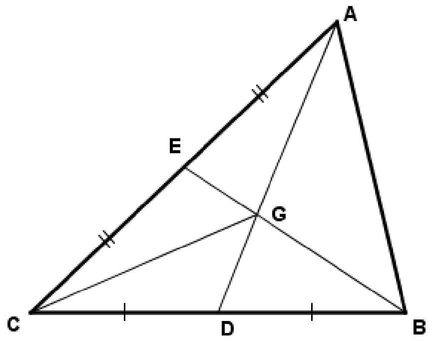
Properties of The Centroid

In the figure, AD and BE are the medians of $\triangle ABC$ intersect at G.



(a) Let the height of $\triangle ADB$ and $\triangle ADB$ be h_1, as shown in Fig.1.
Prove that the area of $\triangle ADB$ = the area of $\triangle ACD$.

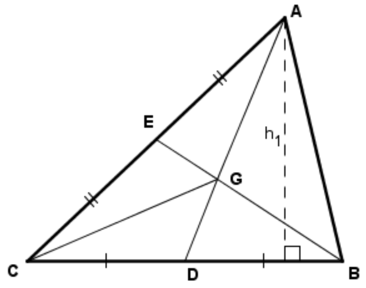


Fig. 1

____

____

____

____

(b) Let the height of $\triangle GDB$ and $\triangle GDC$ be h_2, as shown in Fig.2.
Prove that the area of $\triangle GDB$ = the area of $\triangle GDC$.

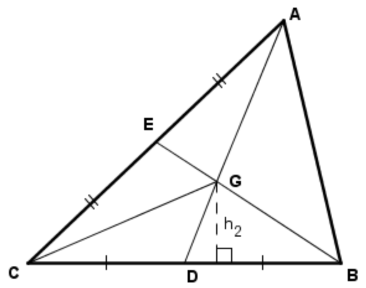


Fig. 2

____

____

____

____

(c) Using the results of (a) and (b), prove that the area of $\triangle AGB$ = the area of $\triangle AGC$.

____

____

____

(d) Prove that the area of $\triangle AGB$ = the area of $\triangle BGC$.

(e) (i) Let the area of $\triangle ABC$ be S. Express the areas of $\triangle AGC$ and $\triangle CGD$ in terms of S.

(ii) Let the height of $\triangle CGD$ and $\triangle AGC$ be h_3, as shown in Fig.3.

Using the result of (e)(i), prove that $AG : GD = 2 : 1$.

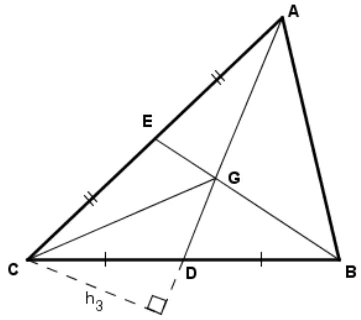


Fig.3