

Lesson Plan: Delving into Integration by Parts

Objective: Students will explore the method of integration by parts, understand its derivation from the product rule, and apply it to solve integrals of products of functions. This lesson aims to deepen students' understanding of the relationship between differentiation and integration and to enhance their problem-solving skills in calculus.

Grade Level: IB Diploma Programme Mathematics - Analysis and Approaches HL (AAHL 5.20).

Duration: 60 minutes

Warm-up Activity (5 minutes)

- Inquiry Question: How do differentiation and integration relate to each other in calculus?
- Activity: A brief discussion to activate prior knowledge about the fundamental theorem of calculus and the relationship between differentiation and integration.

Introduction to Integration by Parts (10 minutes)

- Presentation: Introduce the formula for integration by parts, $\int u dv = uv - \int v du$, and explain its derivation from the product rule of differentiation.
- Discussion: Engage students in a conceptual inquiry about why integration by parts works and its significance in connecting differentiation and integration.

Applying Integration by Parts (20 minutes)

- Group Work: Divide students into small groups and provide examples for applying integration by parts. Include a mix of straightforward examples and more complex ones that require repeated application.
- Guided Practice: Walk through a step-by-step example on the board, choosing functions u and dv strategically to simplify the integration process.

Mini-Investigation: Repeated Integration by Parts (15 minutes)

- Activity: Students will investigate the integral $\int e^x \sin(x) dx$ through repeated application of integration by parts, following structured steps to identify patterns and simplify the integral without including any integrals in the final expression.
- Reflection: Discuss the characteristics of exponential and trigonometric functions that make the integration of their product feasible using integration by parts.

Discussion: Efficiency and Alternatives to Integration by Parts (5 minutes)

- Debatable Inquiry: Is integration by parts always the most efficient method for integrating products of functions? Explore scenarios where it is particularly effective and others where alternative methods might be preferable.

- Exploration: Speculate on the possibility of undiscovered methods that could simplify the integration process further.

Closure and Extension (5 minutes)

- Summary: Recap the key points of the lesson, emphasizing the importance of integration by parts in solving complex integrals.

- Extension: Introduce the optional extension of deriving the integration by parts formula, challenging students to explore the mathematical foundations further.

Homework Assignment:

- Students will complete a set of practice problems that vary in complexity, applying integration by parts and, where necessary, its repeated use.

Materials Needed:

- Whiteboard and markers

- Handouts with the integration by parts formula, example problems, and steps for the mini-investigation

Assessment:

- Formative Assessment: Participation in group work and discussions.

- Summative Assessment: Completion of homework problems and a quiz focusing on integration by parts and its repeated application.