Lesson 3: Representing data graphically

Goals

- Comprehend the word "frequency" to refer to the number of times a particular value occurs in a data set.

- Create and interpret a dot plot to answer statistical questions about a quantitative data set.

- Justify (in writing) whether a dot plot is an appropriate way to display a given data set, paying attention to whether the data set is quantitative or qualitative.

Learning Targets

- I can describe the information presented in tables, dot plots, and bar graphs.

- I can use tables, dot plots, and bar graphs to represent distributions of data.

Lesson Narrative

In this lesson, students represent distributions of quantitative (and optionally qualitative) data after organising them into frequency tables. They construct dot plots for quantitative data (and bar graphs for qualitative data). Using graphical representations of distributions, they continue to develop a spatial understanding of distributions in preparation for understanding the concepts of "centre" and "spread" in future lessons. Students make use of the structure of dot plots to describe distributions and draw conclusions about the data.

Building On

- Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many fewer" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

Addressing

- Recognise a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.

- Display quantitative data in plots on a number line, including dot plots, histograms, and box plots.

- Reporting the number of observations.

- Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

Building Towards

- Understand that a set of data collected to answer a statistical question has a distribution which can be described by its centre, spread, and overall shape.

Instructional Routines

- Collect and Display

- Discussion Supports

Required Materials

Dot stickers
Small circular sticker useful for plotting points on a display.

Sticky notes
Straightedges
A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

Required Preparation

1 sticky note and 1 dot sticker for each student. Straightedges should be made available to create dot plots.

Student Learning Goals

Let's represent data with dot plots and bar graphs.

3.1 Curious about Caps

Warm Up: 5 minutes

The purpose of this warm-up is to reinforce the distinction between statistical and non-statistical questions. Students write a statistical question and articulate why it qualifies as statistical. Students' explanations should focus on the variability in the data used to answer the question. The context will be used again in the next activity, so this also gets students familiar with the object.

Launch

Arrange students in groups of 3–4. Give students 1 minute of quiet work time. Then, tell students to share their question and reasoning within their group. Tell the groups to decide if each question is statistical or non-statistical. If the group disagrees, discuss the question further and revise it until an agreement is reached.

Student Task Statement

Clare collects bottle caps and keeps them in plastic containers.

Write one statistical question that someone could ask Clare about her collection. Be prepared to explain your reasoning.

Student Response

Answers vary. Sample questions:

- In general, how many caps fit in a container? (This would require counting the number of caps in multiple containers, and the number of caps that can fit in a container will vary).

- What is the most common diameter of the bottle caps in the collection? (The caps do not all have the same diameters, and to find out the most common diameter would require collecting measurements of the caps.)

- What is the most common bottle-cap colour in the collection? (This would require tallying up the number of caps of each colour.)

- How long did it typically take to fill up a container? (This would require finding out how long it takes to fill each container, and the amount of time would likely vary.)

Activity Synthesis

The purpose of the discussion is to ensure students are comfortable constructing statistical and non-statistical questions.

Ask students to share their statistical questions and reasoning. If possible, ask students to refer back to the image as they share. Record and display their responses for all to see.

To involve more students in the conversation, consider asking some of the following questions:

- "Were there any questions your group had in common?"

- "Were there any questions your group could not agree were statistical? Why?"

- "Were there any questions your group decided were non-statistical? What made that question non-statistical?"

3.2 Estimating Caps

15 minutes

In this activity, students are motivated to create a dot plot by identifying a statistical question and collecting data from the class to answer the question. When the data is initially presented, it is messy and difficult to analyse in an unorganised form. Students are then asked to choose an appropriate representation to use for the data in a way that can be understood better by using a dot plot. Once the data is organised, students attempt to find a typical value from the data. In addition, students will be practising their estimation skills by guessing the number of items in a jar.

Instructional Routines
- Discussion Supports

Launch

If any groups in the warm-up asked how many caps were included in the jar, use that as a way to transition into this activity.

Tell students to keep their materials closed for this discussion. Give each student a sticky note and a dot sticker. Display the image and questions for all to see.

- Question 1: How many caps are in the jar?

- Question 2: In this class, what is a common estimate for the number of caps in the jar?

- Question 3: What is the teacher's estimate for the number of caps in the jar?

Give students a minute of quiet think time to determine if each question is a statistical question and their reasoning. Invite students to share their responses. Once the class agrees that Question 2 is the only statistical question and has a good reason, ask students to write down an estimate for the number of caps in the jar on a sticky note and remember their answer. Next, ask them to affix their sticky notes on a wall or a board for all to see, and then use the display to answer the statistical question.

Students should recognise that the data cannot be easily interpreted in this format. Discuss other ways the estimates can be displayed so that they can be easily seen and understood. If no students suggest placing the sticky notes along a number line like a dot plot, suggest this idea to them.

Next, look for the smallest and largest estimates (either by asking students or skimming through the sticky notes). Draw or display a number line that spans those two numbers, large enough so that students could affix their dot stickers along it. Have each student put their dot sticker on the right place along the number line. If more than one person made the same estimate, the second person should put theirs higher up on the board in the same horizontal position. When the dot plot is complete, tell students to open their materials and answer the questions.

Student Task Statement
1. Write down the statistical question your class is trying to answer.

2. Look at the dot plot that shows the data from your class. Write down one thing you notice and one thing you wonder about the dot plot.

3. Use the dot plot to answer the statistical question. Be prepared to explain your reasoning.

Student Response
1. In this class, what is a common estimate for the number of caps in the jar?

2. Answers vary. Sample responses:

   I notice that:

   - The dots are mostly grouped around a certain number.

   - There is a wide range of guesses.

   - Nobody guessed fewer than 20.

   I wonder:

   - Why are the guesses so spread out?

   - Where is the middle of the dot plot?

– Where does the real answer fit on the dot plot?

3. Answers vary depending on the class. Sample response: There are a lot of guesses near 60.

Activity Synthesis

The purpose of this discussion is to see why a dot plot can be useful to visualise and quickly understand a large amount of data.

Ask students what they noticed about the display and for any questions they had about the display. Ask students what advantages this display might have over other ways of arranging the sticky notes.

Tell students that the actual number of bottle caps in the jars is 82. Ask students how this number relates to the dot plot created by the class.

In general, the average from a group of individual guesses can be more accurate than any individual guess. This phenomenon is called "wisdom of the crowd" and is relied on for things such as reviews for products sold online.

Representing, Conversing: Discussion Supports. Before the whole-class discussion, give students an opportunity to discuss what advantages a dot plot might have over other ways of arranging the sticky notes. Display sentence frames such as: "This display is useful because…"; "The information that I can see is …"; "This display helps me visualise the data by …" Invite students to practice using these sentence frames together before selecting 2–3 students to share with the class. This will help students make sense of how a dot plot can be used to understand a large amount of data.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

3.3 Been There, Done That!

15 minutes

In the previous activity, the class created a dot plot together. In this activity, students create a dot plot on their own. The work of drawing dot plots is not new, but students are asked to describe their analysis of the data broadly and with limited scaffolding. They also learn to use the term *frequency* to describe the number of occurrences associated with each numerical value or category.

As students work and discuss, identify those who are able to describe the *distribution* of the data clearly and succinctly, as well as students who can articulate why a dot plot is an appropriate representation. Ask them to share later.

Instructional Routines
• Collect and Display

Launch

Keep students in groups of 3–4. Explain that the term **frequency** associated with a particular item represents the number of times an item occurs in the data set. For example, in the set of numbers 1, 2, 1, 4, 5, 1, 1, the number 1 has a frequency of 4 since it appears 4 times in the data set. In the same data set, the number 2 has a frequency of 1 and the number 3 has a frequency of 0 since it does not appear in the data set at all. Additionally explain that the **distribution** refers to a description of how the data is arranged (or distributed) in the dot plot.

Give groups 5–6 minutes to work together to organise the given data and draw their dot plots. Then, give them 2–3 minutes of quiet time to analyse the dot plots and answer the last two questions, and time for a brief small-group discussion about their responses. Reconvene as a whole class afterwards.

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: spread, distribution and frequency.
Supports accessibility for: Memory; Language

Anticipated Misconceptions

When drawing dot plots, some students might use dots of different sizes or neglect to stack the dots in a straight column. Remind students to use uniform dots and to stack them vertically, using a straightedge as a guide, if needed. For some students, the use of graph paper may be helpful.

Student Task Statement

Priya wants to know if basketball players on a men's team and a women's team have had prior experience in international competitions. She gathered data on the number of times the players were on a team before 2016.

men's team

| 3 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

women's team

| 2 | 3 | 3 | 1 | 0 | 2 | 0 | 1 | 1 | 0 | 3 | 1 |

1. Did Priya collect qualitative or quantitative data?

2. Organise the information on the two basketball teams into these tables.
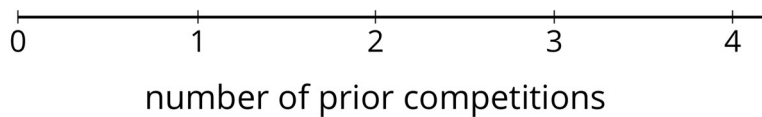
    Men's Basketball Team Players

number of prior competitions	frequency (number)
0	
1	
2	
3	
4	

Women's Basketball Team Players

number of prior competitions	frequency (number)
0	
1	
2	
3	
4	

3.   Make a dot plot for each table.

Men's Basketball Team Players

```
 ├────────┼────────┼────────┼────────┤
 0        1        2        3        4
```
number of prior competitions

Women's Basketball Team Players

```
 ├────────┼────────┼────────┼────────┤
 0        1        2        3        4
```
number of prior competitions
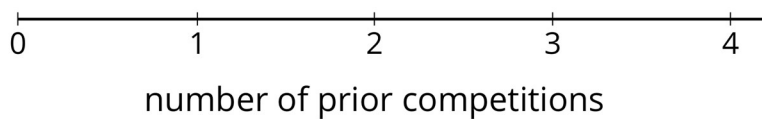
4. Study your dot plots. What do they tell you about the competition participation of:

    a. the players on the men's basketball team?

    b. the players on the women's basketball team?

5. Explain why a dot plot is an appropriate representation for Priya's data.
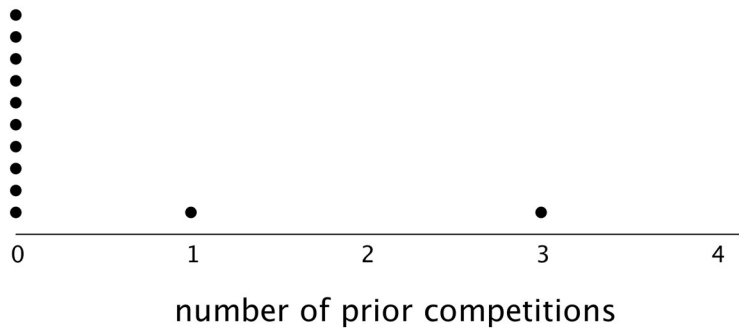
Student Response

1. Priya collected quantitative data.

2. Men's Basketball Team
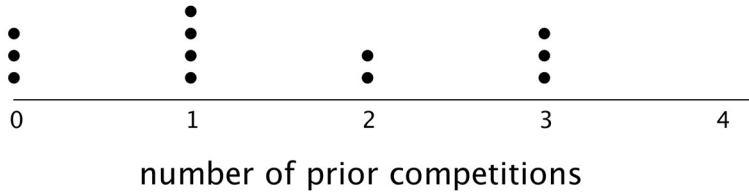
number of prior competitions	frequency (number)
0	10
1	1
2	0
3	1
4	0

Women's Basketball Team

number of prior competitions	frequency (number)
0	3
1	4
2	2
3	3
4	0

3. Men's Team

number of prior competitions

Women's Team

number of prior competitions

4. Answers vary. Sample responses:

   a. The vast majority of players on the men's team are athletes for which this is their first competition. Only 2 of the 12 are returning athletes.

   b. Three-quarters of the players on the women's team are returning athletes. Only 3 players are at their first competition.
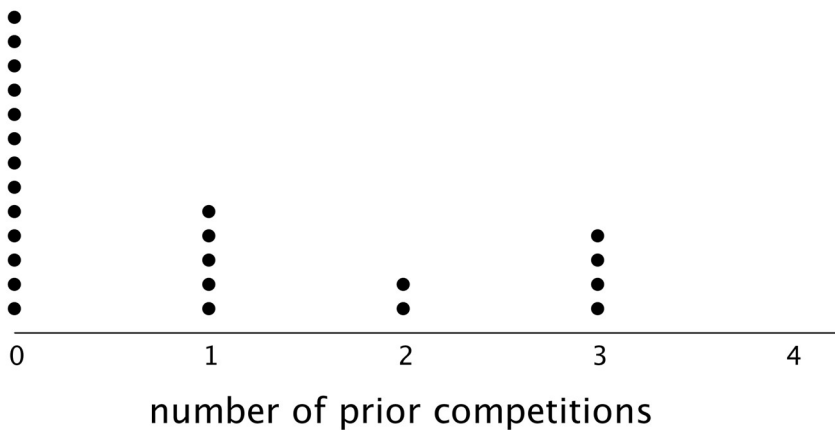
5. Answers vary. Sample response: Priya's data values are numerical, so they can be organised along a number line with a dot plot.

Are You Ready for More?

Combine the data for the players on the men's and women's teams and represent it as a single dot plot. What can you say about the repeat participation of the basketball players?

number of prior competitions

Student Response
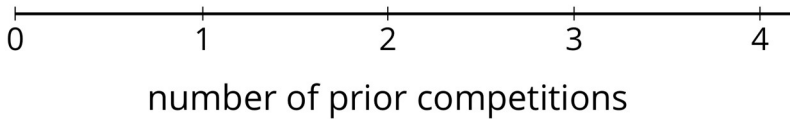
number of prior competitions

Answers vary. Sample response: About half of the basketball players did not play in a previous competition. About 25% of the players had been at one prior competition, and another 25% had 2 or 3 prior competitions.

Activity Synthesis

Select previously identified students to share their comments on the competition participation of the male and female athletes and why a dot plot is an appropriate representation. Then, invite them to compare the features and merits of the representations they used in this lesson. Discuss:

- "In what ways might bar graphs and dot plots be more useful than lists and tables?" (Bar graphs and dot plots both give us a visual snapshot of the data so that we can see patterns or anything unusual.)

- "How are bar graphs and dot plots alike? How are they different?" (They both show frequencies of data vertically–the more frequent a data value occurs, the taller the stack or bar. They are different in that the horizontal axis of a dot plot shows numbers, while the one for a bar graph shows different labels.)

- "Could a dot plot be used to represent the first letter of the last names of the players represented on these teams? Why or why not?" (No, because a dot plot uses a number line. Each point on the number line represents a number, so where a data point is placed matters. A bar graph could be used where the categories could be shown in any order, and the width of the bars overall is flexible.)

Explain to students that a frequency table, bar graph, and dot plot all tell us about the distribution of a data set: each of them lists or shows all the possible values or categories in a data set and how often each one occurs. Throughout the unit, we will investigate distributions closely and use them to learn more about data and the groups or situations they represent.

Reading, Representing: Collect and Display. During the discussion about similarities and differences between bar graphs and dot plots, collect students' responses in a graphic organiser and display it for students to reference later. This will help students recognise language related to dot plots, assisting them with making decisions about appropriate use for different data contexts.
Design Principle(s): Support sense-making; Maximise meta-awareness

3.4 Favourite Summer Sports

Optional: 15 minutes

This activity is marked optional since it goes beyond the expectations of the standards. The activity helps students think about how bar graphs compare to dot plots and are useful in contrast to future lessons working with histograms.

In this activity, students organise qualitative data into frequency tables and represent them as a bar graph. Neither task is new to KS3 students, but students approach data analysis

with a new awareness of data types and they use data to answer questions that are more open-ended. Later, they will contrast the representation for qualitative data with those for quantitative data. Students also learn to use the term *frequency* to describe the number of times a data value occurs.

As students draw and analyse their bar graphs, listen for the questions they might ask one another about drawing decisions. For example, they might wonder about the order in which the categories are displayed, the width of the bars, or whether certain sports might belong in the same category (e.g., swimming and diving). Also notice the arguments students make about whether a dot plot would be a suitable representation for the data.

Launch

Keep students in groups of 3–4. Provide access to straightedges.

Explain to students that they will organise some data and represent them by drawing a bar graph. Ask a few students to share what they know or remember about bar graphs. Draw or display a sample bar graph for all to see. Solicit some comments about what features bar graphs have, what the bars represent, and what their heights tell us. When discussing the heights of the bars, tell students that we can use the term "frequency" for "the number of occurrences."

Give groups 5–6 minutes to work together to organise the given data and draw their bar graph. Then, give them 2–3 minutes of quiet time to analyse the bar graph and answer the last two questions, and time for a brief small-group discussion about their responses. Reconvene as a whole class afterwards.

Action and Expression: Internalise Executive Functions. To support development of organisational skills, check in with students within the first 2-3 minutes of work time. Check to make sure students have developed a way to keep track of their counting. *Supports accessibility for: Memory; Organisation*

Anticipated Misconceptions

When determining the frequencies of different sports students might lose track of their counting. If this happens, urge students to check off each sport as they account for them and then double-check their counts afterwards.

Student Task Statement

Kiran wants to know which three summer sports are most popular in his class. He surveyed his classmates on their favourite summer sport. Here are their responses.

swimming	gymnastics	track and field
volleyball	swimming	swimming
diving	track and field	gymnastics
basketball	basketball	volleyball

track and field	track and field	volleyball
gymnastics	diving	gymnastics
volleyball	rowing	track and field
track and field	football	swimming
gymnastics	track and field	swimming
rowing	diving	football

1. Did Kiran collect qualitative or quantitative data?

2. Organise the responses in a table to help him find which summer sports are most popular in his class.

sport	frequency

3. Represent the information in the table as a bar graph.
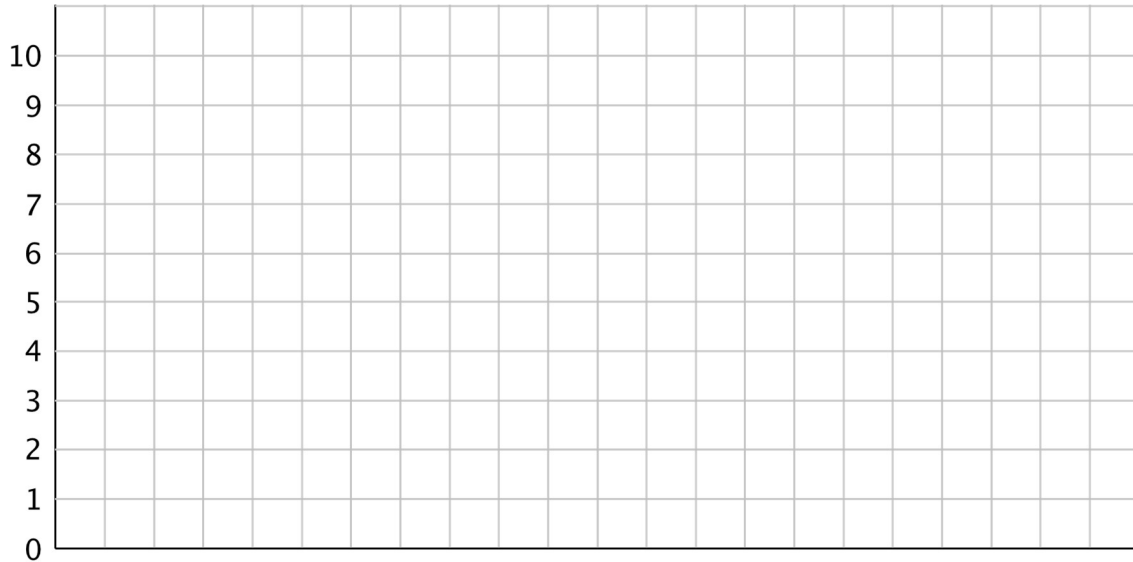
a.   How can you use the bar graph to find how many classmates Kiran surveyed?

b.   Which three summer sports are most popular in Kiran's class?

c.   Use your bar graph to describe at least one observation about Kiran's classmates' preferred summer sports.

4.   Could a dot plot be used to represent Kiran's data? Explain your reasoning.

Student Response

1.   Kiran collected qualitative data.

2.

sport	frequency
basketball	2
diving	3
gymnastics	5
rowing	2
football	2
swimming	5
track and field	7
volleyball	4

3.
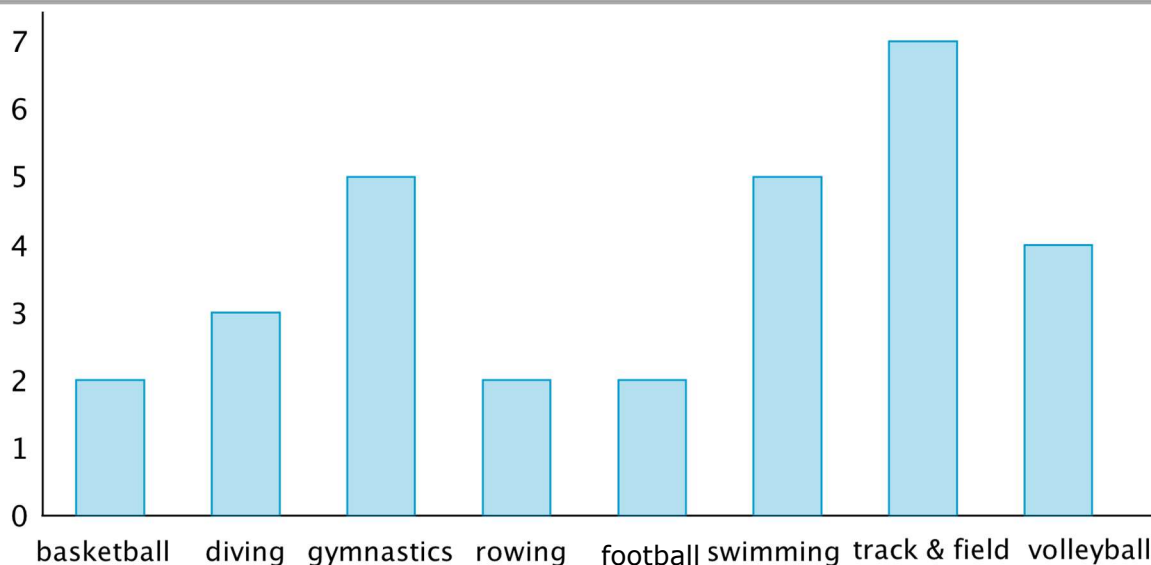
    a.   Add the heights of each bar to find the total number of classmates surveyed. Kiran surveyed 30 classmates.

    b.   Track and field is the most popular summer sport. Swimming and gymnastics tied for the second most popular summer sport.

    c.   Answers vary. Sample observations: More than half the class chose one of these top three sports as their favourite.

4.   No, a dot plot could not be used to represent qualitative data. A dot plot is built on a number line and is to be used with quantitative data.

Activity Synthesis

The purpose of the discussion is for students to recognise when it is appropriate to use a bar graph and how they compare to dot plots.

Focus the whole-class discussion on the decisions that students made when drawing their bar graph and on the last two questions. Some discussion questions:

•   "How did you know which sport to represent first, second, and so on? Does the order in which the sports are listed matter?"

•   "Does the width of the bars matter?"

•   If some students combined two or more sports into a single category: "How might the bar graph change if we combine two sports, say swimming and diving, into a single bar?"

Select a couple of students to share their observations about favourite summer sports in Kiran's class. Ask the class to see if they agree with those observations. If they don't find those conclusions to be reasonable, ask for their reasoning and alternative conclusions.

End by polling the class about whether a dot plot could be used to represent the data. Ask a student who thinks so to explain their reasoning. Dot plots should only be used to represent quantitative data along a number line and not qualitative data.

Lesson Synthesis

In this lesson, we look at how to organise and represent data. When working with data, we're often particularly interested in the distribution of data.

- "What is the distribution of data?"

- "How do we tell the frequency of a data value from a data set? What about from a table?"

- "How do we tell the frequencies of different data values from a dot plot?"

If the optional activity was completed:

- "What kind of graphical representation can we use to show the distribution of qualitative data? What about for quantitative data?"

- "How do we tell the frequencies of different data values from a bar graph?"

- "Both the dot plot and the bar graph are built on a horizontal line. How are the horizontal lines different?"

3.5 Swimmers and Swimming Class

Cool Down: 5 minutes

Student Task Statement
1. Noah gathered information on the home states of the swimmers on Team USA. He organised the data in a table. Would a dot plot be appropriate to display his data? Explain your reasoning.

2. This dot plot shows the ages of students in a swimming class. How many students are in the class?

```
                 •
         •    •  •     •      •
         •  • •  •   • •   •  • • •
   ├───┬───┬───┬───┬───┬───┬───┬───┬───┬───┬───┤
   16  18  20  22  24  26  28  30  32  34  36
                   age in months
```

3. Based on the dot plot, do you agree with each of the following statements? Explain your reasoning.

   a. The class is an adult swimming class.

    b.   Half of the students are between 2 and 3 years old.

Student Response

1.   No. He could use a bar graph because the home states of swimmers are qualitative data.

2.   16 students are in the class.

    a.   Disagree. The unit of measurements is month, and the data shows ages between 18 and 32 months, which means the students are young children, not adults.

    b.   Agree. On the number line, eight of the 16 data points, or half of the class, are placed to the right of 24 months and to the left of 36 months.

Student Lesson Summary

When we analyse data, we are often interested in the **distribution**, which is information that shows all the data values and how often they occur.

In a previous lesson, we saw data about 10 dogs. We can see the distribution of the dog weights in a table such as this one.

weight in kilograms	frequency
6	1
7	3
10	2
32	1
35	2
36	1

The term **frequency** refers to the number of times a data value occurs. In this case, we see that there are three dogs that weigh 7 kilograms, so "3" is the frequency for the value "7 kilograms."

Recall that dot plots are often used to represent quantitative data. Like a frequency table, a dot plot also shows the distribution of a data set. This dot plot, which you saw in an earlier lesson, shows the distribution of dog weights.


dog weights in kilograms
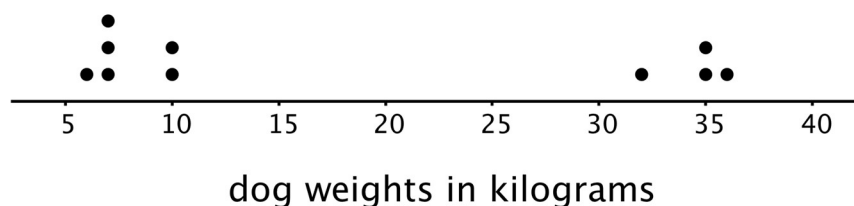
A dot plot uses a horizontal number line. We show the frequency of a value by the number of dots drawn above that value. Here, the two dots above the number 35 tell us that there are two dogs weighing 35 kilograms.

The distribution of qualitative data can also be shown in a table. This table shows the distribution of dog breeds.

breed	frequency
pug	9
beagle	9
German shepherd	12

We often represent the distribution of qualitative data using a bar graph.
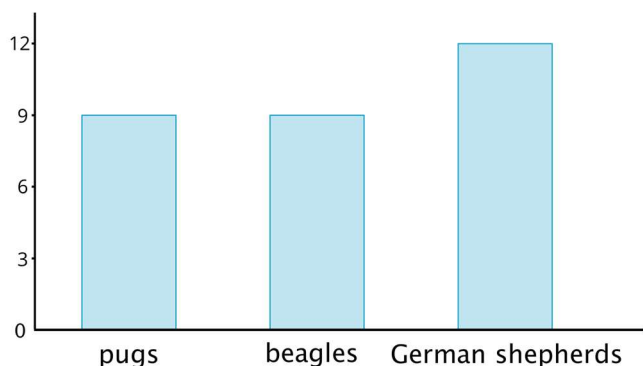
A bar graph also uses a horizontal line. Above it we draw a rectangle (or "bar") to represent each category in the data set. The height of a bar tells us the frequency of the category. There are 12 German shepherds in the data set, so the bar for this category is 12 units tall. Below the line we write the labels for the categories.

In a dot plot, a data value is placed according to its position on the number line. A weight of 10 kilograms must be shown as a dot above 10 on the number line.

In a bar graph, however, the categories can be listed in any order. The bar that shows the frequency of pugs can be placed anywhere along the horizontal line.
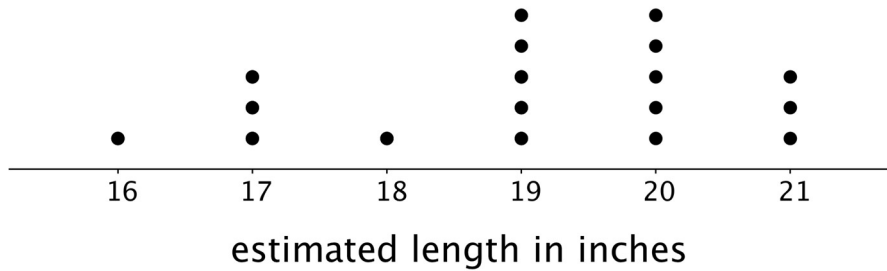
Glossary
- distribution

- frequency

Lesson 3 Practice Problems
Problem 1 Statement

A teacher drew a line segment that was 20 inches long on the blackboard. She asked each of her students to estimate the length of the line segment and used their estimates to draw this dot plot.

estimated length in inches

a. How many students were in the class?

b. Were students generally accurate in their estimates of the length of the line? Explain your reasoning.

Solution

a. 18

b. Responses vary. Sample response: Students tended to underestimate the length of the line, with more than half giving an estimate that was smaller than the actual length of 20. No one overestimated by more than 1 inch, but some students underestimated by as much as 3 or 4 inches.

Problem 2 Statement

Here are descriptions of data sets. Select **all** descriptions of data sets that could be graphed as dot plots.

a. Class size for the classes at a primary school

b. Colours of cars in a car park

c. Favourite sport of each student in a year 7 class

d. Birth weights for the babies born during October at a hospital

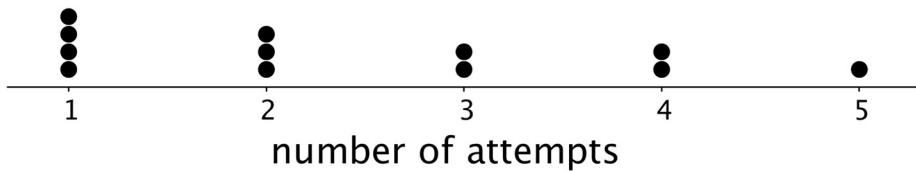Number of goals scored in each of 20 games played by a school football team

Solution ["A", "D", "E"]

Problem 3 Statement

Priya recorded the number of attempts it took each of 12 of her classmates to successfully throw a ball into a basket. Make a dot plot of Priya's data.

| 1 | 2 | 1 | 3 | 1 | 4 | 4 | 3 | 1 | 2 | 5 | 2 |

Solution

number of attempts

Problem 4 Statement

Solve each equation.

    a. $9v = 1$

    b. $1.37w = 0$

    c. $1 = \frac{7}{10}x$

    d. $12.1 = 12.1 + y$

    e. $\frac{3}{5} + z = 1$

Solution

    a. $v = \frac{1}{9}$

    b. $w = 0$

    c. $x = \frac{10}{7}$

    d. $y = 0$

    e. $z = \frac{2}{5}$

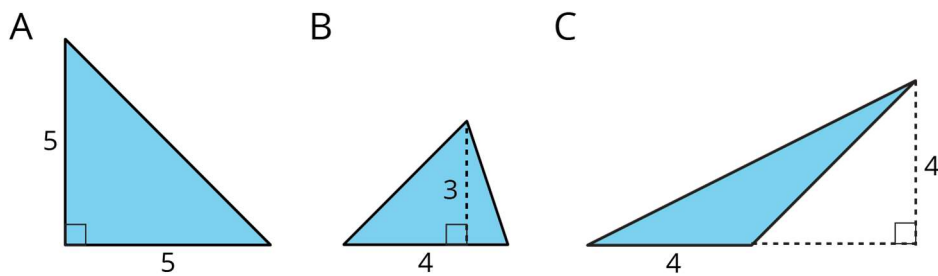### Problem 5 Statement

Find the quotients.

    a. $\frac{2}{5} \div 2$

    b. $\frac{2}{5} \div 5$

    c. $2 \div \frac{2}{5}$

    d. $5 \div \frac{2}{5}$

Solution

    a. $\frac{1}{5}$

    b. $\frac{2}{25}$

    c.   5

    d. $\frac{25}{2}$

Problem 6 Statement

Find the area of each triangle.

A   B   C

Solution

A: 12.5 square units ($5 \times 5 \div 2 = 12.5$)

B: 6 square units ($4 \times 3 \div 2 = 6$)

C: 8 square units ($4 \times 4 \div 2 = 8$)