Forces and Angles

<table>
<tr><td rowspan="4">Inte-
gration</td><td colspan="3">21st Century Theme: Problem solving, Critical thinking, Collaboration</td></tr>
<tr><td rowspan="2">Concepts for STEAM Disciplines</td><td>Mathematics
Vectors
Forces
Trigonometry
Analytical Geometry</td><td>Science Education
Forces</td></tr>
<tr><td>Technology
GeoGebra
3D printing</td><td>Art
-</td></tr>
<tr><td colspan="3">Prerequisite Knowledge

Mathematics
Vector calculation
Trigonometry (Sine, Cosine, Angles)

Science Education
Mechanics, Forces

Information Technologies/Technical Crafting
Media Competency</td></tr>
<tr><td colspan="3">Learning Outcomes

Grade Level: 12-15 years old
Activity duration: 250 minutes

Learning Outcomes for Mathematics
Trigonometry
● Defining sin α, cos α, tan α for 0° ≤α≤ 360°.
● Performing calculations on right-angled and general triangles, on figures and solids (also using the sine and cosine theorem)
● Getting to know polar coordinates
Vectors and analytical geometry of the plane
● Adding vectors and multiplying vectors with real numbers, geometric illustration of these arithmetic operations
● Determining unit vectors and normal vectors
● Working with the scalar product, determining the angle of two vectors
● Describing straight lines using parameter representations and equations, intersecting straight lines
● Solving geometric problems, if necessary including elementary geometry

Learning Outcomes for technology
● Basic use of GeoGebra

Learning Outcomes for Science Education
● Mechanics: Relativity of rest and motion, change of motion due to forces, Newton's equation of motion, rectilinear and circular motion, conservation of momentum.

Learning Outcomes for Engineering/Crafting
● Competence orientation based on process-oriented tasks
● Design process from the need to one's own idea to the completion of the own product
● Reflection on the product
● Identify, formulate and solve problems
● Implement workpieces/work orders in practice
● plan and carry out manual, serial or automated production processes
● Example for technology: functional, spatial, static and kinetic relationships of technical systems, construction of machines, robots, electronics, electrical engineering</td></tr>
</table>

Real Life Situation	**Problem Situation** If a ship is sailing and the wind is blowing from a certain direction the ship will sail faster or slower into a certain direction, depending on the direction of the force of the wind. If a car is rolling down a hill the speed is also dependent on the slope, the direction of the car. You are expected to predict where a ship will sail or where a car will drive if a certain wind is blowing from a certain direction.
	Materials ● Computers with internet access ● 3D printer ● Calculator ● GeoGebra account ● A variety of different springs ● Drawing paper, ideally squared paper ● Pencils and pens ● A couple of 3D printed truncated prisms ● A set of the Logifaces game

	Preparation for the lesson ● Teachers should know how to create GeoGebra classrooms ● Teachers should know how to model geometrical objects in GeoGebra ● Teachers should know how to handle a 3D printer ● Teachers should test whether the 3D printer is working by downloading a 3D model of a truncated prism and 3D print it ● Students should be able to have a GeoGebra account ● A 3D printer should be available and a teachers should be able to explain how to use the 3D printer ● Students should know the rules of the game Logifaces that consists of prisms and truncated prisms

	Resources https://www.geogebra.org/m/fn77anbv https://www.geogebra.org/m/gbh3j8u6

STEM Activity	**Ask** The teacher starts the **first lesson** with an introduction about forces. Students should look at images and discuss what they believe a force is and where which kind of force is applied. Students should discuss the following questions: ● Look at the pictures and state whether there is a physical force or not. ● If you believe that a physical force is behind the situation of the image, which force might that be? ● In case of a force, which direction do you think it is acting? ● Students should put their guesses in the GeoGebra book Students should then try the forces of various springs. Afterwards, students should investigate by searching the internet which situations are connected to forces that can be described by triangles. Students should engage into pairs for this research and discuss the results. The **second lesson** is started by an introduction of the teacher how to create truncated prisms in GeoGebra ● How are prisms generated? ● What is the difference between a prism and a truncated prism? Students should use the virtual Logifaces game to learn about connections between prisms and triangles. The truncated prisms should be printed until the **third lesson** where the pairs each create exercise riddles for each other which should be solved during class.
	Research Students should first do an easy exercise reflecting on forces and read up about forces after doing a short Force-quiz. They then should learn about how to measure forces by learning about springs.

	The first experiment is to pull springs and see whether different springs can be pulled differently and to feel the force of a spring. The research phase of the lesson is carried out by using a computer during the first lesson. How to calculate force vectors should be researched by the students for the further steps.

	Imagine The teacher explains that boats do not always have to go into the direction of the wind, they can also sail into different directions. The teacher shows the GeoGebra resource explaining the force of the wind and the resulting speed of the sailboat to explain the relation. The students should then imagine the resulting force vector as the combination of more than one vector and should think about a riddle for their paired up colleague. For this, they should **play** with either an actual Logifaces game or the online version linked in the Geogebra book. Always two students work together. Each pair creates an exercise that is then switched and has to be solved by the other pair. The pairs can then compare results and validate if their result was correct. Each group takes/investigates one tile. Forces go from low to high and the height resembles the strength of the force. The 60° angle from the base triangle and the height of the corners are used for calculation. The change in height between corners is the strength of a force, vectors always go upwards. Then, the sum of the vectors is calculated. An example is added in the book.

	Plan Students will use GeoGebra 3D to build a 3D model of a Logifaces piece (a prism or truncated prism). This model should correctly reflect the dimensions of the Logifaces piece. This should allow the student to determine the surface area and volume of the Logifaces piece. The 3D model of the Logifaces piece should also be able to unroll the grid. 1. Each student has his or her own computer 2. Each pair of students has a couple of Logifaces blocks to measure the size 3. Each student should develop a scale model of their blocks with GeoGebra 4. The calculations with GeoGebra should be verified by calculations on the real object ● Students now decide on one block and on a sequence of blocks they and to create as a riddle for their partner to solve. ● The students should calculate their personal exercise riddle to have a solution for their paired up colleague. ● The solution should be checked with real Logifaces blocks to make sure there is no error in the calculation.

	Create After students learn how to calculate forces by using triangles as a model, they are asked to create exercises for their partners. Students should also have learned how to create truncated prisms using the GeoGebra resources in the book and should have understood the relevance of the various heights of the edges in truncated prisms. Their task is now to create puzzles for each other: go to GeoGebra, create models of prisms or truncated prisms, 3D print them and give them to each other to calculate the force. ● Students should decide on all needed truncated prisms for their riddle and inspect them on screen ● The 3D models should be downloaded as described in the introduction part of the GeoGebra book

	• The 3D models should be 3D printed until the next session
	Test Does the created riddle work when compared with the real objects? Students should be given truncated prisms with a specific height. Will their truncated prism fit the slope in order to form a prism? Can their partners solve their created truncated prism force exercise?
	Develop Students should develop a wide variety of prism and truncated prism exercises and then form a continuous surface with the parts. If multiple prisms or truncated prisms are added, what is the result of the force in the end?
Materials	The resource of the quiz, the resource to discuss forces and explanations about forces and measuring forces will be given to students in the first lesson as an introduction. More materials are available in the book to be used for GeoGebra classrooms. The resources of the sailing ship will be given to students in the second lesson. Afterwards, students should investigate the prism and truncated prism in the book. As a help how to calculate their exercises, the students might check the resource with the calculation example. Students should be shown how to download and investigate their 3D models. In the third lesson, the students should solve each other's riddles. Additional resources such as found in the chapter “Additional Materials”
Test	*This part will be completed by the students after printing and solving each others exercises.*
Improve	*This part will be completed by the students after each use of the 3D printer.*