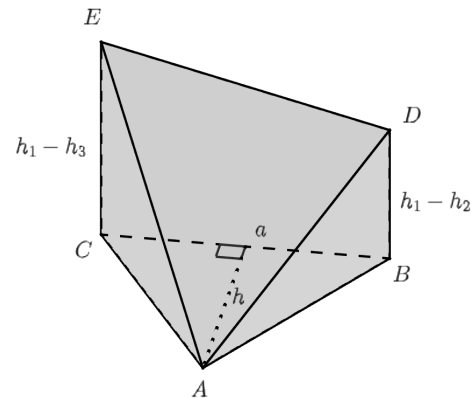
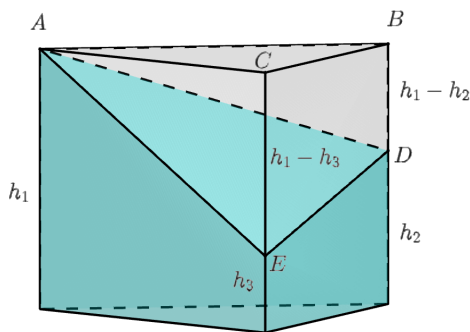
DESCRIPTION

Students prove the following formula for the volume of the truncated prism with triangular base:

$\frac{1}{3}A(h_1 + h_2 + h_3)$, where h_1, h_2 and h_3 are the heights of the prism and A is the area of the base triangle.

Observe that the proof works for an arbitrary base triangle, it is not necessary to have a regular base triangle.

SOLUTIONS / EXAMPLES



Let the three heights of the truncated prism be in the following order:

$$h_1 \geq h_2 \geq h_3.$$

If the truncated prism is cut from the regular prism with height h_1 , a pyramid $ABCDE$ is obtained with edge lengths $|CE| = x = h_1 - h_3$ and $|BD| = y = h_1 - h_2$ (see the figure).

The volume V_{tp} of the truncated prism with heights h_1, h_2 and h_3 will be calculated as the volume V_{rp} of the regular prism of height h_1 minus the volume V_{pyr} of the pyramid $ABCDE$.

The volume of the regular prism: $V_{rp} = A \times h_1$, where $A = \frac{a \times h}{2}$ is the area of the base triangle ABC with edge $BC = a$ and height h (see the figure for the notations).

The base of the pyramid $ABCDE$ is the trapezium $BCED$ with area $A_{tr} = \frac{(x+y)}{2} \times a$. Therefore the volume of the pyramid $ABCDE$ is $V_{pyr} = \frac{A_{tr} \times h}{3} = \frac{1}{3} \times \frac{(x+y)}{2} \times a \times h = \frac{1}{3} \times (x+y) \times \frac{a \times h}{2} = \frac{1}{3} (x+y)A$

Hence the volume of the truncated prism is:

$$V_{tp} = V_{rp} - V_{pyr} = A \times h_1 - \frac{1}{3}A(h_1 - h_3 + h_1 - h_2) = A \times \frac{3h_1 - (2h_1 - h_2 - h_3)}{3} = \frac{1}{3}A(h_1 + h_2 + h_3)$$

Notes on the proof:

Observe that the proof works even if there are vertical edges of the truncated prism of the same length.

The pyramid is a special case of a truncated prism with $h_3 = 0$. The formula is valid even in this case.

PRIOR KNOWLEDGE

Volume, Area, Volume of prisms and pyramids

RECOMMENDATIONS / COMMENTS

We recommend exercise [517 - Heights and Volumes](#) to calculate the volume of the blocks using this formula.