Lesson 3: Making scaled copies

Goals

- Critique (orally and in writing) different strategies (expressed in words and through other representations) for creating scaled copies of a shape.
- Draw a scaled copy of a given shape using a given scale factor.
- Generalise (orally and in writing) that the relationship between the side lengths of a shape and its scaled copy is multiplicative, not additive.

Learning Targets

- I can draw a scaled copy of a shape using a given scale factor.
- I know what operation to use on the side lengths of a shape to produce a scaled copy.

Lesson Narrative

In the previous lesson, students learned that we can use scale factors to describe the relationship between corresponding lengths in scaled shapes. Here they apply this idea to draw scaled copies of simple shapes on and off a grid. They also strengthen their understanding that the relationship between scaled copies is multiplicative, not additive. Students make careful arguments about the scaling process, have opportunities to use tools like tracing paper or index cards strategically.

As students draw scaled copies and analyse scaled relationships more closely, encourage them to continue using the terms *scale factor* and *corresponding* in their reasoning.

Building On

- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Addressing

- Solve problems involving scale drawings of geometric shapes, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- Recognise and represent proportional relationships between quantities.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Stronger and Clearer Each Time
- Clarify, Critique, Correct
- Compare and Connect

- Think Pair Share

Required Materials

Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Required Preparation

Make sure students have access to their geometry toolkits, especially tracing paper and index cards.

Student Learning Goals

Let's draw scaled copies.

3.1 More or Less?

Warm Up: 5 minutes

This warm-up prompts students to use what they know about numbers and multiplication to reason about decimal calculations. The problems are designed to result in an answer very close to the given choices, so students must be more precise in their reasoning than simply rounding and calculating.

Whereas a number talk typically presents a numerical expression and asks students to explain strategies for evaluating it, this activity asks a slightly different question because students don't necessarily need to evaluate the expression. Rather, they are asked to judge whether the expression is greater than or less than a given value. Although this activity is not quite the same thing as a number talk, the discussion might sound quite similar.

Launch

Display the problems for all to see. Give students 2 minutes of quiet think time. Tell students they may not have to calculate, but could instead reason using what they know about the numbers and operation in each problem. Ask students to give a signal when they have an answer and a strategy for every problem.

Anticipated Misconceptions

Students may attempt to solve each problem instead of reasoning about the numbers and operations. If a student is calculating an exact solution to each problem, ask them to look closely at the characteristics of the numbers and how an operation would affect those numbers.

Student Task Statement

For each problem, select the answer from the two choices.

1. The value of $25 \times (8.5)$ is:
    a. More than 205
    b. Less than 205
2. The value of $(9.93) \times (0.984)$ is:
    a. More than 10
    b. Less than 10
3. The value of $(0.24) \times (0.67)$ is:
    a. More than 0.2
    b. Less than 0.2

Student Response

1. More than 205. Since $8 \times 25 = 200$ and $0.5 \times 25 = 12.5$, then the product must be more than 205.
2. Less than 10. Since $9.93 \times 1 = 9.93$ and 0.9 is less than 1, then the product must be less than 10.
3. Less than 0.2. Since 0.24 is less than $\frac{1}{4}$ and 0.68 is less than 0.8, the product must be less than 0.2 which is $\frac{1}{4}$ of 0.8.

Activity Synthesis

Discuss each problem one at a time with this structure:

- Ask students to indicate which option they agree with.
- If everyone agrees on one answer, ask a few students to share their reasoning, recording it for all to see.
- If there is disagreement on an answer, ask students with differing answers to explain their reasoning and come to an agreement on an answer.

3.2 Drawing Scaled Copies

Optional: 10 minutes (there is a digital version of this activity)

Students continue to work with scaled copies of simple geometric shapes, this time on a grid. When trying to scale non-horizontal and non-vertical line segments, students may think of using tracing paper or a ruler to measure lengths and a protractor to measure angles. Make sure they have a chance to see how the structure of the grid can be useful for scaling the lengths of non-vertical and non-horizonal line segments.

To create scaled copies, students need to attend to all parts of the original shape, or else the copy will not be scaled correctly. Use of the grid for scaling non-horizontal and non-vertical line segments is a good example of using tools strategically.

As students work, monitor for students who find a way to scale line segment lengths properly but neglect to consider the size of corresponding angles (especially in making a copy of shape B and D).

Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

Launch

Give students 3 minutes of quiet time to draw and another 3 minutes to share their drawings with a partner, check each other's work, and make revisions. Provide access to their geometry toolkits.

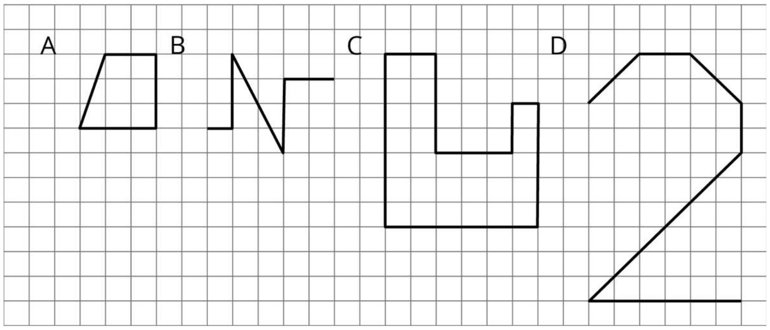
Representation: Internalise Comprehension. Check in with students after the first 2-3 minutes of work time. Check to make sure students have attended to all parts of the original shapes.
Supports accessibility for: Conceptual processing; Organisation

Speaking, Representing: Stronger and Clearer Each Time. Use this routine to support productive discussion when students share their drawings with a partner. Give students time to meet with 2–3 partners, to share and get feedback on their scaled copies. Provide students with prompts for feedback that will help their partners strengthen their ideas and clarify their drawings (e.g., "How did you know how long to make each side length?", "How did you measure to make each angle"?", "How did you use the grid to create your scaled copy?"). Students can borrow ideas and language from each partner to strengthen their work. This provides students with an opportunity to produce verbal mathematical language in service of refining their ideas and their drawings.
Design Principle(s): Optimise output (for justification)

Anticipated Misconceptions

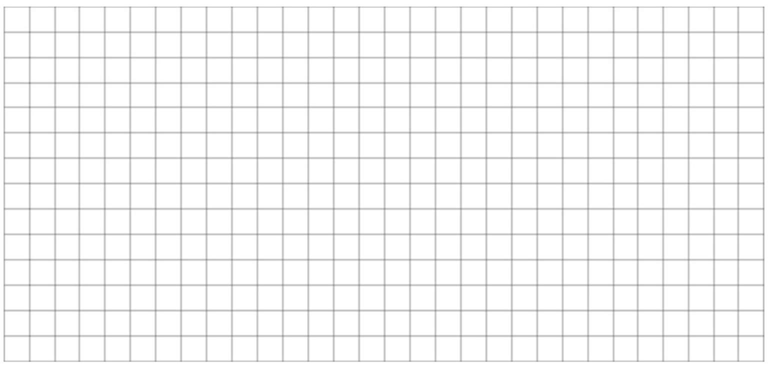
Some students may think that shape C cannot be scaled by a factor of $\frac{1}{2}$ because some vertices will not land on intersections of grid lines. Clarify that the grid helps us see lengths in whole units but line segments we draw on them are not limited to whole units in length.

Student Task Statement

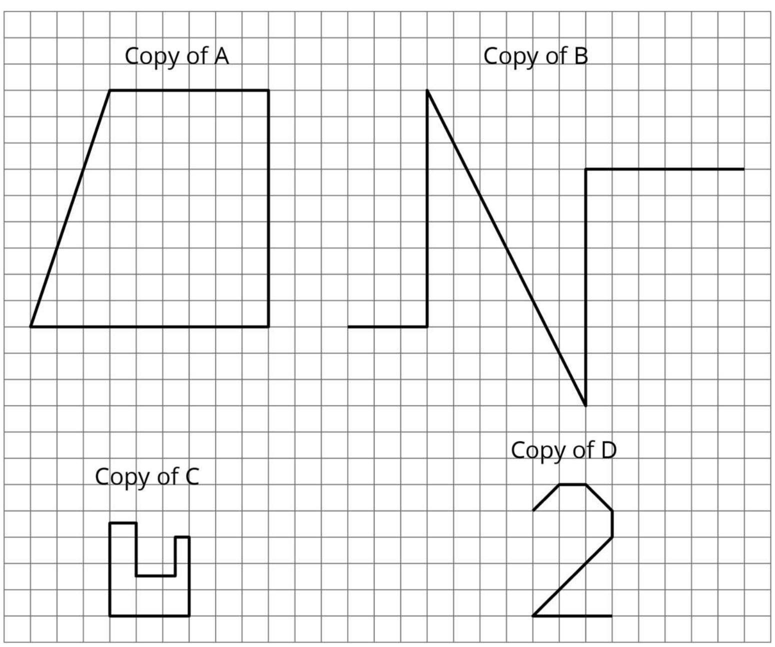


1. Draw a scaled copy of either shape A or B using a scale factor of 3.

2. Draw a scaled copy of either shape C or D using a scale factor of $\frac{1}{2}$.



Student Response



Activity Synthesis

Invite students to share their strategies of how they used the grid (or other tools) to make sure their drawings were scaled copies. Consider asking questions like:

- How did you know how long to make each side in your scaled copy?
- How did you know how big to make each angle in your scaled copy?
- If you made a mistake while drawing your scaled copy, how could you tell?

Model, prompt, and listen for the language students are using to distinguish between scaled and not scaled shapes. Emphasise the usefulness of the grid in drawing and checking right angles, and for drawing and checking lengths of line segments. All correct answers will be the same size and shape, but they could be drawn in different positions on the grid.

3.3 Which Operations? (Part 1)

10 minutes

The purpose of this activity is to contrast the effects of multiplying side lengths versus adding to side lengths when creating copies of a polygon. To find the corresponding side lengths on a scaled copy, the side lengths of a shape are all *multiplied* (or divided) by the same number. However, students often mistakenly think that adding or subtracting the same number to all the side lengths will also create a scaled copy. When students recognise that there is a multiplicative relationship between the side lengths rather than an additive one, they are looking for and making use of structure.

Monitor for students who:

- notice that Diego's copy is no longer a polygon while Jada's still is
- notice that the relationships between side lengths in Diego's copy have changed (e.g., side 1 is twice as long as side 2 in the original but is not twice as long as side 2 in the copy.) while in Jada's copy they have not
- notice that all the corresponding angles have equal sizes (i.e., 90 or 270 degrees)
- describe Jada's copy as having all side lengths divided by 3
- describe Jada's copy as having all side lengths a third as long as their original lengths
- describe Jada's copy as having a scale factor of $\frac{1}{3}$

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Clarify, Critique, Correct
- Think Pair Share

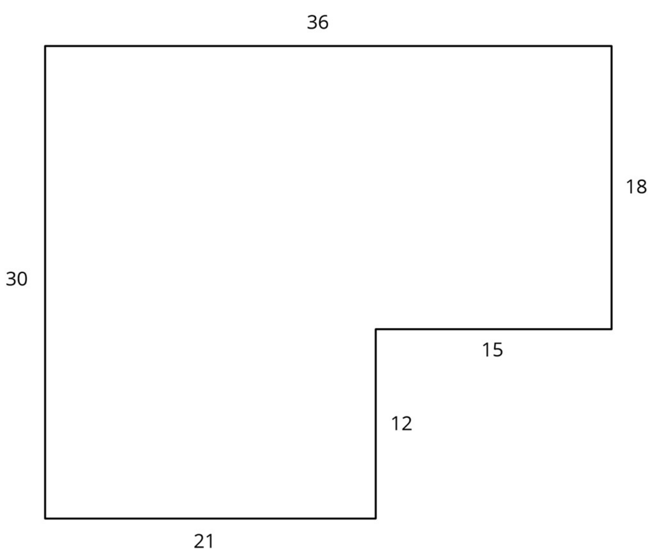
Launch

Give students 2–3 minutes of quiet think time, and then 2 minutes to share their thinking with a partner. Use the strategy "Critique a Partial or Flawed Explanation".

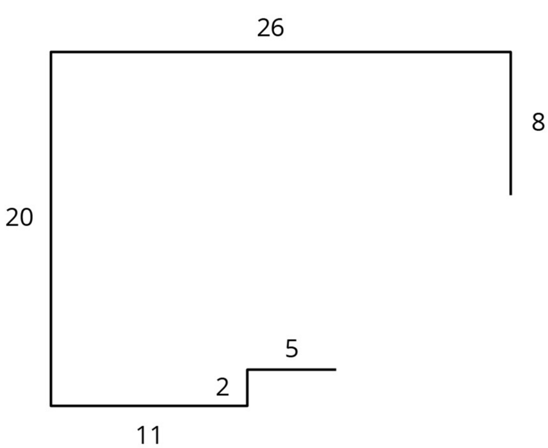
Engagement: Internalise Self-Regulation. Demonstrate giving and receiving constructive feedback. Use a structured process and display sentence frames to support productive feedback. For example, “How did you get…?,” “How do you know…?,” and “That could/couldn’t be true because…”
Supports accessibility for: Social-emotional skills; Organisation; Language

Student Task Statement

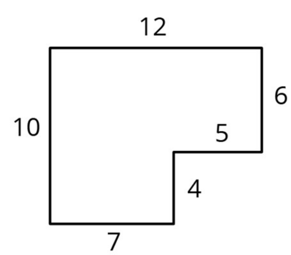
Diego and Jada want to scale this polygon so the side that corresponds to 15 units in the original is 5 units in the scaled copy.



Diego and Jada each use a different operation to find the new side lengths. Here are their finished drawings.



Diego’s drawing



Jada’s drawing

1. What operation do you think Diego used to calculate the lengths for his drawing?

2. What operation do you think Jada used to calculate the lengths for her drawing?

3. Did each method produce a scaled copy of the polygon? Explain your reasoning.

Student Response

1. Since we can get from 15 to 5 by subtracting 10, Diego may have subtracted 10 units from the length of every side. Subtracting 10 from each side length in the original gives Diego's picture.

2. Jada went from 15 to 5 by multiplying by $\frac{1}{3}$ or dividing by 3. Multiplying each side by $\frac{1}{3}$ in the original gives Jada's picture.

3. No, only Jada's method produces a scaled copy. Sample explanation: Subtracting 10 from each length did not work because now the shape is no longer a polygon. There is a big gap between the two sides that should meet. To create a scaled copy, every length needs to be multiplied (or divided) by the same number.

Activity Synthesis

Invite previously-selected students to share their answers and reasoning. Sequence their explanations from most general to most technical.

Before moving to the next activity, consider asking questions like these:

- What is the scale factor used to create Jada's drawing? What about for Diego's drawing? ($\frac{1}{3}$ for Jada's; there isn't one for Diego's, because it is not a scaled copy.)

- What can you say about the corresponding angles in Jada and Diego's drawings? (They are all equal, even though one is a scaled copy and one is not.)

- Subtraction of side lengths does not (usually) produce scaled copies. Do you think addition would work? (Answers vary.)

Note: There are rare cases when adding or subtracting the same length from each side of a polygon (and keeping the angles the same) *will* produce a scaled copy, namely if all side lengths are the same. If not mentioned by students, it is not important to discuss this at this point.

Representing, writing, and speaking: Clarify, Critique, Correct. In this routine, students are given an incorrect or incomplete piece of mathematical work. This may be in the form of a written statement, drawing, problem-solving steps, or another mathematical representation. Students analyse, reflect on, and improve the written work by correcting errors and clarifying meaning. Typical prompts are: "Is anything unclear?" and/or "Are there any reasoning errors?" The purpose of this routine is to engage students in analysing mathematical thinking that is not their own, and to solidify their knowledge through

communicating about conceptual errors and ambiguities in language.
Design Principle(s): Support sense-making; Optimise output (for reasoning)

How It Happens:

1. Play the role of Diego and present the following statement along with his flawed drawing to the class. "I used a scale factor of minus 10, and Jada used a scale factor of one third. So my drawing is a different kind of scaled copy from Jada's."

   Ask students, "What steps did Diego take to make the drawing?" and "Did he create a scaled copy? How do you know?"

2. Give students 1 minute of quiet think time to analyse the statement, and then 3 minutes to work on improving the statement with a partner.

   As pairs discuss, provide these sentence frames for scaffolding: "I believe Diego created the drawing by ___ because ___.", "Diego created/did not create a scaled copy. I know this because ___.", "You can't ___ because ___." Encourage the listener to ask clarifying questions by referring to the statement and the drawings. Allow each partner to take a turn as the speaker and listener.

   Listen for students identifying the type of operation used and justification for whether or not a scaled drawing was produced. Have the pairs reach a mutual understanding and agreement on a correct statement about Diego's drawing.

3. Invite 3 or 4 pairs to present their improved statement to the class, both orally and in writing. Ask students to listen for order/time transition words (first, next, then, etc.), and any elements of justifications (e.g., First, ___ because ___.).

   Here are two sample improved statements:

   "I subtracted 10 from each side length and Jada used a scale factor of one third. So my drawing is not a scaled copy and Jada's is. Jada's is a scaled copy because I know that multiplying—not subtracting—creates a scaled copy. Her drawing created a polygon with no gaps."
   or
   "I subtracted 10 from each side, but I should have realised that in order to scale 15 units in the original down to 5 units in the copy, you have to divide by 3. Jada used a scale factor of one third, which is the same as dividing by 3. My drawing is not a scaled copy and Jada's is because hers is not a polygon with no gaps, and subtracting 10 is not a scale factor."

   Call attention to statements that generalise that the method for finding the side lengths of a scaled copy is by multiplying or dividing, not adding or subtracting. Revoice student thoughts with an emphasis on knowing whether or not they created a scaled polygon.

4. Close the conversation on Diego's drawing, discuss the accuracy of Jada's scaled copy, and then move on to the next lesson activity.

3.4 Which Operations? (Part 2)

10 minutes

In the previous activity, students saw that subtracting the same value from all side lengths of a polygon did not produce a (smaller) scaled copy. This activity makes the case that adding the same value to all lengths also does not produce a (larger) scaled copy, reinforcing the idea that scaling involves multiplication.

This activity gives students a chance to draw a scaled copy without a grid and to use paper as a measuring tool. To create a copy using a scale factor of 2, students need to mark the length of each original line segment and transfer it twice onto their drawing surface, reinforcing—in a tactile way—the meaning of scale factor. The angles in the polygon are right angles (and a 270 degree angle in one case) and can be made using the corner of an index card.

Some students may struggle to shape out how to use an index card or a sheet of paper to measure lengths. Before demonstrating, encourage them to think about how a length in the given polygon could be copied onto an index card and used as an increment for measuring. If needed, show how to mark the 4-unit length along the edge of a card and to use the mark to determine the needed lengths for the copy.

Instructional Routines

- Compare and Connect
- Think Pair Share

Launch

Have students read the task statement and check that they understand which side of the polygon Andre would like to be 8 units long on his drawing. Provide access to index cards, so that students can use it as a measuring tool. Consider not explicitly directing students as to its use to give them a chance to use tools strategically. Give students 5–6 minutes of quiet work time, and then 2 minutes to share their work with a partner.

Anticipated Misconceptions

Some students might not be convinced that making each line segment 4 units longer will not work. To show that adding 4 units would work, they might simply redraw the polygon and write side lengths that are 4 units longer, regardless of whether the numbers match the actual lengths. Urge them to check the side lengths by measuring. Tell them (or show, if needed) how the 4-unit length in Jada's drawing could be used as a measuring unit and added to all sides.

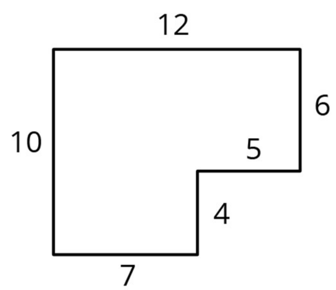
Other students might add 4 units to all sides and manage to make a polygon but changing the angles along the way. If students do so to make the case that the copy will not be scaled, consider sharing their illustrations with the class, as these can help to counter the idea that "scaling involves adding." If, however, students do this to show that adding 4 units all around does work, address the misconception. Ask them to recall the size of corresponding angles in scaled copies, or remind them that angles in a scaled copy are the same size as their counterparts in the original shape.

Student Task Statement

Andre wants to make a scaled copy of Jada's drawing so the side that corresponds to 4 units in Jada's polygon is 8 units in his scaled copy.

1. Andre says "I wonder if I should add 4 units to the lengths of all of the line segments?" What would you say in response to Andre? Explain or show your reasoning.

2. Create the scaled copy that Andre wants. If you get stuck, consider using the edge of an index card or paper to measure the lengths needed to draw the copy.



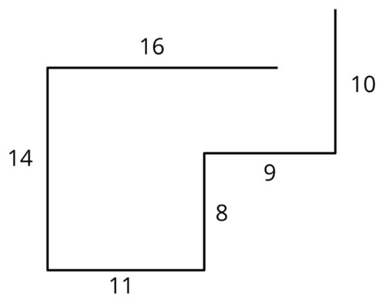
Jada's drawing

Student Response

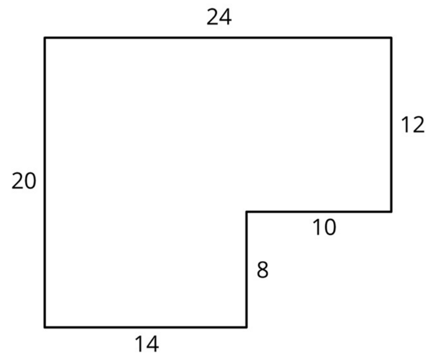
1. Answers vary. Sample reasoning: Adding 4 units would not work because the shape of the copy would be different than the shape of the original. For example, in the original drawing, the top horizontal line segment is 12 units and the two bottom horizontal line segments (5 units and 7 units) also add up to 12 units. If we add 4 units to each line segment, the top horizontal line segment will be 16 units long, and the two bottom horizontal line segments will be 9 units and 11 units, or a total of 20 units. There will be a gap where two line segments should meet, or if we make the two ends meet, the angles will no longer be right angles. See the shape on the left.

2. See the shape on the right.

Adding 4 units to each side



A correctly drawn figure



Are You Ready for More?

The side lengths of triangle B are all 5 more than the side lengths of triangle A. Can triangle B be a scaled copy of triangle A? Explain your reasoning.

Student Response

Yes, if triangle A is equilateral then its side lengths are all the same. Adding 5 to each side, the lengths will still be the same and so triangle B will also be equilateral.

If triangle A is not equilateral then triangle B will not be a scaled copy of triangle A. To see why, notice that adding 5 to a side length of 5 doubles the side length. Adding 5 to a side length that greater than 5 changes the side by a scale factor less than 2 while adding 5 to a side length less than 5 changes the side length by a scale factor less than 2. So if one side length of triangle A is 5, all side lengths have to be 5 or else triangle B will not be a scaled copy of triangle A. This reasoning works for other side lengths than 5. In general, adding 5 to a *greater* side length uses a *smaller* scale factor.

Activity Synthesis

The purpose of the activity is to explicitly call out a potential misunderstanding of how scale factors work, emphasising that scale factors work by multiplying existing side lengths by a common factor, rather than adding a common length to each.

Invite a couple of students to share their explanations or illustrations that adding 4 units to the length of each line segment would not work (e.g. the copy is no longer a polygon, or the copy has angles that are different than in the original shape). Then, select a couple of other students to show their scaled copies and share how they created the copies. Consider asking:

- What scale factor did you use to create your copy? Why?
- How did you use an index card (or a sheet of paper) to measure the lengths for the copy?
- How did you measure the angles for the copy?

Speaking: Compare and Connect. In this routine, students are given a problem that can be approached using multiple strategies or representations, and are asked to prepare a visual display of their method. Students then engage in investigating the strategies (by means of a teacher-led gallery walk, partner exchange, group presentation, etc.), compare approaches, and identify correspondences between different representations. A typical discussion prompt is "What is the same and what is different?", comparing their own strategy to the others. The purpose of this routine is to allow students to make sense of mathematical strategies by identifying, comparing, contrasting, and connecting other approaches to their own, and to develop students' awareness of the language used through constructive conversations.
Design Principle(s): Maximise meta-awareness

How It Happens:

1. Use this routine to compare and contrast different methods for creating scaled copies of Jada's drawing. Before selecting students to share a display of their method with the whole class, first give students an opportunity to do this in a group of 3–4.

   Invite students to quietly investigate each other's work. Ask students to consider what is the same and what is different about each display. Invite students to give a step-by-step explanation of their method using this sentence frame: "In order to create the copy, first I…. Next,…. Then, …. Finally,….". Allow 1–2 minutes for each display and signal when it is time to switch.

2. Next, give each student the opportunity to add detail to their own display for 1-2 minutes. As students work on their displays, circulate the room to identify at least two different methods or two different ways of representing a method. Also look for methods that were only partially successful.

3. Consider selecting 1–2 students to share methods that were only partially successful in producing scaled copies. Then, select a couple of students to share displays of methods that did produce scaled copies.

   Draw students' attention to the approaches used in each drawing (e.g., adding the same value to each side length, not attending to the angles, multiplying by a common factor, not creating a polygon, etc.). Ask students, "Did this approach create a scaled copy? Why or why not?"

4. After the pre-selected students have finished sharing with the whole class, lead a discussion comparing, contrasting, and connecting the different approaches and representations.

   In this discussion, demonstrate using the mathematical language "scale factor", "corresponding", and "multiplicative" to amplify student language.

   Consider using these prompts:

   – "How did the scale factor show up in each method?",

– “Why did the different approaches lead to the same outcome?”,

– “What worked well in _____’s approach/representation? What did not work well?”, and

– “What role does multiplication play in each approach?”

5. Close the discussion by inviting 3 students to revoice the incorrect method for creating a scaled drawing, and then invite 3 different students to revoice the correct method for creating a scaled drawing. Then, transition back to the Lesson Synthesis and Cool Down.

Lesson Synthesis

- How do we draw a scaled copy of a shape?
- Can we create scaled copies by adding or subtracting the same value from all lengths? Why or why not?

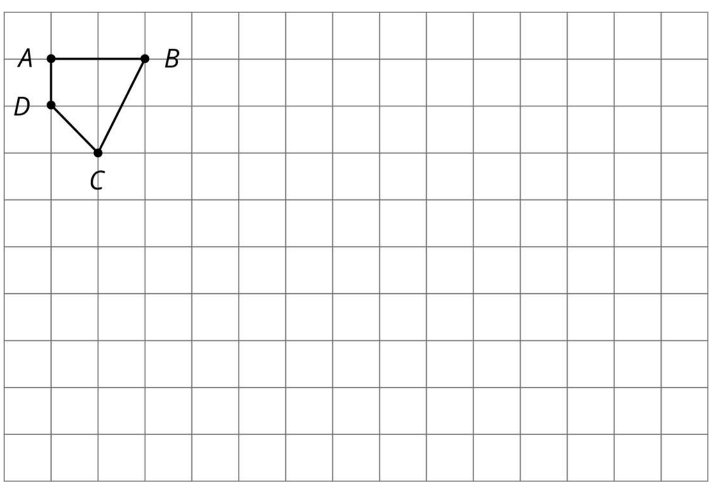
Scaling is a multiplicative process. To draw a scaled copy of a shape, we need to multiply all of the lengths by the scale factor. We saw in the lesson that adding or subtracting the same value to all lengths will not create scaled copies in general.

3.5 More Scaled Copies

Cool Down: 5 minutes

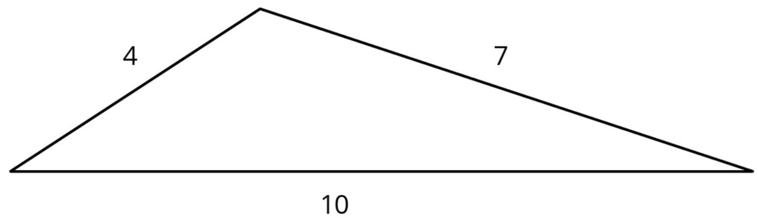
Student Task Statement

1. Create a scaled copy of $ABCD$ using a scale factor of 4.



2. Triangle Z is a scaled copy of triangle M.

M



Select all the sets of values that could be the side lengths of triangle Z.

a. 8, 11, and 14.

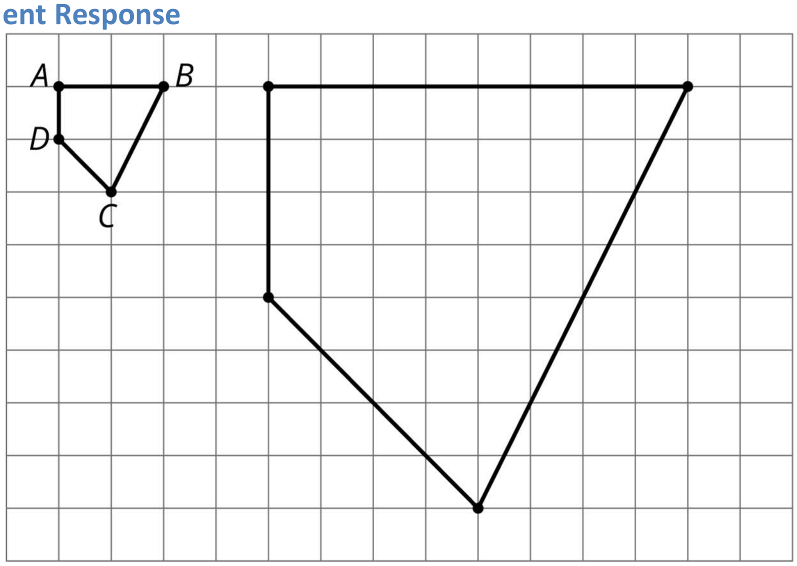
b. 10, 17.5, and 25.

c. 6, 9, and 11.

d. 6, 10.5, and 15.

e. 8, 14, and 20.

Student Response

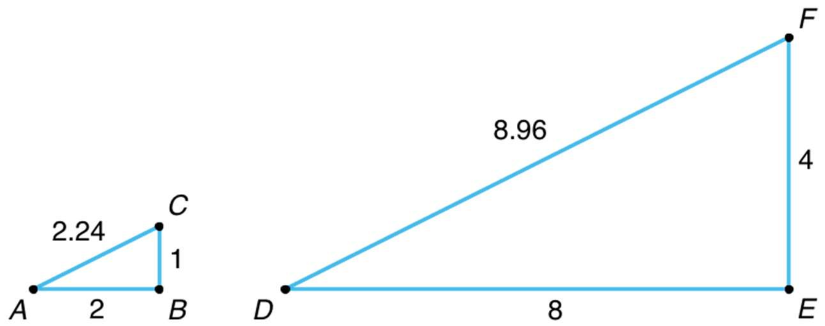
1. 

2. B, D, and E.

Student Lesson Summary

Creating a scaled copy involves *multiplying* the lengths in the original shape by a scale factor.

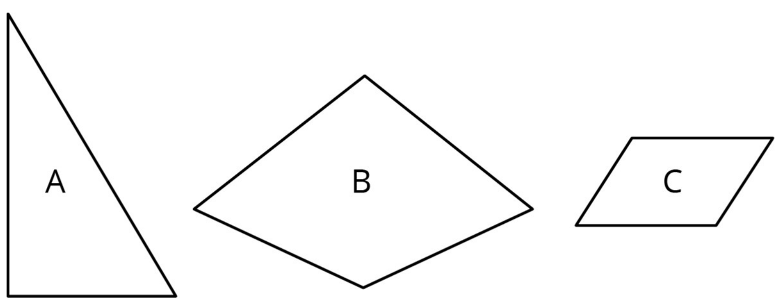
For example, to make a scaled copy of triangle ABC where the base is 8 units, we would use a scale factor of 4. This means multiplying all the side lengths by 4, so in triangle DEF, each side is 4 times as long as the corresponding side in triangle ABC.



Lesson 3 Practice Problems

Problem 1 Statement

Here are 3 polygons.



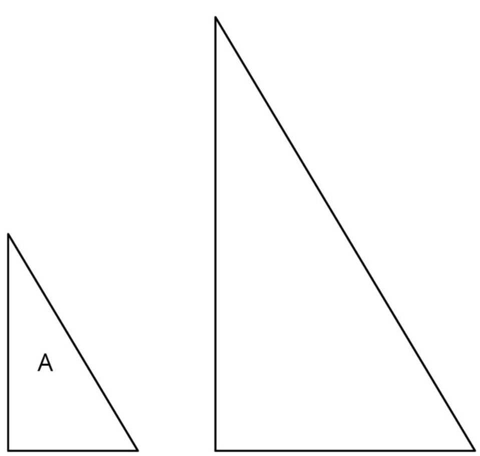
Draw a scaled copy of polygon A using a scale factor of 2.

Draw a scaled copy of polygon B using a scale factor of $\frac{1}{2}$.

Draw a scaled copy of polygon C using a scale factor of $\frac{3}{2}$.

Solution

1.



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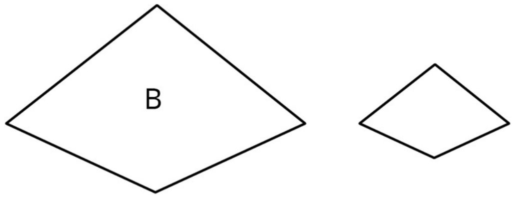

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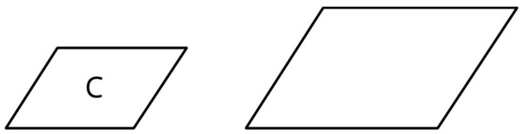


Problem 2 Statement

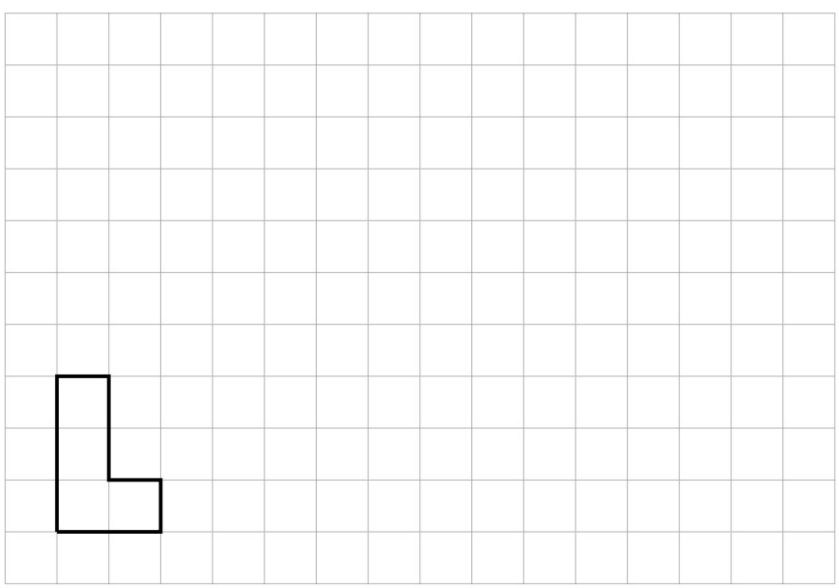
Quadrilateral A has side lengths 6, 9, 9, and 12. Quadrilateral B is a scaled copy of quadrilateral A, with its shortest side of length 2. What is the perimeter of quadrilateral B?

Solution

The scale factor is $\frac{1}{3}$, so the side lengths of quadrilateral B are 2, 3, 3, and 4. Summing these four numbers gives the perimeter of 12.

Problem 3 Statement

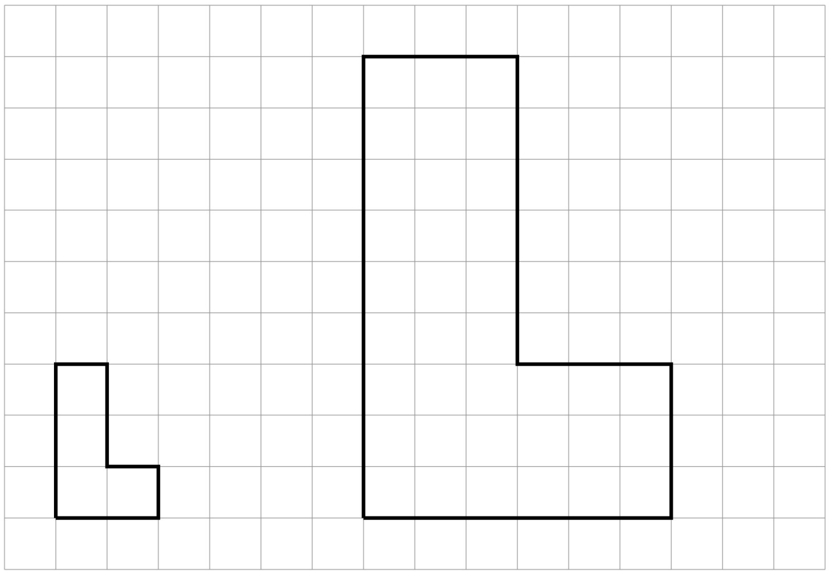
Here is a polygon on a grid.



Draw a scaled copy of this polygon that has a perimeter of 30 units. What is the scale factor? Explain how you know.

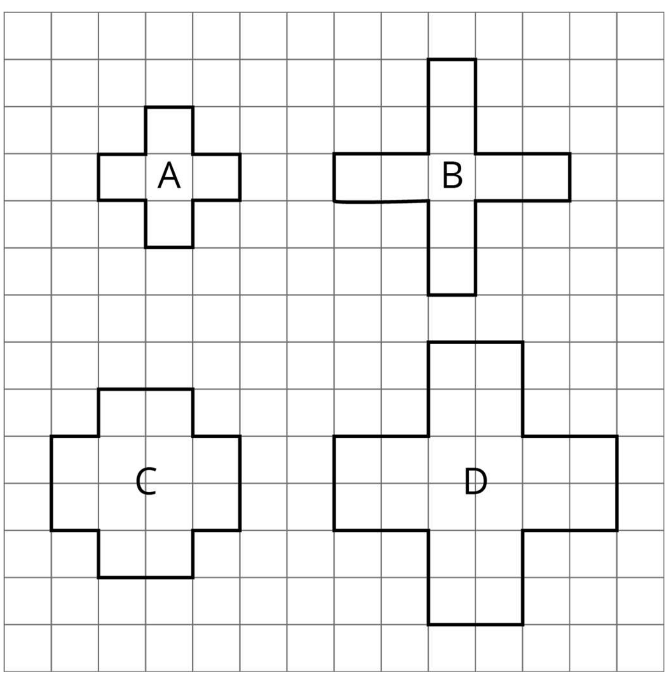
Solution

The perimeter of the original polygon is 10 units. Since the perimeter of a scaled copy is multiplied by the scale factor, a scale factor of 3 needs to be applied to get a copy with a perimeter of 30.



Problem 4 Statement

Priya and Tyler are discussing the shapes shown below. Priya thinks that B, C, and D are scaled copies of A. Tyler says B and D are scaled copies of A. Do you agree with Priya, or do you agree with Tyler? Explain your reasoning.



Solution

Answers vary. Sample response: I agree with neither one. Only D is a scaled copy of A. In D, the length of each line segment of the plus sign is twice the matching line segments in A. In B and C, some line segments are double the matching lengths in A but some are not.