Lesson 19: Tables, equations and graphs, oh my!

Goals

- Create a verbal description and a graph to represent the relationship shown in an equation and table.

- Identify tables and equations that represent the same relationship and justify (orally) the match.

- Interpret and critique (orally) different representations of the same relationship, i.e. table, equation, graph, and verbal description

Learning Targets

- I can create a table and a graph that represent the relationship in a given equation.

- I can explain what an equation tells us about the situation.

Lesson Narrative

In this culminating lesson, students look at several examples of equations that represent important relationships from real-world situations. In the first activity, students examine all 9 of the relationships briefly, matching an equation and a table that represent the same relationship. In the following activities, each student works with one of the relationships in more detail: interpreting the equation, continuing the table, and creating a graph. This gives students an opportunity to become an expert on one of these relationships and then use multiple representations to explain their understanding to others.

Alignments

Addressing

- Write, read, and evaluate expressions in which letters stand for numbers.

- Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

- Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyse the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Instructional Routines

- Group Presentations

- Compare and Connect

- Take Turns

Required Materials

Graph paper
Sticky notes
Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: graph paper and pens, whiteboard space and pens, shared online drawing tool, access to a document camera.

Student Learning Goals

- Let's explore some equations from real-world situations.

19.1 Matching Equations and Tables

10 minutes

The purpose of this activity is to familiarise students with the 9 relationships they will continue to work with for the rest of this lesson. In this activity, students match equations and tables that represent the same relationship, without knowing what real-world situations the relationships represent. Students can make use of structure as they narrow down which tables could possibly match each equation, such as recognising whether the values for the dependent variable should be greater or less than the corresponding values for the independent variable, based on the operation in the equation.

Instructional Routines

- Take Turns

Launch

Arrange students in groups of 2. Ask students to take turns: one partner identifies a match and explains why they think it is a match. The other partner's job is to listen and make sure they agree. If they don't agree, the partners discuss until they come to an agreement. The students swap roles for the next equation. If necessary, demonstrate this protocol before students start working. Also, consider demonstrating productive ways to agree or disagree, for example, by explaining your mathematical thinking or asking clarifying questions.

Consider allowing students to use calculators to ensure inclusive participation in the activity.

Anticipated Misconceptions

Some students may struggle to relate the variables in the equation to the columns of the table. Remind them that when we have one variable expressed in terms of the other variable, we call the former the dependent variable and the latter the independent variable. For example, in the equation $a = b + 6$ we say that b is the independent variable and a is the dependent variable, because a is expressed in terms of b.

Student Task Statement

Match each equation with a table that represents the same relationship. Be prepared to explain your reasoning.

$S - 2 = T$

$G = J + 13$

$P = I - 47.50$

$C + 273.15 = K$

$e = 6s$

$m = 8.96V$

$y = \dfrac{1}{12}x$

$t = \dfrac{d}{2.5}$

$g = 28.35z$

Table 1:

independent variable	dependent variable
20	8
58.85	23.54
804	321.6

Table 2:

independent variable	dependent variable
5	18

36	49
75	88

Table 3:

independent variable	dependent variable
2.5	22.4
20	179.2
75	672

Table 4:

independent variable	dependent variable
20	$1\frac{2}{3}$
36	3
804	67

Table 5:

independent variable	dependent variable
58.85	11.35
175.5	128
804	756.5

Table 6:

independent variable	dependent variable
2.5	275.65
20	293.15
58.85	332

Table 7:

independent variable	dependent variable
5	3
20	18
36	34

Table 8:

independent variable	dependent variable
2.6	73.71
20	567
36	1020.6

Table 9:

independent variable	dependent variable
2.6	15.6
36	216
58.85	353.1

Student Response

Table 1: $t = \dfrac{d}{2.5}$

Table 2: $G = J + 13$

Table 3: $m = 8.96V$

Table 4: $y = \dfrac{1}{12}x$

Table 5: $P = I - 47.50$

Table 6: $C + 273.15 = K$

Table 7: $S - 2 = T$

Table 8: $g = 28.35z$

Table 9: $e = 6s$

Activity Synthesis

Much of the discussion will take place between partners. Invite students to share how they identified tables and equations that match.

- What characteristics of the equation or table helped you narrow down the potential matches?

- Were there any matches you and your partner disagreed about? How did you work to reach an agreement?

19.2 Getting to Know an Equation

15 minutes

In the previous activity students looked at 9 different equations. In this activity, each student focuses on just one of those equations to learn about the real-world situation it represents. They interpret the variables and values in terms of the situation. Adding to the table they saw in the previous activity gives students an opportunity to practice both evaluating expressions and solving equations.

This activity also asks students to create a graph of their assigned relationship on graph paper. In previous activities, students were given a grid with the axes already labelled and numbered whenever they were asked to create a graph of a relationship. Here they must decide for themselves how to scale each axis.

Launch

Decide whether you want to assign the equations or allow students to select which equation they will work with. Make sure there is at least one student working with each equation.

Give students 5–10 minutes of quiet work time followed by partner and whole-class discussion. If there is more than one student working with the same equation, consider letting them share their answers and reasoning with each other, after they have had some time to work on their own.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts. Check in with students within the first 2-3 minutes of work time. Look for students who need additional support deciding how to scale each axis for their table. Invite students to share strategies they have used so far, as well as any questions they have before continuing.
Supports accessibility for: Memory; Organisation

Anticipated Misconceptions

Some students may struggle to understand what each step is asking them to do. Consider preparing an example to share with them, such as this for the equation $2.54i = c$, and point out which part of the example corresponds to the question they are on.

1. The number of centimetres is the product of the number of inches and 2.54.

independent variable: length (inches)	dependent variable: length (centimetres)
5	12.7
36	91.44
75	190.5
60	
	300

2. If something is 36 inches long, this is equivalent to 91.44 when measured in centimetres.

3. In the table above, fill in 152.4 in the right column, next to the 60, because $2.54 \times 60 = 152.4$. Fill in 118.11 in the left column, next to the 300, because $2.54 \times 118.11 \approx 300$

4. A graph with "length (inches)" on the horizontal axis, "length (centimetres)" on the vertical axis, and points at $(5, 12.7)$, $(36, 91.44)$, $(75, 190.5)$, $(60, 152.4)$, and $(118.11, 300)$

Some students may struggle with setting up their graph from scratch on graph paper. Prompt them to think about the maximum value they want to represent on each axis and what number they could count by to get to that maximum value in the amount of space they have.

Student Task Statement

The equations in the previous activity represent situations.

- $S - 2 = T$ where S is the number of sides on a polygon and T is the number of triangles you can draw inside it (from one vertex to the others, without overlapping)

- $G = J + 13$ where G is a day in the Gregorian calendar and J is the same day in the Julian calendar

- $P = I - 47.50$ where I is the amount of income and P is the profit after £47.50 in expenses

- $C + 273.15 = K$ where C is a temperature in degrees Celsius and K is the same temperature in Kelvin

- $e = 6s$ where e is the total edge length of a regular tetrahedron and s is the length of one side

- $m = 8.96V$ where V is the volume of a piece of copper and m is its mass

- $y = \frac{1}{12}x$ where x is the number of eggs and y is how many dozens that makes

- $t = \frac{d}{2.5}$ where t is the amount of time it takes in seconds to jog a distance of d metres at a constant speed of 2.5 metres per second

- $g = 28.35z$ where g is the mass in grams and z is the same amount in ounces

Your teacher will assign you one of these equations to examine more closely.

1. Rewrite your equation using words. Use words like product, sum, difference, quotient, and term.

2. In the previous activity, you matched equations and tables. Copy the values from the table that matched your assigned equation into the first 3 rows of this table. Make sure to label what each column represents.

independent variable: _____	dependent variable: _____
60	
	300

3. Select one of the first 3 rows of the table and explain what those values mean in this situation.

4. Use your equation to find the values that complete the last 2 rows of the table. Explain your reasoning.

5. On graph paper, create a graph that represents this relationship. Make sure to label your axes.

Student Response

Answers vary. Sample response: For the equation $S - 2 = T$

1. The number of triangles in a polygon is the difference of the number of sides of the polygon and 2.

2. The first three rows of this table:

independent variable: number of sides of the polygon	dependent variable: number of triangles inside
5	3
20	18
36	34
60	58
302	300

3. In a polygon with 5 sides, you can draw 3 triangles from one vertex that do not overlap.

4. The last two rows of the table above, because $60 - 2 = 58$ and $300 + 2 = 302$.

5. A graph with "number of sides of the polygon" on the horizontal axis, "number of triangles inside" on the vertical axis, and points at $(5,3), (20,18), (36,34), (60,58),$ and $(302,300)$.

Activity Synthesis

Continue to the next activity. The discussion of these relationships occurs after students have created their visual displays.

19.3 Sharing Your Equation with Others

15 minutes

The purpose of this activity is for students to use multiple representations to share with each other what they have learned about their assigned relationship.

Instructional Routines
*   Group Presentations

*   Compare and Connect

Launch

Distribute tools for making a visual display. Give students 5–10 minutes of quiet work time, followed by a whole-class discussion or gallery walk. If time permits, consider having students research more information about their situation to add to their displays.

Representing, Conversing: Compare and Connect. Use this routine to help students consider audience when preparing to display their work. Display the list of items that should be included each display. Ask students, "what kinds of details could you include on your display to help a reader understand what you have learned about your assigned relationship?" Record ideas and display for all to see. Examples of these types of details or

annotations include: the order in which representations are organised on the display, attaching written notes or details to certain representations, using specific vocabulary or phrases, or using colour or arrows to show connections between representations. If time allows, after the gallery walk, ask students to describe specific examples of additional details that other groups used that helped them to interpret and understand their displays.
Design Principle(s): Maximise meta-awareness; Optimise output

Student Task Statement

Create a visual display of your assigned relationships that includes:

- your equation along with an explanation of each variable

- a verbal description of the relationship

- your table

- your graph

If you have time, research more about your relationship and add more details or illustrations to help explain the situation.

Student Response

Answers vary.

Activity Synthesis

Conduct a gallery walk so students have a chance to read and discuss each other's displays. Ask students to reflect on the following prompts:

- What is the same about their relationship and your relationship? What is different?

- What is the independent variable in their relationship? What is the dependent variable?

- What could they add to the display to make their explanation of the relationship even clearer?

Have students use sticky notes to leave questions or comments for the person who created the display. Give students a moment at the end to review any questions or comments left on their display.