Lesson 1: Size of divisor and size of quotient

Goals

- Comprehend the terms “dividend” and “divisor” (in spoken language) to refer to the numbers in a division problem.
- Explain (orally) how to estimate quotients, by comparing the size of the dividend and divisor.
- Generalise about the size of a quotient, i.e., predicting whether it is a very large number, a very small number, or close to 1.

Learning Targets

- When dividing, I know how the size of a divisor affects the quotient.

Lesson Narrative

The first three lessons of this unit help students make sense of division situations. In this opening lesson, students begin thinking about the relationships between the numbers in a division equation. They see that they can estimate the size of the quotient by reasoning about the relative sizes of the *divisor* and the *dividend*.

Students begin exploring these relationships in concrete situations. For example, they estimate how many thinner and thicker objects are needed to make a stack of a given height, and how many segments of a certain size make a particular length.

Later, they generalise their observations to division expressions. Students become aware that dividing by a number that is much smaller than the dividend results in a quotient that is larger than 1, that dividing by a number that is much larger than the dividend gives a quotient that is close to 0, and that dividing by a number that is close to the dividend results in a quotient that is close to 1.

Building On

- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Building Towards

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Instructional Routines

- Collect and Display
- Discussion Supports

- Number Talk
- Poll the Class

Required Materials

Pre-printed slips, cut from copies of the blackline master

All in Order – Set 1 $800 \div 10\,000$	All in Order – Set 1 $800 \div 10\,000$	All in Order – Set 1 $800 \div 10\,000$
All in Order – Set 1 $800 \div 801$	All in Order – Set 1 $800 \div 801$	All in Order – Set 1 $800 \div 801$
All in Order – Set 1 $800 \div 1\,250$	All in Order – Set 1 $800 \div 1\,250$	All in Order – Set 1 $800 \div 1\,250$
All in Order – Set 1 $800 \div \frac{1}{10}$	All in Order – Set 1 $800 \div \frac{1}{10}$	All in Order – Set 1 $800 \div \frac{1}{10}$
All in Order – Set 1 $800 \div 250$	All in Order – Set 1 $800 \div 250$	All in Order – Set 1 $800 \div 250$
All in Order – Set 1 $800 \div 2.5$	All in Order – Set 1 $800 \div 2.5$	All in Order – Set 1 $800 \div 2.5$
All in Order – Set 1 $800 \div 0.0001$	All in Order – Set 1 $800 \div 0.0001$	All in Order – Set 1 $800 \div 0.0001$
All in Order – Set 1 $800 \div 799.5$	All in Order – Set 1 $800 \div 799.5$	All in Order – Set 1 $800 \div 799.5$

All in Order – Set 2 $75 \div 25$	All in Order – Set 2 $75 \div 25$	All in Order – Set 2 $75 \div 25$
All in Order – Set 2 $1\,000 \div 25$	All in Order – Set 2 $1\,000 \div 25$	All in Order – Set 2 $1\,000 \div 25$
All in Order – Set 2 $625 \div 25$	All in Order – Set 2 $625 \div 25$	All in Order – Set 2 $625 \div 25$
All in Order – Set 2 $5\,000\,000 \div 25$	All in Order – Set 2 $5\,000\,000 \div 25$	All in Order – Set 2 $5\,000\,000 \div 25$
All in Order – Set 2 $6.25 \div 25$	All in Order – Set 2 $6.25 \div 25$	All in Order – Set 2 $6.25 \div 25$
All in Order – Set 2 $0.625 \div 25$	All in Order – Set 2 $0.625 \div 25$	All in Order – Set 2 $0.625 \div 25$
All in Order – Set 2 $24 \div 25$	All in Order – Set 2 $24 \div 25$	All in Order – Set 2 $24 \div 25$
All in Order – Set 2 $25.25 \div 25$	All in Order – Set 2 $25.25 \div 25$	All in Order – Set 2 $25.25 \div 25$

Required Preparation

Print and cut up slips containing expressions from the blackline master. Consider copying Set 1 and Set 2 on paper of different colours. Prepare 1 copy (8 slips of Set 1 and 8 slips of Set 2) for every 3 students.

Student Learning Goals

Let’s explore quotients of different sizes.

1.1 Number Talk: Size of Dividend and Divisor

Warm Up: 5 minutes

This number talk prompts students to notice how the values of the dividend and divisor affect the size of the quotient. To mentally evaluate the series of expressions, students think carefully about the numbers and rely on what they know about division, structure, and properties of operations.

The focus of the activity is less on the strategies for evaluating quotients and more on what happens when we divide the same number by different-sized numbers. Students should notice that dividing by a number much smaller than the dividend results in a large quotient and that dividing by a number much larger than the quotient results in a small quotient.

Instructional Routines

- Collect and Display
- Discussion Supports
- Number Talk

Launch

Reveal one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the task.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Anticipated Misconceptions

Some students may cross out the zeros without being aware of what doing so means. They may or may not reach the correct quotients. If students cross out zeros, ask the class during discussion to explain what they believe is happening mathematically when zeros are crossed out. Clarify any confusion accordingly.

Student Task Statement

Find the value of each expression mentally.

$5\,000 \div 5$

$5\,000 \div 2\,500$

$5\,000 \div 10\,000$

$5\,000 \div 500\,000$

Student Response

- 1 000. Possible reasoning: 5 thousands divided by 5 is 1 thousand.
- 2. Possible reasoning: There are 2 groups of 2 500 in 5 000.
- $\frac{1}{2}$ (or 0.5). Possible reasoning: 5 000 is half of 10 000, and 5 000 divided into 10 000 groups means 0.5 in each group.
- $\frac{1}{100}$ (or 0.01). Possible reasoning: $5\,000 \div 1\,000 = 5$ and $500\,000 \div 1\,000 = 500$ and $5 \div 500 = \frac{5}{500}$, which is $\frac{1}{100}$.

Activity Synthesis

Invite a couple of students to share their answer and strategies for each problem. Record and display their explanations for all to see. Refer to *Collect and Display*. After evaluating all four expressions, ask students:

- "What do you notice about the value of each expression as the *divisor* (the number we use to divide) gets larger?"
- "Why is the value of the expression getting smaller each time?"

Highlight explanations that support two ways of thinking about division:

1. Dividing means breaking the *dividend* into a certain number of equal parts, and when there are more parts, the size of each part gets smaller.
2. Dividing means breaking the dividend into parts of a particular size, and when the size of each part gets larger, the number of parts gets smaller.

To involve more students in the conversation, consider asking as the students share their ideas:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone solve the problem the same way but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

1.2 All Stacked Up

15 minutes

This lesson aims to give students a concrete setting for thinking about division. Students estimate how many of each given object are needed to make a stack of a given height. To do so, they use what they know about the sizes of familiar objects (boxes, bricks, notebooks, and coins) and their intuition that it takes more of a thinner object and fewer of a thicker object to reach the same height. Later, they will use this idea to think about division more generally.

We often refer to certain objects (coins, books, etc.) as having a thickness rather than a height. Clarify that "thickness" and "height" refer to the same dimension in these examples. The images of the boxes and the bricks show stacks with more items at the base. Clarify that we are concerned only with a stack with one item per layer.

As students discuss in groups, monitor for those who:

- Can explain clearly why there is a relationship between the height of the object being stacked and the height of a stack.
- Can explain clearly why the situation can be represented with a division expression.

Instructional Routines

- Discussion Supports
- Poll the Class

Launch

Arrange students in groups of 3–4. Give students 4–5 minutes quiet think time and then time to discuss their solutions with their group.

Representation: Internalise Comprehension. Begin the activity with concrete or familiar contexts. Provide students with access to some cardboard boxes, bricks, notebooks, or coins (whatever is available) for them to explore, but not measure.
Supports accessibility for: Visual-spatial processing

Anticipated Misconceptions

Some students may compare the height of objects in centimetres to the height of a stack in metres. Remind them to attend to units of measurement when making their estimates.

If students struggle to make estimates for the first set of questions, ask them which object we would need the most of and the least of to reach 2 metres. Then, prompt them to reason about the relative heights of those objects.

Student Task Statement

1. Here are several types of objects. For each type of object, estimate how many are in a stack that is 2 metres high. Be prepared to explain your reasoning.

Cardboard boxes

Bricks

Notebooks

Coins

2. A stack of books is 72 centimetres tall. Each book is 2 centimetres thick. Which expression tells us how many books are in the stack? Be prepared to explain your reasoning.

   - 72×2
   - $72 - 2$
   - $2 \div 72$
   - $72 \div 2$

3. Another stack of books is 43 centimetres tall. Each book is $\frac{1}{2}$ centimetre thick. Write an expression that represents the number of books in the stack.

Student Response

1. Estimates vary. Sample responses and reasoning:

   a. About 4 boxes. I estimated that each box is about 0.5 metres or 50 centimetres tall, so it would take 4 of them to reach 2 metres.

   b. About 20 bricks. Assuming a brick is about 10 centimetres thick.

   c. About 80 books, assuming a notebook is about 2.5 centimetres thick. $80 \times 2.5 = 200$.

   d. About 960 pennies. I estimated each penny to be about 0.2 centimetres, so it would take 5 pennies to make 1 centimetre, and 5×200, or 1000 pennies, to make 2 metres.

2. $72 \div 2$

3. $43 \div \frac{1}{2}$ (or 86)

Activity Synthesis

Ask a few students to share their estimates and explanations for the first set of questions. After a student gives an estimate, poll the class to see if their estimate is less, greater, or about the same. Record for all to see the range of estimates of each object. After all four objects are discussed, ask students:

- "How did your estimates for the number of objects change as the object got thinner?" (The estimates got larger.)
- "Why might that be?" (As the thickness decreased, more objects were needed to make a height of 2 metres.)

Select previously identified students to share their responses and reasoning to the last two questions. Highlight that all situations in this activity involve division. Point out that:

- In the case of the 2 metres (200 centimetres) stack, the relationship can be represented as: 200 ÷ height = number of objects.
- The height of each object is the *divisor* (the number we use to divide) and the number of objects is the *quotient* (or result of division).
- The greater the divisor, the smaller the quotient, and vice versa.

Consider using this applet: https://ggbm.at/RJQyS6av to further illustrate the relationship between the size of the divisor and the size of the quotient.

Speaking, Listening: Discussion Supports. Use this routine to amplify mathematical uses of language to communicate about divisors, dividends, quotients, and expressions. Remind students to use these words when stating their ideas. Ask students to chorally repeat the phrases that include these words in context.
Design Principle(s): Support sense-making; Optimise output (for explanation)

1.3 All in Order

20 minutes

In this sorting activity, students continue to explore the relationship between dividends, divisors, and quotients.

First, they study two sets of division expressions and arrange them in order—from the largest to smallest—based on the size of the quotients. The first set of quotients has the same dividend (800). The second set has the same divisor (25).

Next, students estimate the size of quotients relative to 0 and 1. They approximate the size of divisors such that the quotients are close to 0, close to 1, or much greater than 1.

As students work, monitor how they think about placing the expressions. Select students who could explain their rationale clearly so that they could share later.

Also notice the expressions students find difficult to put in order. Expressions with fractional or decimal divisors, or expressions in which the dividends and divisors are very close to each other (e.g. 800 ÷ 800.1 and 800 ÷ 799.5), may be particularly challenging. This is an opportunity to make use of the structure in the relationship between the three parts in a division, i.e. to see that as the divisor gets larger, even if only by a very small amount, the quotient necessarily gets smaller.

In the extension, students have the option to work in groups of 2. Instead of writing a list of expressions, partners may take turns writing expressions that have values increasingly closer to 1 without equalling 1.

Instructional Routines

- Collect and Display

Launch

If not already done in the preceding activity, review the terms “quotient” and “divisor.” Display a division equation labelled with these terms so students can refer to it as needed.

Arrange students in groups of 3–4 and pair each group with another group so that they could check each other's work. Give each group 2 sets of pre-cut slips from the blackline master. Give students 6–8 minutes to sort the two sets of expressions, a couple of minutes to review another group's work, and another 5 minutes to record their sorted lists and complete the last question.

If desired, collect the slips after students record their lists so that they could be reused.

Representation: Internalise Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Provide select students fewer cards to start with and introduce the remaining cards once students have completed their initial sort. Some students may benefit from an additional set of cards with easier values to get them started.
Supports accessibility for: Conceptual processing; Organisation Conversing, Representing, Writing: Collect and Display. Listen for and collect the mathematical language students use as they discuss each expression. Record words and phrases such as “the quotient is close to 1” and “the divisor is larger than the dividend,” on a display for all to see. Remind students to borrow words, phrases or expressions from the display as needed. This will help students use mathematical language during paired and group discussions.
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

Anticipated Misconceptions

Students may try to compute the value of each expression because they are unsure how to begin otherwise. Suggest that they compare two quotients at a time, starting with those that have very different divisors. Ask, for instance, "Which is greater, $800 \div 250$ or $800 \div 2.5$?" Urge them to use the patterns they saw earlier about how the size of a divisor affects the quotient.

Student Task Statement

Your teacher will give you two sets of papers with division expressions.

1. Without computing, estimate the quotients in each set and order them from greatest to least. Be prepared to explain your reasoning.

   Pause here for a class discussion.

   Record the expressions in each set in order from the greatest value to the least.

   a. Set 1

   b. Set 2

2. Without computing, estimate the quotients and sort them into the following three groups. Be prepared to explain your reasoning.

$30 \div \frac{1}{2}$

$30 \div 0.45$

$9 \div 10$

$9 \div 10\,000$

$18 \div 19$

$18 \div 0.18$

$15\,000 \div 1\,500\,000$

$15\,000 \div 14\,500$

- Close to 0
- Close to 1
- Much larger than 1

Student Response

a. Set 1: $800 \div 0.0001$, $800 \div \frac{1}{10}$, $800 \div 2.5$, $800 \div 250$, $800 \div 799.5$, $800 \div 801$, $800 \div 1\,250$, $800 \div 10\,000$

b. Set 2: $5\,000\,000 \div 25$, $1\,000 \div 25$, $625 \div 25$, $75 \div 25$, $25.25 \div 25$, $24 \div 25$, $6.25 \div 25$, $0.625 \div 25$

- Close to 0: $15\,000 \div 1\,500\,000$ and $9 \div 10\,000$
- Close to 1: $9 \div 10$, $18 \div 19$, and $15\,000 \div 14\,500$
- Much larger than 1: $30 \div 0.45$, $18 \div 0.18$, and $30 \div \frac{1}{2}$

Are You Ready for More?

Write 10 expressions of the form $12 \div ?$ in a list ordered from least to greatest. Can you list expressions that have value near 1 without equaling 1? How close can you get to the value 1?

Student Response

Answers vary. Sample response: $12 \div 13$, $12 \div 12.5$, $12 \div 12.1$, $12 \div 12.05$, $12 \div 12.01$, $12 \div 12.001$. I can get closer and closer to the value of 1 by dividing 12 by a number that is closer and closer to 12 but is not exactly 12.

Activity Synthesis

Ask a couple of groups to share how they sorted the two sets of expressions. Discuss expressions that students found hard to place (e.g., those involving decimal, fractional, or very large divisors), and select other students to share how they made their decisions.

Next, discuss ways to reason about the size of quotients relative to 0 and 1. The terms "dividend," "divisor," and "quotient" may not be intuitive to students. Support them by keeping a reference (a division equation labelled with these terms) displayed or referring to numerical examples. Encourage students to use these terms as they respond to the following questions, which focus on reasoning about the size of quotients relative to 1 and 0:

- "How did you decide that a quotient is close to 0?" (The divisor is much larger than the dividend.)
- "How did you decide that a quotient is close to 1?" (If the divisor is close to the dividend, then it would be closer to 1.)
- "Is there a way to tell if a quotient is less than 1 or more than 1?" (If the divisor is smaller than the dividend, then the quotient is more than 1. Otherwise, it is less than 1.)
- "Suppose a divisor is less than the dividend. How can we tell if the quotient is just a little larger than 1 or much larger than 1?" (If the dividend and divisor are very far apart in size, then the quotient is much larger than 1.)

Lesson Synthesis

In this lesson, we explored situations that involve division and the numbers in divisions. We noticed that the size of the *dividend* (the number that we are dividing) and the size of the *divisor* (the number we use to divide) both affect the quotient(the result of dividing). Let's recall our observations.

- "What happens to the quotient when we divide by smaller and smaller numbers?" (The quotient gets increasingly larger.)
- "Which would result in a smaller quotient: dividing a number by 0.5 or dividing it by 5? Why?" (5, because we would need fewer 5s than 0.5s to reach the size of the dividend.)

We also compared the size of a divisor to that of a dividend and saw how they affect the quotient. Ask students:

- "What can we say about the quotient when the divisor is much smaller than the dividend?" (The quotient will be much greater than 1.)
- "What about when we divide a number by another number that is much larger?" (The quotient will be close to 0.)

- “What can we say about the quotient when the divisor and dividend are about the same size?” (The quotient will be close to 1.)

1.4 Result of Division

Cool Down: 5 minutes

Student Task Statement

Without computing, decide whether the value of each expression is much smaller than 1, close to 1, or much larger than 1.

1. $1\,000\,001 \div 99$
2. $3.7 \div 4.2$
3. $1 \div 835$
4. $100 \div \frac{1}{100}$
5. $0.006 \div 6\,000$
6. $50 \div 50\frac{1}{4}$

Student Response

1. Much larger than 1
2. Close to 1
3. Much smaller than 1
4. Much larger than 1
5. Much smaller than 1
6. Close to 1

Student Lesson Summary

Here is a division expression: $60 \div 4$. In this division, we call 60 the *dividend* and 4 the *divisor*. The result of the division is the quotient. In this example, the quotient is 15, because $60 \div 4 = 15$.

We don’t always have to make calculations to have a sense of what a quotient will be. We can reason about it by looking at the size of the dividend and the divisor. Let’s look at some examples.

- In $100 \div 11$ and in $18 \div 2.9$ the dividend is larger than the divisor. $100 \div 11$ is very close to $99 \div 11$, which is 9. The quotient $18 \div 2.9$ is close to $18 \div 3$ or 6.

In general, when a larger number is divided by a smaller number, the quotient is greater than 1.

- In 99 ÷ 101 and in 7.5 ÷ 7.4 the dividend and divisor are very close to each other. 99 ÷ 101 is very close to 99 ÷ 100, which is $\frac{99}{100}$ or 0.99. The quotient 7.5 ÷ 7.4 is close to 7.5 ÷ 7.5, which is 1.

  In general, when we divide two numbers that are nearly equal to each other, the quotient is close to 1.

- In 10 ÷ 101 and in 50 ÷ 198 the dividend is smaller than the divisor. 10 ÷ 101 is very close to 10 ÷ 100, which is $\frac{10}{100}$ or 0.1. The division 50 ÷ 198 is close to 50 ÷ 200, which is $\frac{1}{4}$ or 0.25.

  In general, when a smaller number is divided by a larger number, the quotient is less than 1.

Lesson 1 Practice Problems

1. **Problem 1 Statement**

   Order from smallest to largest:

   – Number of pennies in a stack that is 1 ft high

   – Number of books in a stack that is 1 ft high

   – Number of pound notes in a stack that is 1 ft high

   – Number of slices of bread in a stack that is 1 ft high

   Solution

   Number of books, number of slices of bread, number of pennies, number of pound notes

2. **Problem 2 Statement**

   Use each of the numbers 4, 40, and 4000 once to complete the sentences.

   a. The value of _ ÷ 40.01 is close to 1.

   b. The value of _ ÷ 40.01 is much less than 1.

   c. The value of _ ÷ 40.01 is much greater than 1.

Solution

a. 40

b. 4

c. 4000

3. **Problem 3 Statement**

Without computing, decide whether the value of each expression is much smaller than 1, close to 1, or much greater than 1.

a. $100 \div \frac{1}{1000}$

b. $50\frac{1}{3} \div 50\frac{1}{4}$

c. $4.7 \div 5.2$

d. $2 \div 7\,335$

e. $2\,000\,001 \div 9$

f. $0.002 \div 2\,000$

Solution

a. Much greater than 1

b. Close to 1

c. Close to 1

d. Much smaller than 1

e. Much greater than 1

f. Much smaller than 1

4. **Problem 4 Statement**

A rocking horse has a weight limit of 60 pounds.

a. What percentage of the weight limit is 33 pounds?

b. What percentage of the weight limit is 114 pounds?

c. What weight is 95% of the limit?

Solution

a. 55%

b. 190%

c. 57 pounds

5. **Problem 5 Statement**

Compare using $>$, $=$, or $<$.

a. 0.7 _____ 0.70

b. $0.03 + \frac{6}{10}$ _____ $0.30 + \frac{6}{100}$

c. 0.9 _____ 0.12

Solution

a. 0.7 = 0.70 because $\frac{7}{10} = \frac{70}{100}$.

b. $0.03 + \frac{6}{10} > 0.30 + \frac{6}{100}$ because $0.63 > 0.36$.

c. 0.9 > 0.12 because $\frac{9}{10} = \frac{90}{100}$.

6. **Problem 6 Statement**

Diego has 90 songs on his playlist. How many songs are there for each genre?

a. 40% rock

b. 10% country

c. 30% hip-hop

d. The rest is electronica

Solution

a. 36, because $(0.4) \times 90 = 36$.

b. 9, because $(0.1) \times 90 = 9$.

c. 27, because $(0.3) \times 90 = 27$.

d. 18, because $(0.2) \times 90 = 18$.

7. **Problem 7 Statement**

A garden hose emits 9 quarts of water in 6 seconds. At this rate:

a. How long will it take the hose to emit 12 quarts?

b. How much water does the hose emit in 10 seconds?

Solution

a. 8 seconds

b. 15 quarts