Grades 5-8 (AS), 9-12 (S) Duration: 30-40 min Tools: one block / student or pair Individual / Pair work Keywords: Area, Rectangle, Trapezium, Algebraic Expressions, Proof	410 - Heights and Areas **MATHS / 2D GEOMETRY**	LOGIFACES METHODOLOGY Erasmus+ TEACHER Logifaces 2019-1-HU01-KA201-0612722019-1

DESCRIPTION

After calculating the areas of the vertical faces of each block (see exercise 409 - Area of Rectangular Faces), students look for the rule that connects the code of the block and the sum of the areas of the vertical faces.

LEVEL 1 Students observe and formulate the connection. They can use the table shown in the solution.

LEVEL 2 Students prove the connection in the case of blocks 111, 222 or 333.

LEVEL 3 Students prove the connection in general.

SOLUTIONS / EXAMPLES

Denote the three heights of the block by h_1, h_2, h_3 and the length of the base edge by a.

Observation of the connection:

LEVEL 1 Based on the calculation of the sums of the areas, we can conclude that there is a connection between the sum of the heights and the sum of the areas:

Code of block	Sum of heights (in standard units) $h_1 + h_2 + h_3$	Sum of the areas of the faces (in standard units)
111	1+1+1=3	4+4+4=12 (=**3**x4)
112	1+1+2=4	4+6+6=16 (=**4**x4)
122	1+2+2=5	6+6+8=20 (=**5**x4)
113	1+1+3=5	4+8+8=20 (=**5**x4)
222	2+2+2=6	8+8+8=24 (=**6**x4)
123, 132	1+2+3=6	6+8+10=24 (=**6**x4)
133	1+3+3=7	8+8+12=28 (=**7**x4)
223	2+2+3=7	8+10+10=28 (=**7**x4)
233	2+3+3=8	10+10+12=32 (=**8**x4)
333	3+3+3=9	12+12+12=36 (=**9**x4)

Based on the results in the table, the formula for the sum of the areas of the vertical faces is $(h_1 + h_2 + h_3) \times 4$, where 4 is the length of the base edge. If working with a general base length, a, the formula becomes $(h_1 + h_2 + h_3) \times a$.

<u>Proof of the connection:</u>

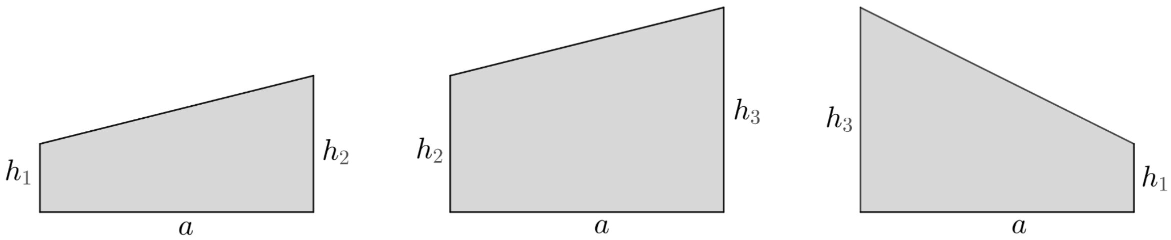
LEVEL 2 These blocks have rectangular vertical faces, hence the sum of their areas is $h_1 \times a + h_2 \times a + h_3 \times a = (h_1 + h_2 + h_3) \times a$.

LEVEL 3 We present two proofs. In the first proof we do not assume that students are familiar with the area formula of a trapezium and present a geometric argument. The second proof is more algebraic and uses the area formula of a trapezium.

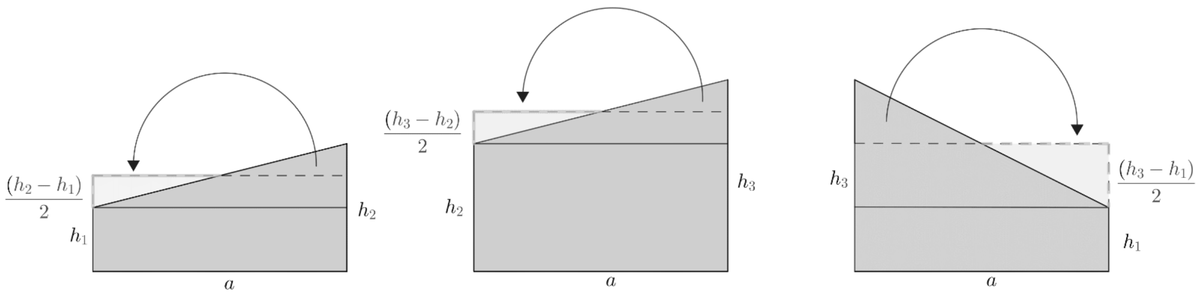
Please note that in both cases rectangles are treated as special trapeziums, where $h = h_1 = h_2$, hence the arguments and formulas stay correct.

<u>Proof 1:</u>

The vertical faces of the block are the trapeziums shown in the figure:



Divide the trapeziums into a rectangle and a triangle as shown in the figure. Then cut the triangles into two parts, and fit them to form rectangles.



Then the sum of the areas of the three rectangles can be calculated as follows:

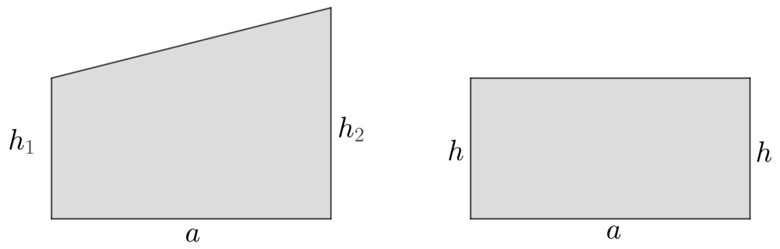
$$ah_1 + a \times \frac{h_2 - h_1}{2} + ah_2 + a \times \frac{h_3 - h_2}{2} + ah_1 + a \times \frac{h_3 - h_1}{2} = a\left(\frac{2h_1 + h_2 - h_1 + 2h_2 + h_3 - h_2 + 2h_1 + h_+ - h_1}{2}\right) = a\left(\frac{2h_1 + 2h_2 + 2h_3}{2}\right)$$
$$= a(h_1 + h_2 + h_3)$$

This explains the observed connection (as $a = 4$).

<u>Proof 2:</u>

To prove the connection between the sum of the heights and the sum of the areas of the vertical faces, note that in a trapezium face, the heights (h_1, h_2) of the Logifaces blocks are the bases of the trapezium and the base edge (a) of the block is the height of the trapezium. Therefore, the area of a trapezium is $A = \frac{1}{2}(h_1 + h_2) \times a$.

Note again that the formula of the area of the trapezium yields also for the area of a rectangular face: $A = a \times h = \frac{1}{2}(h_1 + h_2) \times a$, where $h = h_1 = h_2$ is the length of the vertical edge of the rectangular face.



Hence the sum of the areas of the vertical faces of a Logifaces block with heights h_1, h_2, h_3 is

$$\frac{1}{2}(h_1 + h_2) \times a + \frac{1}{2}(h_1 + h_3) \times a + \frac{1}{2}(h_2 + h_3) \times a = \frac{1}{2}(2h_1 + 2h_2 + 2h_3) \times a = (h_1 + h_2 + h_3) \times a$$

This explains the observed connection (as $a = 4$).

PRIOR KNOWLEDGE

Area formulas of rectangle and trapezium, Algebraic expressions

RECOMMENDATIONS / COMMENT

Exercise 409 - Area of Rectangular Faces is recommended before this exercise. That exercise included the calculation of the areas of the vertical faces.

The proof of the connection (Level 3 exercise) is recommended to be discussed with the whole class.