Mathematics Education Innovation

Lesson 2: Circular grid

Goals

- Comprehend that “a point on the circle” (in written and spoken language) refers to a point that lies on the edge of the circle and not in the circle’s interior.
- Create enlargements of polygons using a circular grid given a scale factor and centre of enlargement.
- Explain (orally) how an enlargement affects the size, side lengths and angles of polygons.

Learning Targets

- I can apply enlargements to shapes on a circular grid when the centre of enlargement is the centre of the grid.

Lesson Narrative

The previous lesson introduced the general idea of an enlargement as a method for producing scaled copies of geometric shapes. This lesson formally introduces a method for producing enlargements. An enlargement has a centre and a *scale factor*. For an enlargement with centre P and scale factor 2, for example, the centre does not move. Meanwhile each point Q stays on ray PQ but its distance from P doubles (because the scale factor is 2).

A circular grid is an effective tool for performing an enlargement. A circular grid has circles with radius 1 unit, 2 units, and so on all sharing the same centre. Students experiment with enlargements on a circular grid, where the centre of enlargement is the common centre of the circles. By using the structure of the grid, they make several important discoveries about the images of shapes after an enlargement including:

- Each grid circle maps to a grid circle.
- Line segments map to line segments and, in particular, the image of a polygon is a scaled copy of the polygon.

The next several lessons will examine enlargements on a rectangular grid and with no grid, solidifying student understanding of the relationship between a polygon and its enlarged image. This echoes similar work in the previous unit investigating the relationship between a shape and its image under other transformations.

As with previous geometry lessons, students should have access to geometry toolkits so they can make strategic choices about which tools to use.

Building On

- Reason with shapes and their attributes.

- Recognise angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

Addressing

- Understand congruence and similarity using physical models, transparencies, or geometry software.

Instructional Routines

- Collect and Display
- Discussion Supports
- Notice and Wonder

Required Materials

Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Student Learning Goals

Let’s enlarge shapes on circular grids.

2.1 Notice and Wonder: Concentric Circles

Warm Up: 5 minutes

The goal of this warm-up is to introduce the circular grid which students will examine in greater detail throughout this unit. The circles in the grid all have the same centre and the distance between consecutive circles is the same. The circular grid is particularly useful for showing enlargements where the centre of enlargement is the centre of the grid.

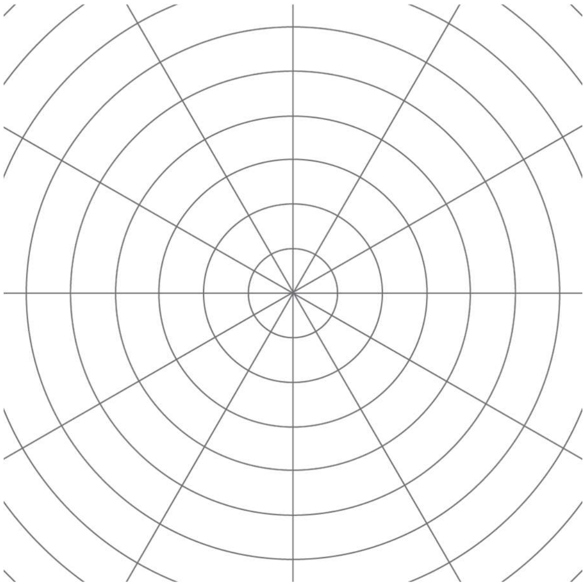
Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement



What do you notice? What do you wonder?

Student Response

Answers vary. Sample responses:

- Do the circles have the same centre?
- Is the centre of the circles where the lines meet?
- Why are there 6 lines meeting in the centre?
- Is the distance between the consecutive circles the same?
- How many pieces is each circle divided into? (This can be taken two ways depending whether you are talking about the one dimensional or two dimensional object.)

Activity Synthesis

Ask students to share their responses, highlighting these features of the picture:

- The circles share the same centre
- The centre of the circles is the point where the lines meet
- The distance from one circle to the next is always the same (the radius of each successive circle is one unit more than its predecessor)

Students may also notice that the angle made by successive rays from the centre is always 30 degrees. Some things students may wonder include

- When is this grid useful?

- Why are the circles equally spaced?
- Why are the lines there?

2.2 A Droplet on the Surface

15 minutes (there is a digital version of this activity)

The purpose of this activity is to begin to think of an enlargement with a scale factor as a rule or operation on points in the plane. Students work on a circular grid with centre of enlargement at the centre of the grid. They examine what happens to different points on a given circle when the enlargement is applied and observe that these points all map to another circle whose radius is scaled by the scale factor of the enlargement. For example, if the scale factor is 3 and the points lie on a circle whose radius is 2 grid units, then the enlarged points will all lie on a circle whose radius is 6 units. Students need to explain their reasoning for finding the scale factor.

Students discover that the circular grid is a powerful tool for representing enlargements and they will continue to use the circular grid as they study what happens when enlargements are applied to shapes other than grid circles.

In the digital activity, students encounter some new tools and the circular grid.

Instructional Routines

- Discussion Supports

Launch

Ask students if they have ever seen a pebble dropped in a still pond, and select students to describe what happens. (The pebble becomes the centre of a sequence of circular ripples.) Display the image from the task statement, and ask students to think about how it is like a pebble dropped in a still pond. Demonstrate that distance on the circular grid is measured by counting units along one of the rays that start at the centre, P. Use *Discussion Supports* to draw students' attention to a few important words in the task:

- "When we say 'on the circle,' we mean on the curve or on the edge. (We do not mean the circle's interior.)"
- "Remember that a ray starts at a point and goes forever in one direction. Their rays should start at P and be drawn to the edge of the grid."

If using the digital activity, you may want to demonstrate enlarging a point before having students begin the task.

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of these terms. Include the following terms and maintain the display for reference throughout the unit: circular grid, "on the circle," ray, scale factor, and

distance. Include a circular grid on the display, give examples, and label the features.
Supports accessibility for: Conceptual processing; Language

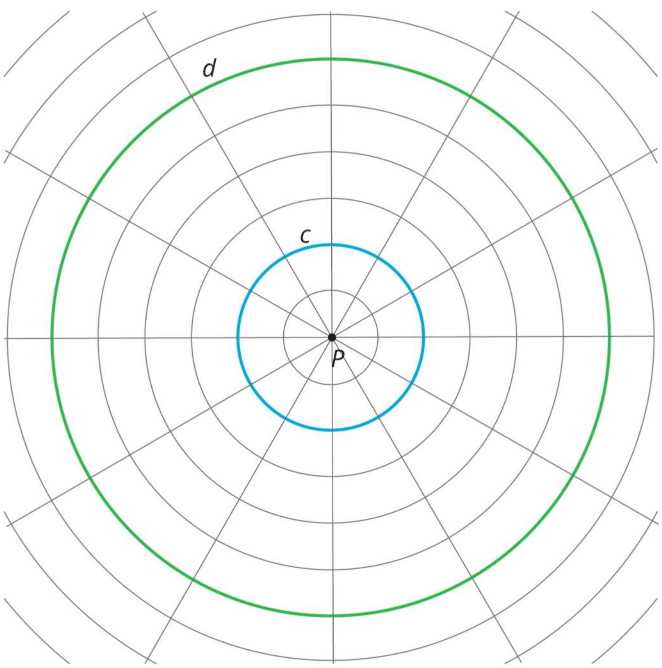
Anticipated Misconceptions

For question 5, students might think the scale factor is 4, because the distance between the smaller and larger circle for each point increases by 4. If this happens, ask students how many grid units circle c is from the centre (2) and how many grid units circle d is from the centre (6). Then remind them that *scale factor* means a number you multiply by.

Student Task Statement

The larger circle d is an enlargement of the smaller circle c. P is the **centre of enlargement**.

1. Draw four points *on* the smaller circle (not inside the circle!), and label them E, F, G, and H.
2. Draw the rays from P through each of those four points.
3. Label the points where the rays meet the larger circle E', F', G', and H'.



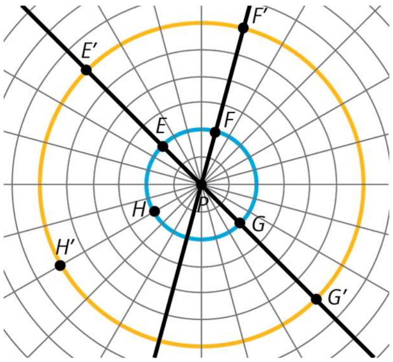
4. Complete the table. In the row labelled S, write the distance between P and the point on the smaller circle in grid units. In the row labelled L, write the distance between P and the corresponding point on the larger circle in grid units.

	E	F	G	H
S				
L				

5. The centre of enlargement is point *P*. What is the *scale factor* that takes the smaller circle to the larger circle? Explain your reasoning.

Student Response

1–3. Answers vary. Sample response:



4.

	E	*F*	*G*	*H*
c	2	2	2	2
d	6	6	6	6

5. The scale factor is 3 because the distance for the small circle is multiplied by 3 to find the distance in the large circle.

Activity Synthesis

Ask students if they made a strategic choice of points, such as points that lie on the grid lines coming from the centre point *P*. Why are these points good choices for enlarging?

Ask students what they think would happen if a circle were enlarged about its centre with a scale factor of 2 or 4. (The result would be a circle with twice the radius and 4 times the radius, respectfully, all sharing the same centre.)

Two important observations coming from the lesson are:

1. The scale factor for this enlargement is 3 so distances from the centre of the circles triple when the enlargement is applied.

2. The large circle is the enlargement of the small circle, that is each point on the circle with radius 6 units is the enlarged image of a point on the circle of radius 2 units. (To find which one, draw the line from the point to the centre and see where it intersects the circle of radius 2 units.)

Speaking, Listening: Discussion Supports. As students share the scale factor that takes the smaller circle to the larger circle, press for details in students' reasoning by asking how they know the scale factor is 3. Listen for students' explanations that reference the table

with distances between the centre of enlargement and points on the circles. Amplify statements that use precise language such as, "The distance between point P and any point on the smaller circle is multiplied by a scale factor of 3 to get the distance between point P and the corresponding point on the larger circle." This will a support rich and inclusive discussion about how the scale factor affects the distance between the centre of enlargement and points on the circle.
Design Principle(s): Support sense-making

2.3 Quadrilateral on a Circular Grid

15 minutes (there is a digital version of this activity)

This activity continues studying enlargements on a circular grid, this time focusing on what happens to points lying on a polygon. Students first enlarge the vertices of a polygon as in the previous activity. Then they examine what happens to points on the sides of the polygon. They discover that when these points are enlarged, they all lie on a side of another polygon. Just as the image of a grid circle is another circle, so the enlargement of a polygon is another polygon. Moreover, the enlarged polygon is a scaled copy of the original polygon. These important properties of enlargements are not apparent in the definition.

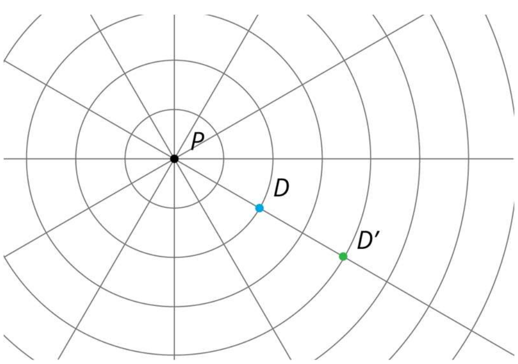
Monitor for students who notice that the sides of the scaled polygon $A'B'C'D'$ are parallel to the sides of $ABCD$ and that $A'B'C'D'$ is a scaled copy of $ABCD$ with scale factor 2. Also monitor for students who notice the same structure for $EFGH$ except this time the scale factor is $\frac{1}{2}$. Invite these students to share during the discussion.

Instructional Routines

- Collect and Display

Launch

Provide access to geometry toolkits. Tell students that they are going to enlarge some points. Before they begin, demonstrate the mechanics of enlarging a point using a centre of enlargement and a scale factor. Tell students, "In the previous activity, each point was enlarged to its image using a scale factor of 3. The enlarged point was three times as far from the centre as the original point. When we enlarge point D using P as the centre of enlargement and a scale factor of 2, that means we're going to take the distance from P to D and place a new point on the ray PD twice as far away from P." Display for all to see:



If using the digital activity, demonstrate the mechanics of enlarging using the applet. You can also use the measurement tool to confirm.

Representation: Internalise Comprehension. Begin with a physical demonstration of the process of enlarging a point using a centre of enlargement and a scale factor to support connections between new situations and prior understandings. Consider using these prompts: "How does this build on the previous activity in which the main task was to find distances and scale factor?" or "How does the point D' correspond to the points D and P?"
Supports accessibility for: Conceptual processing; Visual-spatial processing Conversing, Reading: Collect and Display. Circulate and listen to students as they make observations about the polygon with a scale factor of 2 and the polygon with a scale factor of $\frac{1}{2}$. Write down the words and phrases students use to compare features of the new polygons to the original polygon. As students review the language collected in the visual display, encourage students to clarify the meaning of a word or phrase. For example, a phrase such as "the new polygon is the same as the original polygon but bigger" can be clarified with the phrase "the new polygon is a scaled copy with scale factor 2 of the original polygon." A phrase such as "the polygons have the same angles" can be clarified with the phrase "each angle in the original polygon is the same as the corresponding angle in the new polygon." This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximise meta-awareness

Anticipated Misconceptions

Students may think only grid points can be enlarged. In fact, any point can, but they may have to measure or estimate the distances from the centre. Grid points are convenient because you can measure by counting.

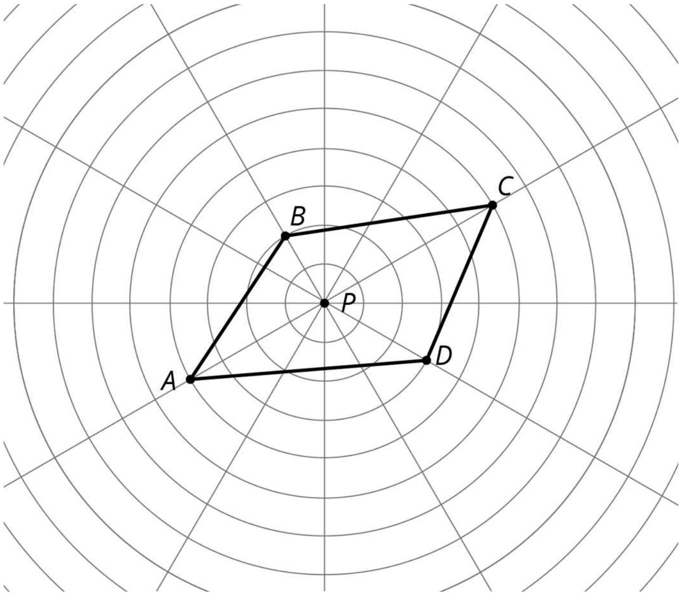
Student Task Statement

Here is a polygon $ABCD$.

1. Enlarge each vertex of polygon $ABCD$ using P as the centre of enlargement and a scale factor of 2. Label the image of A as A', and label the images of the remaining three vertices as B', C', and D'.

2. Draw line segments between the enlarged points to create polygon $A'B'C'D'$.

3. What are some things you notice about the new polygon?



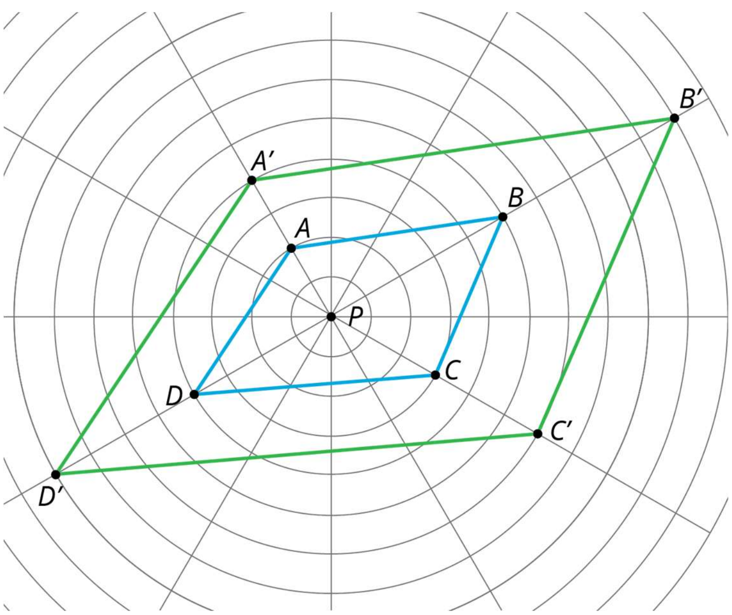
4. Choose a few more points on the sides of the original polygon and transform them using the same enlargement. What do you notice?

5. Enlarge each vertex of polygon $ABCD$ using P as the centre of enlargement and a scale factor of $\frac{1}{2}$. Label the image of A as E, the image of B as F, the image of C as G and the image of D as H.

6. What do you notice about polygon $EFGH$?

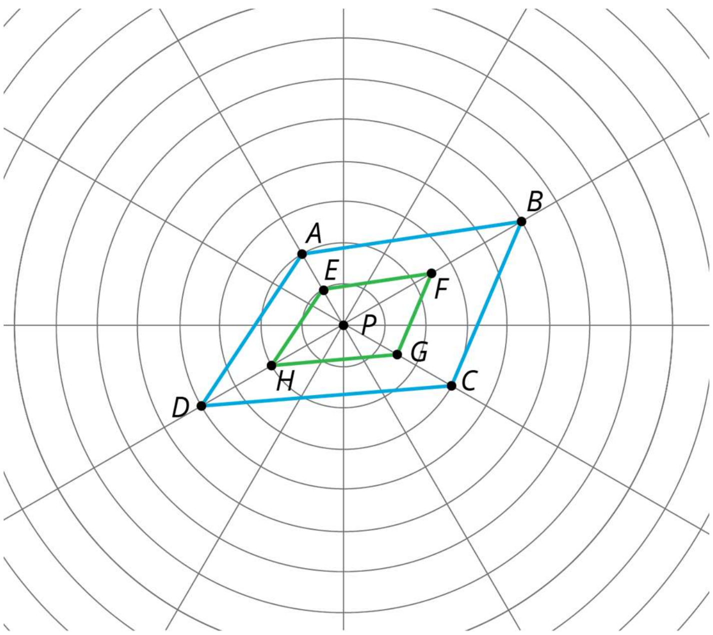
Student Response

1–5.



6. Answers vary. Possible responses:

- The new polygon is a scaled copy of the original polygon.
- Each side of the new polygon is parallel to the corresponding side on the original polygon.
- Each angle in the original shape is congruent to the corresponding angle in the enlarged shape.
- Each side of the new polygon is half the length of the corresponding side in the original polygon.



Are You Ready for More?

Suppose P is a point not on line segment $\overline{WX}$. Let $\overline{YZ}$ be the enlargement of line segment $\overline{WX}$ using P as the centre with scale factor 2. Experiment using a circular grid to make predictions about whether each of the following statements must be true, might be true, or must be false.

1. $\overline{YZ}$ is twice as long $\overline{WX}$.
2. $\overline{YZ}$ is five units longer than $\overline{WX}$.
3. The point P is on $\overline{YZ}$.
4. $\overline{YZ}$ and $\overline{WX}$ intersect.

Student Response

1. Must be true.

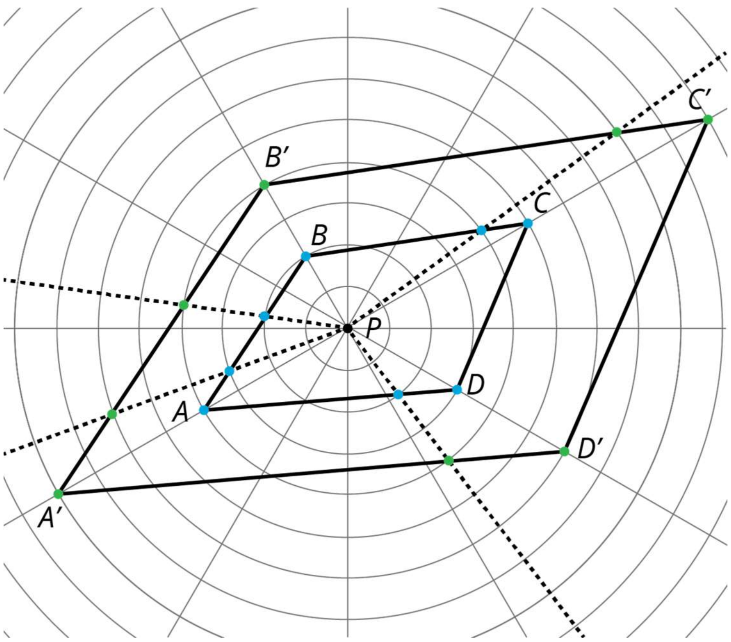
2. Might be true. (True if $\overline{WX}$ has length 5)

3. Must be false.

4. Might be true. (True, for example, if $\overline{WX}$ and $\overline{YZ}$ are both on the same line)

Activity Synthesis

Display the original shape and its image under enlargement with scale factor 2 and centre P.



Ask selected students to share what they notice about the new polygon. Ensure that the following observations are made. Encourage students to verify each assertion using geometry tools like tracing paper, a ruler, or a protractor.

- The new shape is a scaled copy of the original shape.
- The sides of the new shape are twice the length of the sides of the original shape.
- The corresponding line segments are parallel.
- The corresponding angles are congruent.

Ask students what happened to the additional points they enlarged on polygon $ABCD$. Note that a good *strategic* choice for these points are points where $ABCD$ meets one of the circles: in these cases, it is possible to double the distance from that point to the centre without measuring. The additional points should have landed on a side of the enlarged polygon (because of measurement error, this might not always occur exactly). The important takeaway from this observation is that enlarging the polygon's vertices, and then connecting them, gives the image of the entire polygon under the enlargement.

2.4 A Quadrilateral and Concentric Circles

Optional: 10 minutes (there is a digital version of this activity)

This activity continues work on enlargements of polygons on a circular grid. The new twist in this activity is that the radial lines from the centre of the circular grid have been removed. This means that when they enlarge each point, students will need to use a ruler or other straightedge to connect that point to the centre of the circular grid. If there is extra time, they can experiment enlarging points other than the vertices and check that the enlargement of a side of the polygon is still a line segment (though there may be small deviations due to measurement error).

Launch

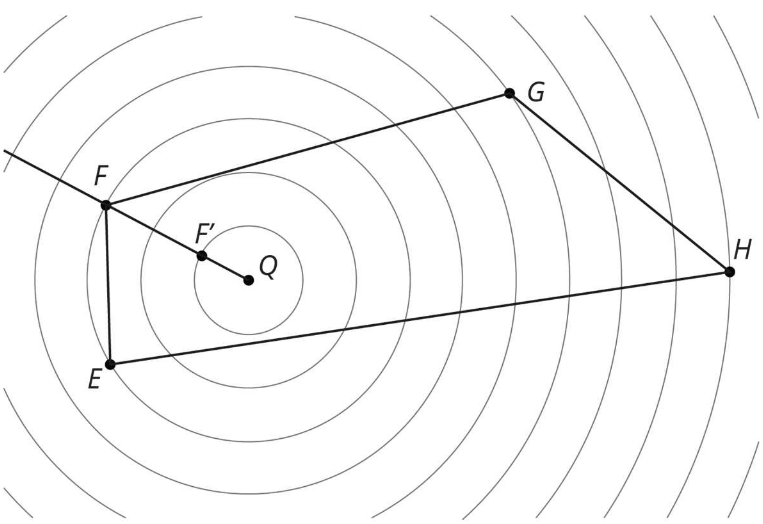
Ask students to quietly read the problem, and then ask them how this problem is alike and different from the previous one. It is alike because it shows a quadrilateral and concentric circles, and we are asked to enlarge the quadrilateral using the centre of the circles as the centre of enlargement. It is different because there is only one radial line through the centre, because the scale factor is now $\frac{1}{3}$, and because one of the points is already enlarged.

Tell students to study how the location of F' was determined, and then to enlarge the remaining points.

Anticipated Misconceptions

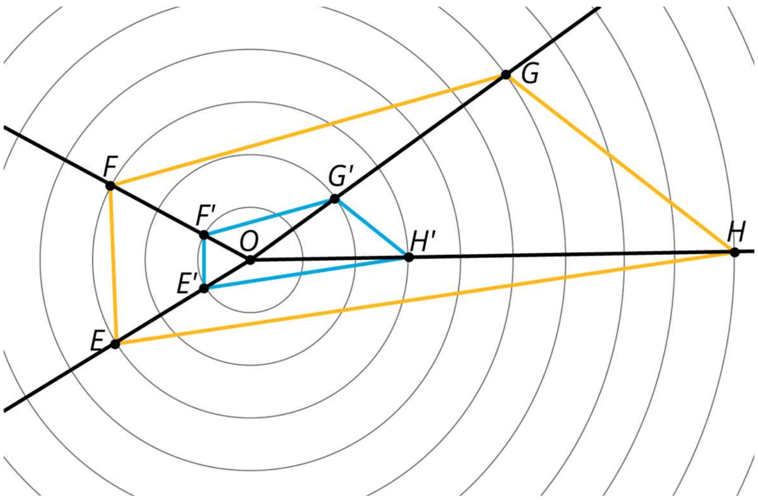
Students may be bothered because the enlarged quadrilateral looks off-centre and the distance between corresponding sides of the quadrilaterals depends on the side. Ensure them that the image is correct and ask them to focus on the parallel corresponding sides of the shapes or ask them if the enlarged quadrilateral appears to be a scaled copy of the original (it does).

Student Task Statement



Enlarge polygon $EFGH$ using Q as the centre of enlargement and a scale factor of $\frac{1}{3}$. The image of F is already shown on the diagram. (You may need to draw more rays from Q in order to find the images of other points.)

Student Response



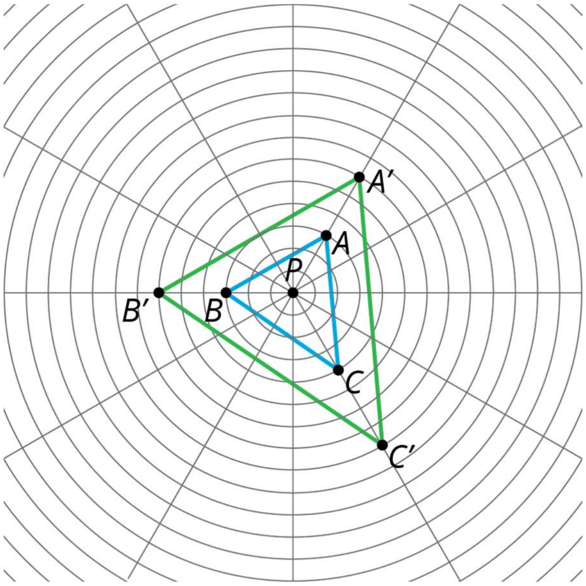
Activity Synthesis

Highlight the need to add line segments joining E, F, G, H to the centre in order to find the image of those points under the enlargement. Also highlight that the scale factor of $\frac{1}{3}$ resulted in an image that was smaller than the original shape instead of larger. You might ask students what scale factor would result in no change? That is, for what scale factor would the image land right on top of the original shape? They can likely name "1" as the scale factor that would accomplish this. So, scale factors that are greater than 1 result in an image larger than the original, and scale factors less than 1 result in an image smaller than the original.

Lesson Synthesis

- "What are some important properties of the circular grid?"
- "How does it help to perform enlargements?"

Highlight the fact that the circular grid is mainly useful when the centre of enlargement is the centre of the grid. When the scale factor is 3, for example, the circle with radius 1 grid unit maps to the circle with radius 3 grid units. More generally, each grid circle maps to a grid circle whose radius is three times as large.



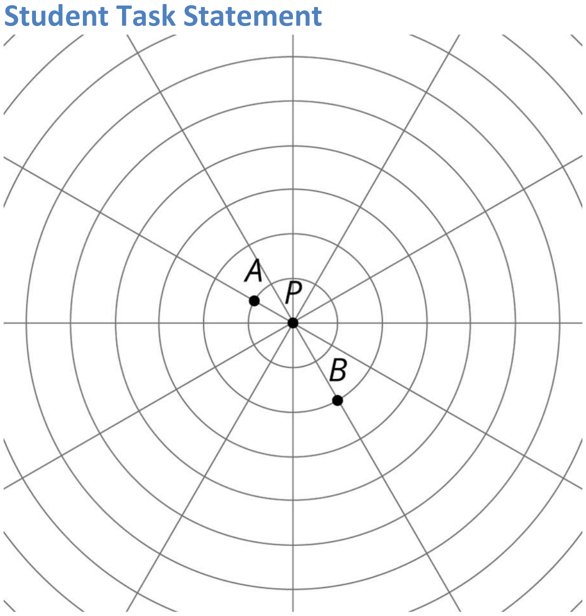
To apply an enlargement to a polygon, we can enlarge the vertices and then add appropriate line segments. For example, triangle $A'B'C'$ is the enlargement of triangle ABC with scale factor 2 and centre of enlargement P:How does triangle $A'B'C'$ compare to triangle ABC? Make sure students see that it is a scaled copy with scale factor 2.

2.5 Enlarging points on a circular grid

Cool Down: 5 minutes

Students apply enlargements with scale factors larger than 1 to points on a circular grid that lie on radial lines.

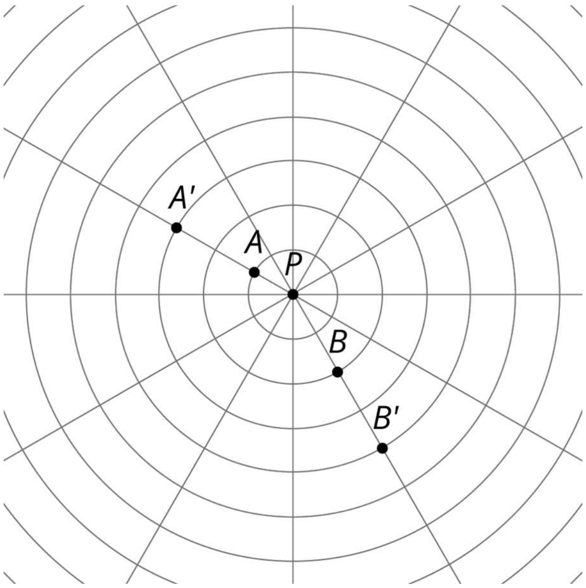
Student Task Statement



1. Enlarge A using P as the centre of enlargement and a scale factor of 3. Label the new point A'.

2. Enlarge B using P as the centre of enlargement and a scale factor of 2. Label the new point B'.

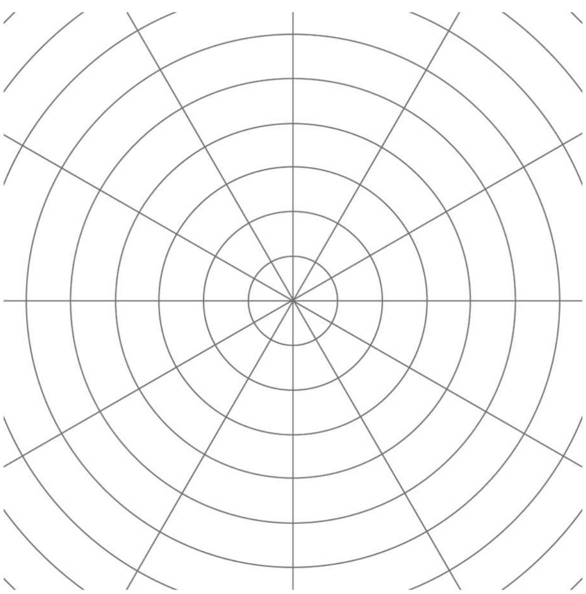
Student Response



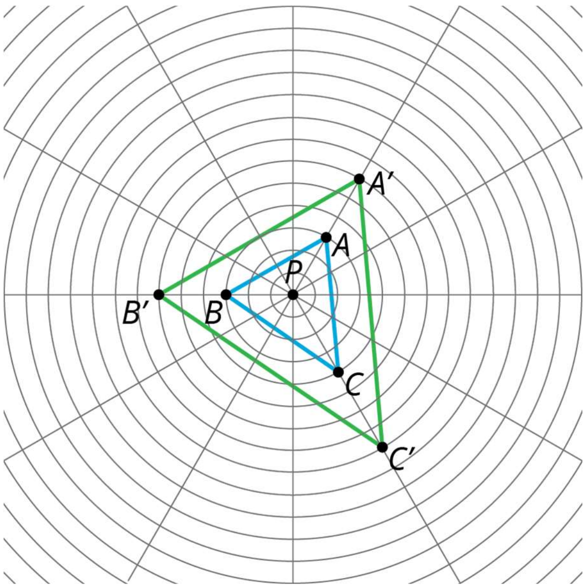
Student Lesson Summary

A circular grid like this one can be helpful for performing **enlargements**.

The radius of the smallest circle is one unit, and the radius of each successive circle is one unit more than the previous one.



To perform an enlargement, we need a **centre of enlargement**, a scale factor, and a point to enlarge. In the picture, P is the centre of enlargement. With a scale factor of 2, each point stays on the same ray from P, but its distance from P doubles:



Since the circles on the grid are the same distance apart, line segment PA' has twice the length of line segment PA, and the same holds for the other points.

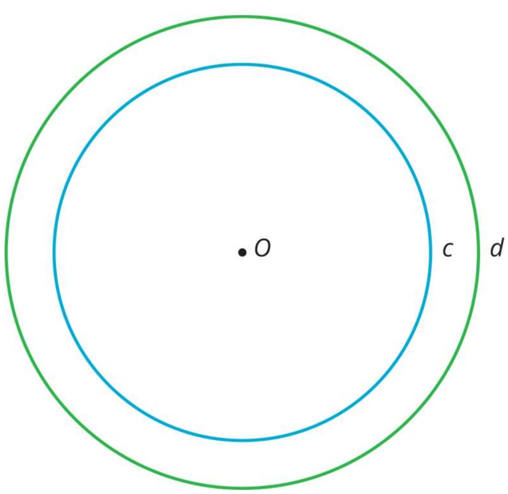
Glossary

- centre of an enlargement
- enlargement

Lesson 2 Practice Problems

1. **Problem 1 Statement**

   Here are circles c and d. Point O is the centre of enlargement, and the enlargement takes circle c to circle d.



a. Plot a point on circle c. Label the point P. Plot where P goes when the enlargement is applied.

b. Plot a point on circle d. Label the point Q. Plot a point that the enlargement takes to Q.

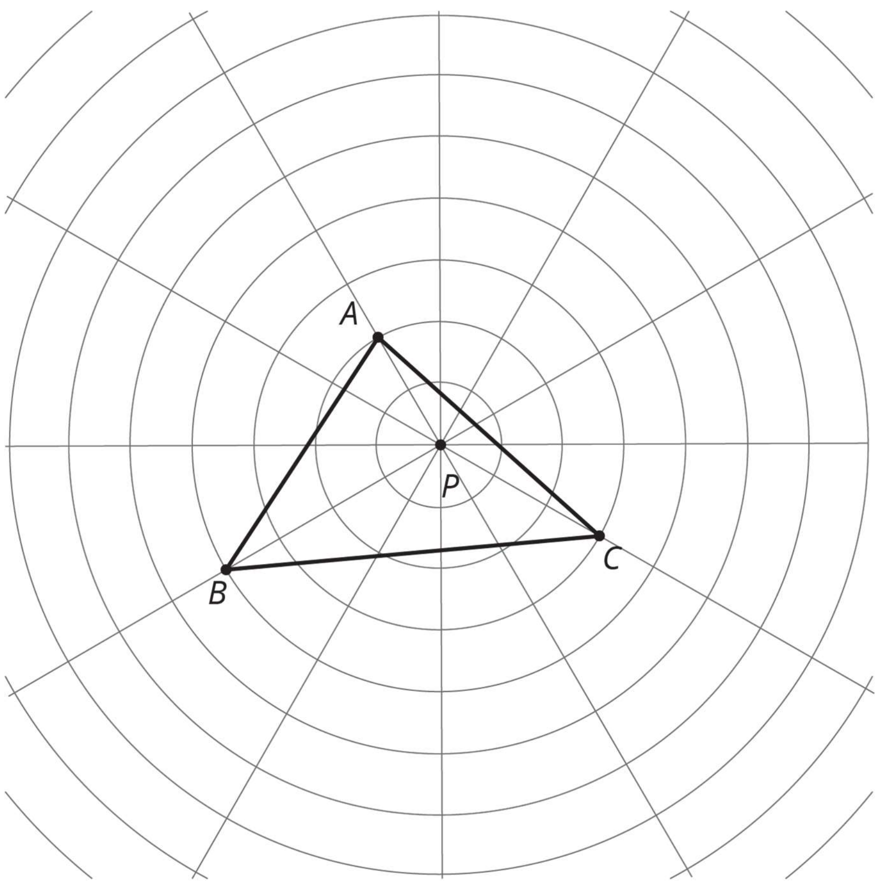
Solution

a. Plot any point P, then draw a ray from O through P. The point where this ray intersects circle d is P'.

b. Plot any point Q, then draw a ray from O through Q. The point where this ray intersects circle c is Q'.

2. **Problem 2 Statement**

Here is triangle ABC.



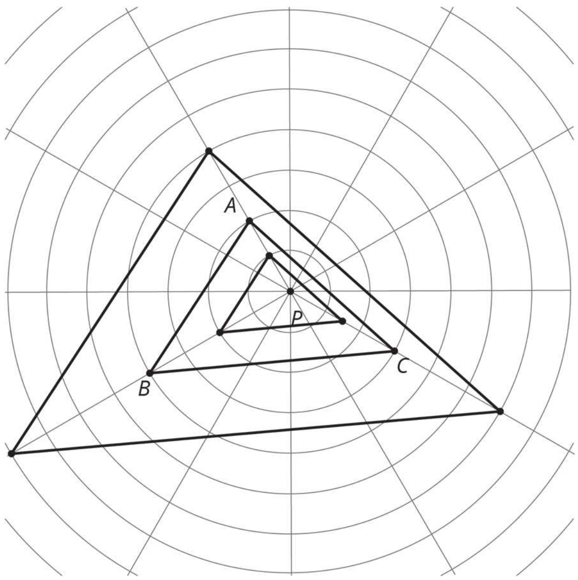
a. Enlarge each vertex of triangle ABC using P as the centre of enlargement and a scale factor of 2. Draw the triangle connecting the three new points.

b. Enlarge each vertex of triangle ABC using P as the centre of enlargement and a scale factor of $\frac{1}{2}$. Draw the triangle connecting the three new points.

c. Measure the longest side of each of the three triangles. What do you notice?

d. Measure the angles of each triangle. What do you notice?

Solution



a. Triangle $A'B'C'$ has each respective point at the same ray. A' is 4 units from the origin, B' is 8 units from the origin, and C' is 6 units from the origin.

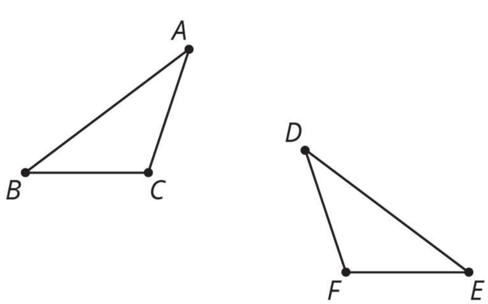
b. Triangle $A''B''C''$ has each respective point at the same ray. A'' is 1 unit from the origin, B'' is 2 units from the origin, and C'' is 1.5 units from the origin.

c. The longest side of the largest triangle is twice as long as the longest side of triangle ABC, which is twice as long as the smallest triangle.

d. The angles in all three triangles have the same sizes.

3. **Problem 3 Statement**

Describe a transformation that you could use to show the polygons are congruent.

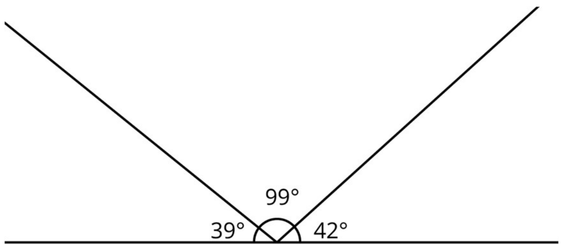


Solution

Reflect triangle ABC in a vertical line and translate so A meets D.

4. **Problem 4 Statement**

The line has been partitioned into three angles.



Is there a triangle with these three angles? Explain.

Solution

Yes