Grades 9-12 (A)

Duration: 30 min

Tools: one 9 pcs Set / pair

Individual / Pair work

Keywords: Faces, Favourable outcome, Total outcome, Addition rule of probability

622 - Addition Rule of Probability

MATHS / PROBABILITY

LOGIFACES METHODOLOGY

Erasmus+

TEACHER

Logifaces

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DESCRIPTION

LEVEL 1 Students discuss the following observation: fixing for example the block 223 and choosing one other block from the 16 pcs Set at random, the following occurs:

Probability that the chosen block can be fit to	
a) the top face	$\frac{7}{15}$
b) any vertical face	$\frac{9}{15}$
c) any face	$\frac{11}{15}$

The sum of the probabilities in parts a) and b) is $\frac{16}{15}$, which is more than the probability in part c). What is the explanation of this phenomenon?

LEVEL 2 Students discuss the following observation: fixing for example the block 223 and choosing one other block from the 16 pcs Set at random, the following occurs:

Probability that the chosen block can be fit to	
a) the top face	$\frac{7}{15}$
b) the vertical face 22	$\frac{2}{15}$
c) the vertical face 23	$\frac{6}{15}$
d) the vertical face 32	$\frac{6}{15}$
e) neither of the faces	$\frac{4}{15}$

The sum of the probabilities is $\frac{25}{15}$, because the sum of the numbers of favourable outcomes is 25, which is more than the number of blocks in the Set. What is the explanation of this phenomenon?

SOLUTIONS / EXAMPLES

LEVEL 1 Reason: there are some blocks (marked with bold letters) that are counted in the favourable outcomes both in part a) and b), see the table below. The notation 112(2) means that there are two blocks of type 112.

part	probability	favourable outcomes
a)	$\frac{7}{15}$	112(2), **122**, **223(1)**, **233(3)**
b)	$\frac{9}{15}$	**122**, **223(1)**, **233(3)**, 123(2), 132(2)
c)	$\frac{11}{15}$	112(2), 122, 223(1), 233(3), 123(2), 132(2)

The addition rule of probability states for two events A and B that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Here the events are the following:
A = the chosen block can be fit to the top face
B = the chosen block can be fit to some vertical face
$A \cup B$ = the chosen block can be fit to any face
$A \cap B$ = the chosen block can be fit both to the top face and to a vertical face

LEVEL 2 Reason: there are some blocks that are counted in the favourable outcomes more than once in parts a)-e), see the table below.

part	probability	favourable outcomes
a)	$\frac{7}{15}$	112(2), 122, 223(1), 233(3)
b)	$\frac{2}{15}$	122, 223(1)
c)	$\frac{6}{15}$	223(1), 233(3), 123(2)
d)	$\frac{6}{15}$	223(1), 233(3), 132(2)
e)	$\frac{4}{15}$	111, 113, 133, 333

The following notations are used in the solution:

A = the chosen block can be fit to the top face
B_{22} = the chosen block can be fit to the vertical face 22
B_{23} = the chosen block can be fit to the vertical face 23
B_{32} = the chosen block can be fit to the vertical face 32
E = the chosen block cannot be fit to the block 223

The event E is mutually exclusive from the remaining events, and in fact it is the complementary event of the event $A \cup B_{22} \cup B_{23} \cup B_{32}$ = the chosen block can be fit to some face. Hence $P(A \cup B_{22} \cup B_{23} \cup B_{32}) + P(E) = 1$ and the sum of the number of blocks corresponding to the two events is 15, the number of total outcomes.

The probability $P(A \cup B_{22} \cup B_{23} \cup B_{32})$ can be derived from the Addition rule of probability for the four events A, B_{22}, B_{23}, B_{32}:

$$P(A \cup B_{22} \cup B_{23} \cup B_{32}) = P(A) + P(B_{22}) + P(B_{23}) + P(B_{32})$$
$$- \left[P(A \cap B_{22}) + P(A \cap B_{23}) + P(A \cap B_{32}) + P(B_{22} \cap B_{23}) + P(B_{22} \cap B_{32}) + P(B_{23} \cap B_{32})\right]$$
$$+ \left[P(A \cap B_{22} \cap B_{23}) + P(A \cap B_{22} \cap B_{32}) + P(A \cap B_{23} \cap B_{32}) + P(B_{22} \cap B_{23} \cap B_{32})\right]$$
$$- P(A \cap B_{22} \cap B_{23} \cap B_{32})$$
$$= \frac{7}{15} + \frac{2}{15} + \frac{6}{15} + \frac{6}{15} - \left[\frac{2}{15} + \frac{4}{15} + \frac{4}{15} + \frac{1}{15} + \frac{1}{15} + \frac{4}{15}\right] + \left[\frac{1}{15} + \frac{1}{15} + \frac{4}{15} + \frac{1}{15}\right] - \frac{1}{15} = \frac{11}{15}.$$

The Addition rule of probability can also be examined for other blocks. The probabilities for the other blocks can be found in the solution of exercise 621 - Fitting Faces.

PRIOR KNOWLEDGE

The traditional model of probability, Addition rule of probability

RECOMMENDATIONS / COMMENTS

Exercise 617 - Can you Match them? can be considered as a warm-up exercise to this one.
Exercise 618 - Variety with Formulas consists of more general questions of this type and an overview to answer the questions with fewer calculations.