Lesson 15: Error intervals

Goals

- Comprehend that manufacturers often define a maximum acceptable percentage error for characteristics of their products.
- Determine what information is needed to solve a problem involving percentage error. Ask questions to elicit that information.
- Generate values that fall within the acceptable range for a measurement, given a maximum percentage error.

Learning Targets

- I can find a range of possible values for a quantity if I know the maximum percentage error and the correct value.

Lesson Narrative

This lesson is optional. In this lesson, students find and analyse intervals of possible error based on maximum possible errors.

This material gives students a solid foundation for future work in statistics. It is not necessary at this stage to emphasise the idea of a margin of error which defines a range of possible values. It is enough for students to see the values falling into that range, as preparation for future learning.

Addressing

- Use proportional relationships to solve multistep ratio and percentage problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percentage increase and decrease, percentage error.

Instructional Routines

- Information Gap Cards
- Three Reads
- Poll the Class
- Think Pair Share

Required Materials

Four-function calculators
Pre-printed slips, cut from copies of the blackline master

Info Gap: Quality Control Problem Card 1 A factory worker tested the speedometer of a new car by driving it a certain distance at a constant speed. The percentage error was too large, so the car was sent back to be fixed. What speed did the speedometer show?	Info Gap: Quality Control Data Card 1 • The car went 10 miles. • The car drove for 8 minutes. • To pass the test, the speedometer must show within 2% of the correct speed. • The speedometer was showing a speed slower than the car was actually going.
Info Gap: Quality Control Problem Card 2 A machine is supposed to fill each bottle with the same amount of juice. A particular bottle was tested and the percentage error was not too large. How much juice was in the bottle?	Info Gap: Quality Control Data Card 2 • Each bottle is supposed to be filled with 450 millilitres of juice. • To pass the test, the bottle must be filled to within 1.5% of the correct amount. • The bottle was slightly overfilled.

Required Preparation

Print and cut cards from the Info Gap: Quality Control activity blackline master. One copy of the blackline master will be used for every 4 students.

Student Learning Goals

Let's solve more problems about percentage error.

15.1 A Lot of Iron Ore

Warm Up: 10 minutes

The purpose of this warm-up is to begin to understand the idea that a maximum percentage error defines an interval of values that a quantity can lie within. Students are asked to give possible readings on a scale that has a possible error of up to 1%. Different answers are possible.

Instructional Routines

- Poll the Class

Launch

Give students 3 minutes quiet work time, followed by whole-class discussion.

Student Task Statement

An industrial scale is guaranteed by the manufacturer to have a percentage error of no more than 1%. What is a possible reading on the scale if you put 500 kilograms of iron ore on it?

Student Response

Answers vary. The scale may show any value between 495 and 505.

Activity Synthesis

Draw a blank number line and then put three tick marks in the middle and label them 490, 500, 510. Poll the class for possible measurements, and plot each on the number line. Some students may give answers outside the error interval. Record those with the others and flag them mentally for discussion. Make sure everyone agrees with all of the possible measurements. Students might limit their answers to the extreme values 495 and 505, which have an error of exactly 1%. Be sure to solicit other answers if those are the only two offered.

Summarise the results using the greatest and least possible measurements (495 and 505, respectively) and point out how all of the possible measurements fall between these two values.

15.2 Saw Mill

Optional: 10 minutes

This activity uses a quality control situation to work with percentage error. It is very common that products in a factory are checked to make sure that they meet certain specifications. In this case, boards are cut to a specific length. If they are too long or too short they are rejected. Students should make sense of the decision to be based on percentage error and not on measurement error. If we know the correct length and an acceptable percentage error, then we can find out which lengths are acceptable and which should be rejected.

Instructional Routines

- Three Reads
- Think Pair Share

Launch

Arrange students in groups of 2. Allow students 3--5 minutes of quiet work time followed by partner and whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge of the meaning of percentage error and how to convert a percentage to its decimal form. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing Reading, Writing: Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, ask students to read the problem with the goal of comprehending the situation (e.g., A saw mill cuts wood boards. Boards are inspected. Some boards are rejected.). If needed, discuss the meaning of unfamiliar terms at this time (e.g., saw mill, quality control, inspector, etc.). Use the second read to identify the important quantities by asking students what can be counted or measured (e.g., board length, percentage error, tolerance interval). In the third read, ask students to brainstorm possible mathematical solution strategies to complete the task. This will help students connect the language in the word problem and the reasoning needed to solve the problem while keeping the intended level of cognitive demand in the task.
Design Principle(s): Support sense-making

Student Task Statement

1. A saw mill cuts boards that are 16 ft long. After they are cut, the boards are inspected and rejected if the length has a percentage error of 1.5% or more.
   a. List some board lengths that should be accepted.
   b. List some board lengths that should be rejected.
2. The saw mill also cuts boards that are 10, 12, and 14 feet long. An inspector rejects a board that was 2.3 inches too long. What was the intended length of the board?

Student Response

1. Answers vary. Acceptable responses:
   a. For a 16 ft board, 1.5% is 0.24 ft or 2.88 in. Acceptable lengths are between 15.76 ft and 16.24 ft.
   b. Unacceptable lengths are less than 15.76 ft or more than 16.24 ft.
2. The rejected board could have been an intended 10 ft or 12 ft board since 1.5% of 10 ft is 0.15 ft or 1.8 in and 1.5% of 12 ft is 0.18 ft or 2.16 in. It could not have been 14 ft since 1.5% of 14 ft is 0.21 ft or 2.52 in.

Activity Synthesis

The purpose of the discussion is for students to understand why a tolerance based on percentage error may be acceptable.

Consider asking these discussion questions:

- "A 16 foot board is also 192 inches long. What is the maximum number of inches allowed for an acceptable board of this length?" (2.88 inches)

- "Why should the mill accept boards that are longer or shorter than 16 feet exactly?" (It may be difficult to cut the boards to that exact length, especially as quickly as a saw mill probably needs to cut them, so some range of acceptable lengths is probably allowed.)
- "Why does it make sense for the range of acceptable lengths to be listed as a percentage error rather than on a fixed length?" (While 2.88 inches may be ok for a board that should be about 192 inches long, if the board was supposed to be 5 inches long and it was allowed to be 2.88 inches longer, it would be more than 50% longer than intended.)

15.3 Info Gap: Quality Control

Optional: 20 minutes

This info gap activity uses a quality control situation working with percentage error. It is very common that products in a factory are checked to make sure that they meet certain specifications. In this case the odometer of a car is tested and the amount of liquid in a bottle that is automatically filled is checked. It makes sense that the decision should be based on percentage error and not on absolute error.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need. It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need.

Instructional Routines

- Information Gap Cards

Launch

Provide access to calculators. Tell students they will continue to work with percentage errors in realistic scenarios. Explain the Info Gap structure and consider demonstrating the protocol if students are unfamiliar with it. There are step-by-step instructions in the student task statement.

Arrange students in groups of 2. In each group, distribute a problem card to one student and a data card to the other student. After you review their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.
Supports accessibility for: Memory; Organisation Conversing: Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece

of information)”, and “Why do you need to know . . . (that piece of information)?"
Design Principle(s): Cultivate Conversation

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner *“What specific information do you need?”* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.
3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, swapping roles with your partner.

Student Response

1. The speedometer must have shown a speed less than 73.5 miles per hour. The car was actually moving 1.25 miles per minute, because $10 \div 8 = 1.25$. This is the same speed as 75 miles per hour, because there are 60 minutes in an hour, and $1.25 \times 60 = 75$. Since 2% of 75 is 1.5, that means the speedometer must have been incorrect by more than 1.5 miles per hour. The speedometer was showing slower than the car was

actually moving, so the speedometer must have shown less than $75 - 1.5$, or 73.5 miles per hour.

2. The bottle must have more than 450 but less than 456.75 millilitres of juice in it. Since the bottle was slightly overfilled, we can multiply 1.015×450 to get the maximum amount of juice that could be in the bottle and still be acceptable.

Activity Synthesis

The purpose of the discussion is to recognise what information is needed when dealing with tolerances based on percentage error.

After students have completed their work, share the correct answers and ask students to discuss the different ways they solved this problem. Some guiding questions:

- "What information did you and your partner have to figure out?"
- "What different calculations did you have to make for the two situations?"
- "Why might a car manufacturer not want to sell a car that shows a speed lower than what the car is actually going?" (It could be dangerous for the driver if they think they are going slower than they actually are. They are also more prone to getting speeding tickets if they think they are going under the speed limit based on the speedometer, but are actually going faster.)
- "Why might a juice manufacturer not want to have too much more juice in the bottle than it says on the label?" (If too many bottles are overfilled, the company may be losing money by giving away extra juice without charging the customer more.)

Conversing: This activity uses Information Gap to give students a purpose for discussing information necessary to solve problems involving equivalent ratios.
Design Principle(s): Cultivate Conversation

Lesson Synthesis

This lesson was about error intervals.

- “Why might companies accept a percentage error in measurement for their products?” (It may be very difficult to get the measurements exact in an efficient way, so some small error is allowed.)
- “Why might a percentage error be used rather than an absolute measurement error?” (The same absolute measurement error may have a much greater impact for smaller measurements than for larger measurements. A percentage error allows the company to use a single rule for multiple measurements rather than writing a new rule for each measurement.)
- “How do you find the range of values that are acceptable when you know the target measurement and a percentage error that is allowed?” (Multiply the target

measurement by the percentage error, then add and subtract that value from the target measurement to get the two endpoints for the interval that is allowed.)

15.4 An Angler's Dilemma

Cool Down: 5 minutes

This cool-down assesses student's understand of a range of possible values for measurements based on a percentage error tolerance.

Student Task Statement

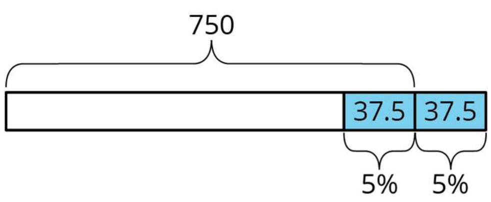
A fisherman weighs an ahi tuna (a very large fish) on a scale and gets a reading of 135 pounds. The reading on the scale may have an error of up to 5%. What are two possible values for the actual weight of the fish?

Student Response

Acceptable answers are between 128.25 pounds and 141.75 pounds. 5% of 135 is 6.75 ($0.05 \times 135 = 6.75$), so $135 - 6.75 = 128.25$ and $135 + 6.75 = 141.75$ define the range for the scale.

Student Lesson Summary

Percentage error is often used to express a range of possible values. For example, if a box of cereal is guaranteed to have 750 grams of cereal, with a margin of error of less than 5%, what are possible values for the actual number of grams of cereal in the box? The error could be as large as $(0.05) \times 750 = 37.5$ and could be either above or below than the correct amount.



Therefore, the box can have anywhere between 712.5 and 787.5 grams of cereal in it, but it should not have 700 grams or 800 grams, because both of those are more than 37.5 grams away from 750 grams.

Lesson 15 Practice Problems

Problem 1 Statement

Jada measured the height of a plant in a science experiment and finds that, to the nearest $\frac{1}{4}$ of an inch, it is $4\frac{3}{4}$ inches.

a. What is the largest the actual height of the plant could be?

b. What is the smallest the actual height of the plant could be?

c. How large could the percentage error in Jada's measurement be?

Solution

a. At most $4\frac{7}{8}$ inches tall (if it were taller, then $4\frac{3}{4}$ would not be the nearest quarter inch measurement)

b. At least $4\frac{5}{8}$ inches tall

c. About 2.6% ($0.125 \div 4\frac{3}{4}$)

Problem 2 Statement

The reading on a car's speedometer has 1.6% maximum error. The speed limit on a road is 65 miles per hour.

a. The speedometer reads 64 miles per hour. Is it possible that the car is going over the speed limit?

b. The speedometer reads 66 miles per hour. Is the car definitely going over the speed limit?

Solution

a. Yes, the car might be going more than 65 mph. 1.6% of 64 is 1.024, so the car could be going 65.024 mph which is over the speed limit.

b. No, the car might be going less than 65 mph. 1.6% of 66 is 1.056, so the car could be going as slow as 64.944 mph which is less than the speed limit.

Problem 3 Statement

Water is running into a bathtub at a constant rate. After 2 minutes, the tub is filled with 2.5 gallons of water. Write two equations for this proportional relationship. Use w for the amount of water (gallons) and t for time (minutes). In each case, what does the constant of proportionality tell you about the situation?

Solution

$w = 1.25t$; Every minute the amount of water increases by 1.25 gallons.

$t = 0.8w$; Every 0.8 minutes the amount of water increases by 1 gallon.

Problem 4 Statement

Noah picked 3 kg of cherries. Jada picked half as many cherries as Noah. How many total kg of cherries did Jada and Noah pick?

a. $3 + 0.5$

b. $3 - 0.5$

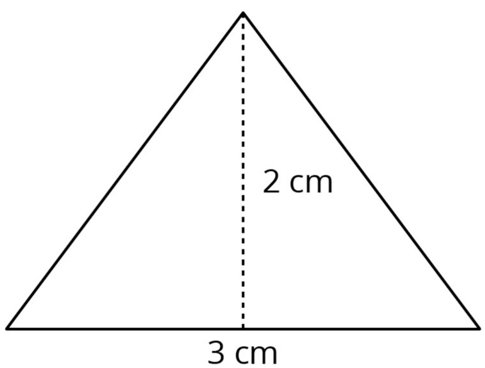
c. $(1 + 0.5) \times 3$

d. $1 + 0.5 \times 3$

Solution C

Problem 5 Statement

Here is a shape with some measurements in cm.



a. Complete the table showing the area of different scaled copies of the triangle.

scale factor	area (cm^2)
1	
2	
5	
s	

b. Is the relationship between the scale factor and the area of the scaled copy proportional?

Solution

a. Complete the table showing the area of different scaled copies of the triangle.

scale factor	area (cm^2)
1	3
2	12
5	75
s	$3s^2$

b. No, the relationship between the scale factor and the area of the scaled copy is not proportional.