Lesson 10: On or off the line?

Goals

- Determine (in writing) a point that satisfies two relationships simultaneously, using tables or graphs.
- Interpret (orally and in writing) points that lie on one, both, or neither line on a graph of two simultaneous equations in context.

Learning Targets

- I can identify ordered pairs that are solutions to an equation.
- I can interpret ordered pairs that are solutions to an equation.

Lesson Narrative

This lesson builds upon earlier work with linear equations in two variables in two types of contexts: contexts like distance versus time, where there is an initial value and a rate of change, and contexts like budgets, where there is an equation constraining the possible combinations of two quantities. In this lesson, students consider pairs of linear equations in each type of context and interpret the meaning of points on the graphs of the equations.

In the first activity, students are given two constraints on the number of quarters and dimes in someone's pocket: a constraint on the total value and a constraint on the total number of coins. In the second activity, students study two graphs that represent two different students producing locker signs at different rates. In each case students interpret the meaning of various points on and off the lines, including the point of intersection.

Alignments

Addressing

- Analyse and solve linear equations and pairs of simultaneous linear equations.
- Analyse and solve pairs of simultaneous linear equations.

Building Towards

- Analyse and solve pairs of simultaneous linear equations.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Co-Craft Questions
- Discussion Supports
- Which One Doesn’t Belong?

Student Learning Goals

Let’s interpret the meaning of points in a coordinate plane.

10.1 Which One Doesn’t Belong: Lines in the Plane

Warm Up: 5 minutes

The purpose of this warm-up is to elicit ways students can describe different characteristics that arise when more than one line is graphed in a coordinate plane.

Instructional Routines

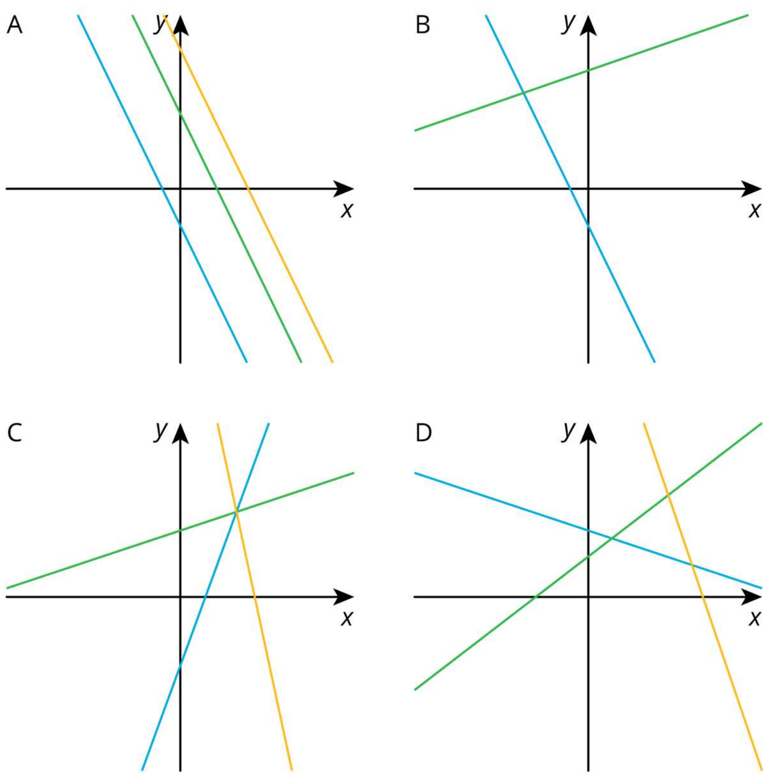
- Which One Doesn’t Belong?

Launch

Arrange students in groups of 2–4. Display the image of all four graphs for all to see. Give students 1 minute of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask the groups to offer at least one reason why *each* graph doesn’t belong.

Student Task Statement

Which one doesn’t belong? Explain your reasoning.



Student Response

Answers vary. Possible responses:

- Graph A is the only one with no intersection points (they are all parallel), or, graph A is the only one that appears to be the same line translated vertically in two different ways.
- Graph B is the only one with an intersection point that has a negative coordinate or the only one with two lines.
- Graph C is the only one with three lines through a single point.
- Graph D is the only one with multiple intersection points.

Activity Synthesis

After students have conferred in groups, invite each group to share one reason why a particular graph might not belong. Record and display the responses for all to see. After each response, ask the rest of the class if they agree or disagree. Since there is no single correct answer to the question of which graph does not belong, attend to students' explanations and ensure the reasons given are correct. Encourage students to use concepts and language introduced in previous lessons about lines such as gradient and intercepts. In particular, draw students' attention to any intersections of the lines.

10.2 Pocket Full of Change

15 minutes

In previous lessons, students have set two expressions equal to one another to find a common value where both expressions are true (if it exists). A system of two equations asks a similar question: at what common pair of values are both equations true? In this activity, students focus on a context involving coins and use multiple representations to think about the context in different ways. The goal of this activity is not for students to write equations or learn the language "system of equations," but rather investigate the mathematical structure with two stated facts using familiar representations and context while reasoning about what must be true.

Monitor for students using different representations, such as a graph, table, or words, as they solve the final problem to share during the whole-class discussion.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Discussion Supports

Launch

Before looking at the task, tell students, "I have $2 in my pocket. What might be in my pocket?" Students will likely guess that you have two $1 bills, but ask what *else* it might be.

Some answers could be 8 quarters; 200 pennies; a $2 bill; or 20 nickels, 2 quarters, and 5 dimes. You will have to explain first that there are 100 cents in $1, 25 cents in a quarter, 10 cents in a dime and 5 cents in a nickel and that a penny is a cent, or ask students what they think the relationships are. The following table could be displayed to aid with this:

Name	Value
Penny	1 cent
Nickel	5 cents
Dime	10 cents
Quarter	25 cents
Half dollar	50 cents
Dollar	100 cents

Read the problem context together. Ensure students understand that we know that Jada has exactly $2 in her pocket, that she only has quarters and dimes, and that she has exactly 17 coins. Give 1–2 minutes for students to read and complete the first problem. Display the table for all to see and ask students for values to fill in the table.

Give students 5–7 minutes of quiet work time to finish the remaining problems followed by a whole-class discussion.

Anticipated Misconceptions

Students may be confused about the last row of the table. Tell them they can enter any values that make sense in the context and are not already on the table.

Student Task Statement

Jada told Noah that she has $2 worth of quarters and dimes in her pocket and 17 coins all together. She asked him to guess how many of each type of coin she has.

1. Here is a table that shows some combinations of quarters and dimes that are worth $2. Complete the table.

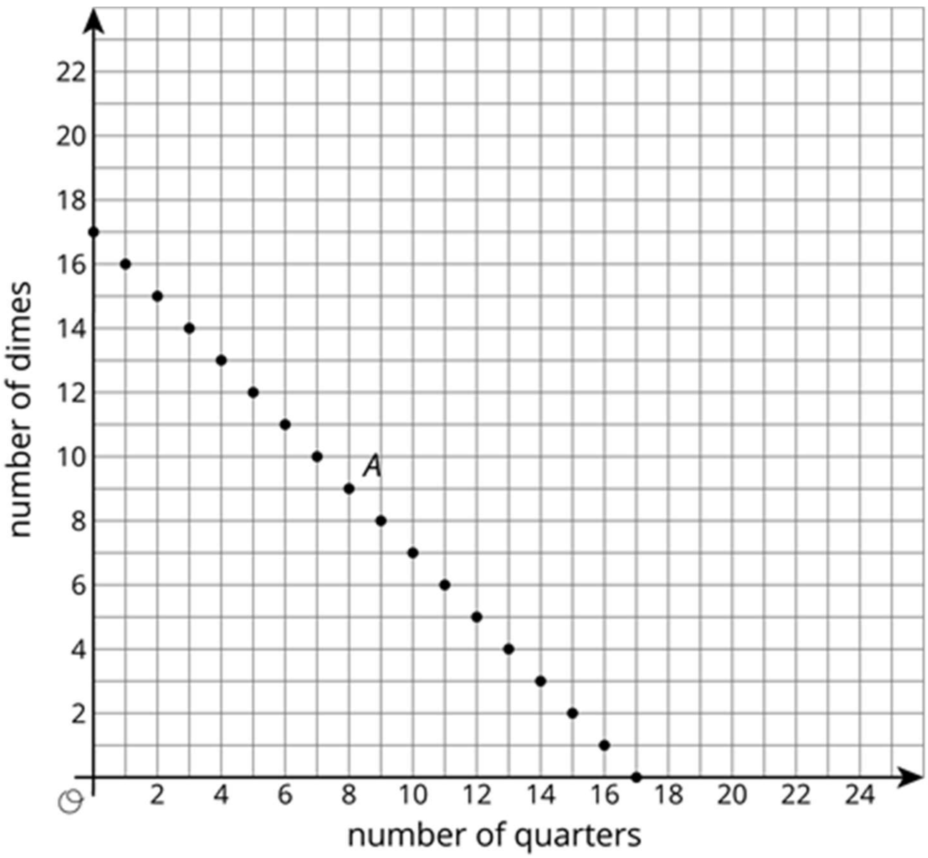
number of quarters	number of dimes
0	20
4	
	0

	5

2. Here is a graph of the relationship between the number of quarters and the number of dimes when there are a total of 17 coins.

   a. What does Point A represent?

   b. How much money, in dollars, is the combination represented by Point A worth?



3. Is it possible for Jada to have 4 quarters and 13 dimes in her pocket? Explain how you know.

4. How many quarters and dimes must Jada have? Explain your reasoning.

Student Response

1. (0,20); (4,10); (8,0); (6,5); (2,15)

   a. A represents 8 quarters and 9 dimes.

   b. These coins would be worth \$2.90, because $8 \times 0.25 + 9 \times 0.10 = 2.90$.

2. No. Even though 4 quarters and 13 dimes is 17 coins, they are not worth \$2. (They are worth \$2.30.)

3. 2 quarters and 15 dimes. This is 17 coins that are worth \$2 because $2 \times 0.25 + 15 \times 0.10 = 2.00$. It is the only combination of quarters and dimes that appears both in the table and as a point on the graph.

Activity Synthesis

The goal of this discussion is to focus students on what must be true based on the two facts they know: that the coins total \$2 and that there are 17 coins. Begin the discussion by asking students about some things they know *cannot* be true about the coins in Jada's pocket and how they know. Students may respond that Jada cannot have 20 dimes and 0 quarters because that is not 17 coins or other variations where either the coins do not total \$2, there are not exactly 17 coins, or neither are true.

Select previously identified students to share how they answered the last problem. If possible, begin a sequence with a student who added on to the table representation to figure out the solution, followed by one who added onto the graph. Include any students who wrote out their reasoning in words last. After each student shares, connect to the idea that their solution is one where *both* facts are true.

Representation: Internalise Comprehension. Represent the same information through different modalities by using physical objects to represent abstract concepts. Students may benefit from access to coins (or paper copies of coins) to use as representations to visualise multiple combinations that add up to \$2.
Supports accessibility for: Conceptual processing; Visual-spatial processing Speaking, Conversing: Discussion Supports. Use this routine to prepare students for the whole-class discussion. Give students 2 minutes of quiet think time to consider, "What are some things you know cannot be true about the coins in Jada's pocket? How do you know?" before inviting them to share their thinking with a partner. Encourage students to ask each other clarifying questions such as, "Can you describe that a different way?", or "How do you know this cannot be true?" This will provide students with an opportunity to clarify their thinking before taking part in a whole-class discussion.
Design Principle(s): Optimise output; Cultivate conversation

10.3 Making Signs

10 minutes

In the previous activity, the system of equations was represented in words, a table, and a graph. In this activity, the system of equations is partially given in words, but key elements are only provided in the graph. Students have worked with lines that represent a context before. Now they must work with two lines at the same time to determine whether a point lies on one line, both lines, or neither line.

Instructional Routines

- Co-Craft Questions

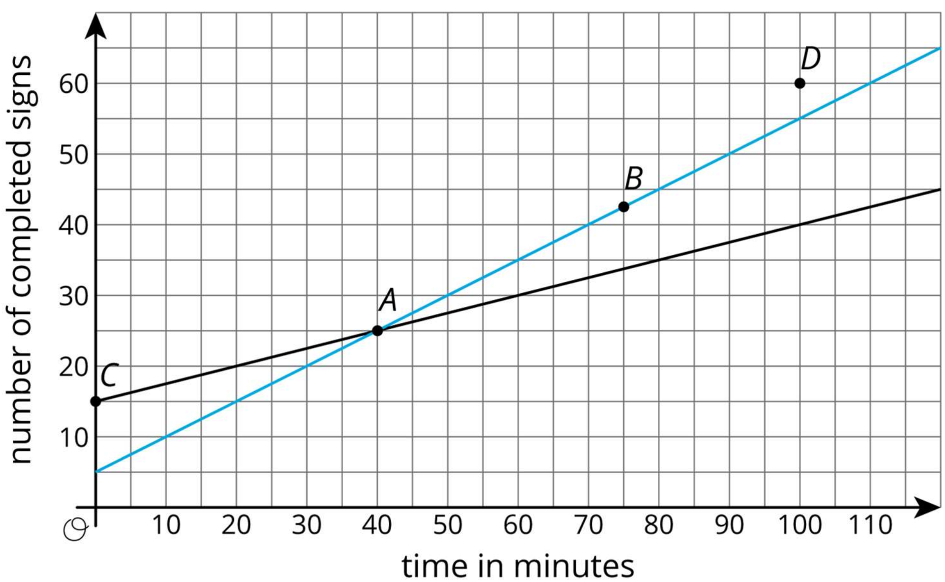
Launch

Arrange students in groups of 2. Give students 1 minute to read the problem and answer any questions they have about the context. Tell students to complete the table one row at a time with one person responding for Clare and the other responding for Andre. Give students 2–3 minutes to finish the table followed by whole-class discussion.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts for students who benefit from support with organisational skills in problem solving. Check in with students within the first 2–3 minutes of work time to ensure that they have understood the directions. If students are unsure how to begin, suggest that they consider each statement for Clare first, and then for Andre.
Supports accessibility for: Organisation; Attention Writing, Speaking: Co-Craft Questions. Display the graph that shows Clare and Andre's progress of making signs, without showing the table. Ask students to write down possible mathematical questions that can be asked about the situation, and then share with a partner. As students discuss, listen for and amplify questions that notice and wonder about the points A, B, C, and D; the location of these points; and why the points were possibly highlighted and what it means in context of the situation. Finally, reveal the table from the activity or use questions generated by the students and allow time to work on them. This will help students make sense of a graph with two lines at the same time by generating questions using mathematical language.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

Student Task Statement

Clare and Andre are making signs for all the lockers as part of the decorations for an event. Yesterday, Andre made 15 signs and Clare made 5 signs. Today, they need to make more signs. Each person's progress today is shown in the coordinate plane.



Based on the lines, mark the statements as true or false for each person.

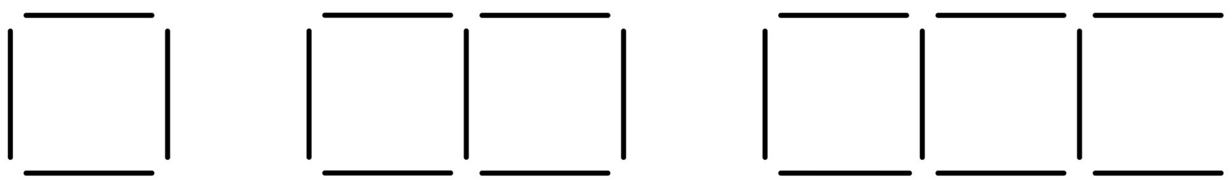
point	what it says	Clare	Andre
A	At 40 minutes, I have 25 signs completed.		
B	At 75 minutes, I have 42 and a half signs completed.		
C	At 0 minutes, I have 15 signs completed.		
D	At 100 minutes, I have 60 signs completed.		

Student Response

A: True, True. B: True, False. C: False, True. D: False, False

Are You Ready for More?

- 4 toothpicks make 1 square
- 7 toothpicks make 2 squares
- 10 toothpicks make 3 squares



Do you see a pattern? If so, how many toothpicks would you need to make 10 squares according to your pattern? Can you represent your pattern with an expression?

Student Response

For each new square you will need 3 toothpicks. If you want to make x squares you will need $3x + 1$ toothpicks. So 10 squares needs $(3 \times 10) + 1 = 31$ toothpicks.

Activity Synthesis

Display the graphs from the task statement. The goal of this discussion is for students to realise that points that lie on one line can be interpreted as statements that are true for Clare, and points that lie on the other line as statements that are true for Andre. Ask students:

- “What is true for Clare and Andre after 20 minutes?” (Clare has 15 signs completed and Andre has 20 signs completed.)
- “What, then, do you know about the point (20,15) and the equation for Clare's graph?” (The point (20,15) is a solution to the equation for Clare's graph.)

- "What do you know about the point (20,20) and the equation for Andre's graph?" (The point (20,20) is a solution to the equation for Andre's graph.)

Invite groups to share their reasoning about points A–D. Conclude by pointing out to students that, in this context, there are many points true for Clare and many points true for Andre but only one point true for both of them. Future lessons will be about how to figure out that point.

Lesson Synthesis

Tell students to think about how they found the ordered pair that makes two relationships true using tables and graphs today. Ask:

- "What are some advantages of tables? If you used two tables to describe the two relationships, how would you know whether a common point exists? If it did exist, how would you find it?" (Tables are good for knowing the exact values for individual points. If the common point is listed in each table, it may be easy to notice, but it may be missing from at least one table or difficult to find if the tables are large and unordered. If the common point is listed in each table, one row of the table should match in both columns.)
- "What are some advantages of graphs?" (Graphs give a better overall picture of the relationships and usually makes estimating (if not finding exactly) the common point easier.)
- "When using graphs, where are the points whose coordinates do *not* make a given relationship true? Do the coordinates of those points show up in a table of values?" (Points that are off of the line do not make the given relationship true. They can be above or below the line. The coordinates of these points do not show up in a table representing the given relationship.)

If time allows, invite students to make up their own stories with two quantities and two relationships to swap with a partner. Have each partner create either two tables of values, two graphs, or one of each to describe the situation and answer a question about the values of the two quantities that make both relationships true.

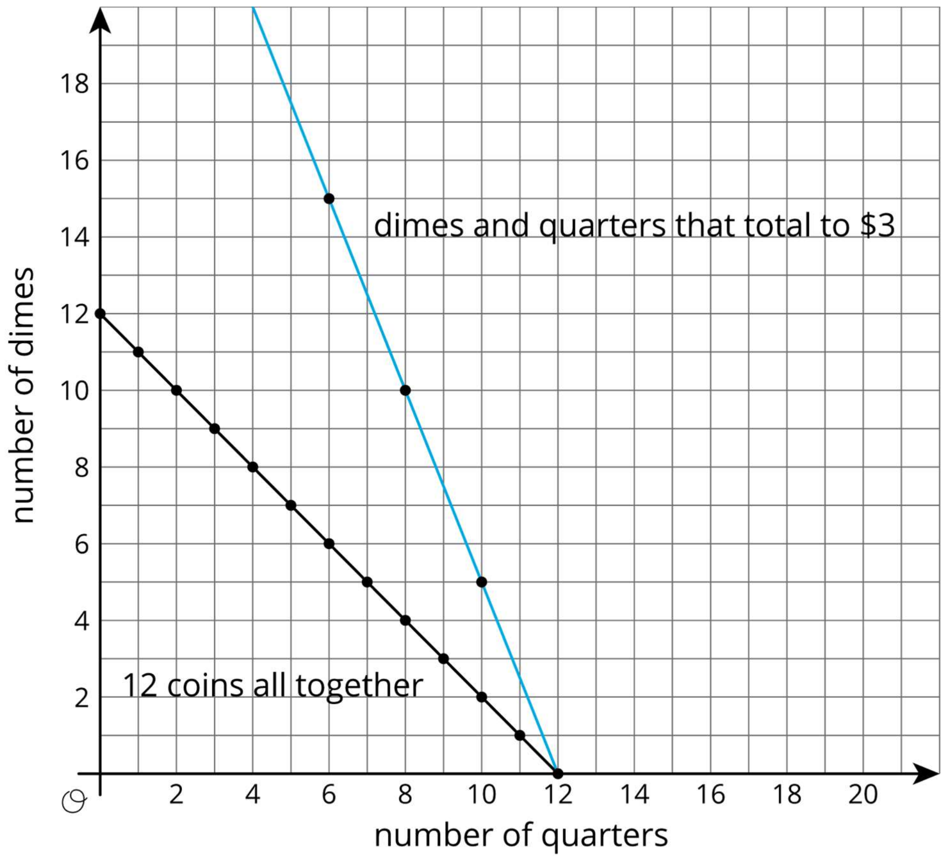
10.4 Another Pocket Full of Change

Cool Down: 5 minutes

This cool-down assesses whether students understand how to read graphs and interpret points on lines as pairs of values that make given relationships true.

Student Task Statement

On the coordinate plane shown, one line shows combinations of dimes and quarters that are worth $3. The other line shows combinations of dimes and quarters that total to 12 coins.



1. Name one combination of 12 coins shown on the graph.

2. Name one combination of coins shown on the graph that total to $3.

3. How many quarters and dimes would you need to have both 12 coins and $3 at the same time?

Student Response

1. Answers vary. Sample responses: 6 quarters and 6 dimes. 11 quarters and 1 dime.

2. Answers vary. Sample responses: 6 quarters and 15 dimes. 10 quarters and 5 dimes.

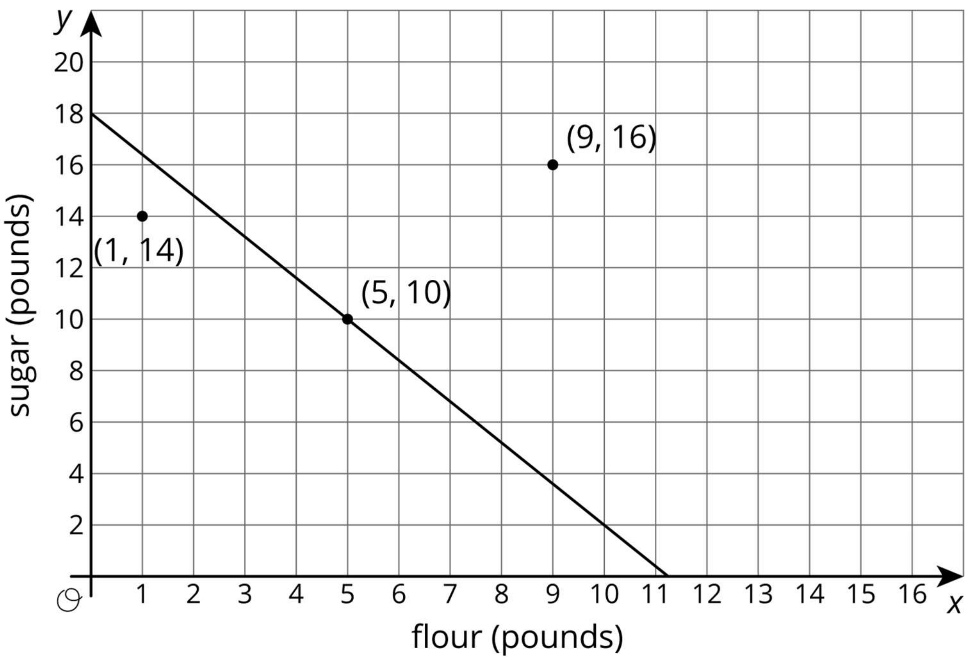
3. 12 quarters and 0 dimes.

Student Lesson Summary

We studied linear relationships in an earlier unit. We learned that values of x and y that make an equation true correspond to points (x, y) on the graph. For example, if we have x pounds of flour that costs £0.80 per pound and y pounds of sugar that costs £0.50 per pound, and the total cost is £9.00, then we can write an equation like this to represent the relationship between x and y:

$0.8x + 0.5y = 9$

Since 5 pounds of flour costs £4.00 and 10 pounds of sugar costs £5.00, we know that $x = 5$, $y = 10$ is a solution to the equation, and the point $(5,10)$ is a point on the graph. The line shown is the graph of the equation:

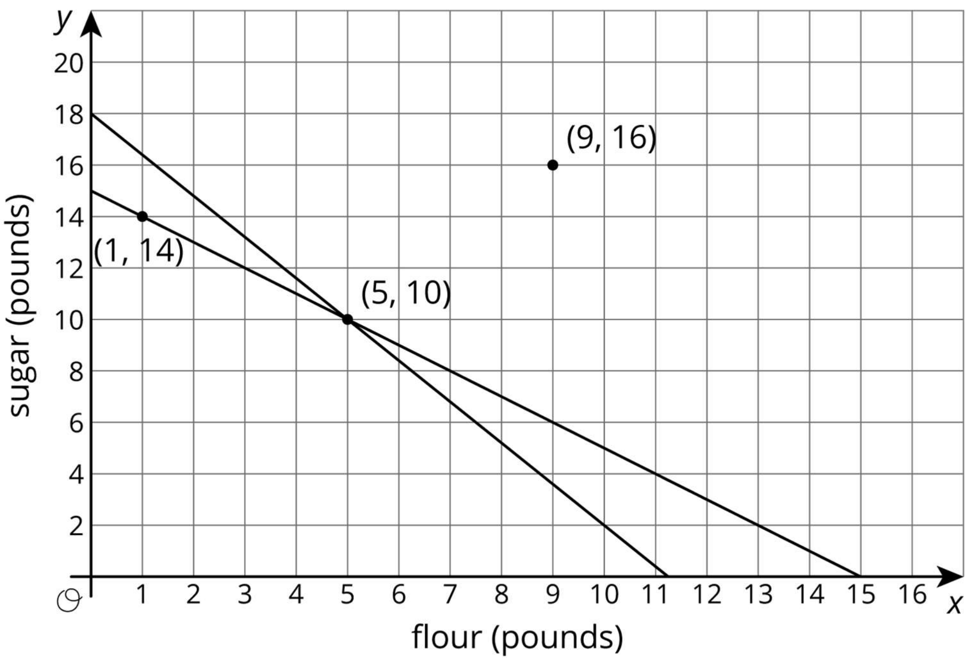


Notice that there are two points shown that are not on the line. What do they mean in the context? The point $(1,14)$ means that there is 1 pound of flour and 14 pounds of sugar. The total cost for this is $0.8 \times 1 + 0.5 \times 14$ or £7.80. Since the cost is not £9.00, this point is not on the graph. Likewise, 9 pounds of flour and 16 pounds of sugar costs $0.8 \times 9 + 0.5 \times 16$ or £15.20, so the other point is not on the graph either.

Suppose we also know that the flour and sugar together weigh 15 pounds. That means that

$x + y = 15$

If we draw the graph of this equation on the same coordinate plane, we see it passes through two of the three labelled points:



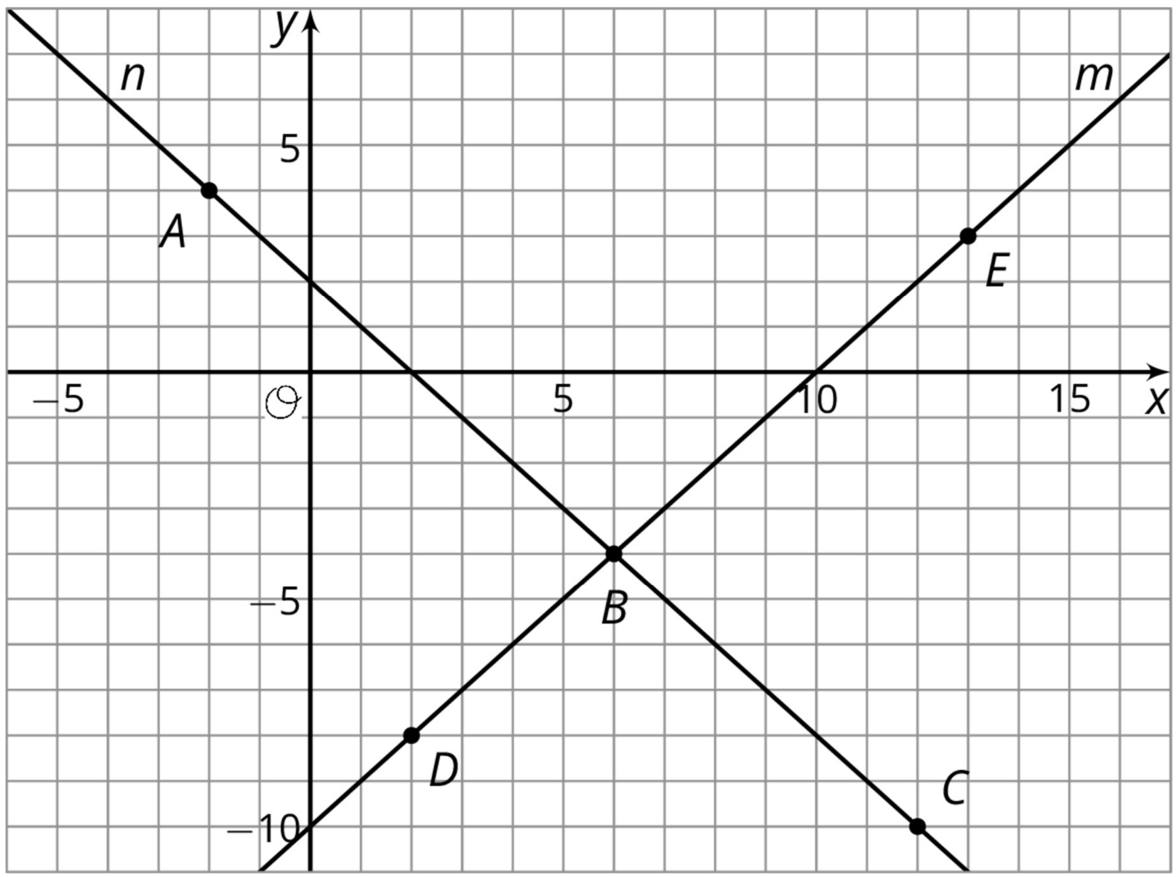
The point (1,14) is on the graph of $x + y = 15$ because $1 + 14 = 15$. Similarly, $5 + 10 = 15$. But $9 + 16 \neq 15$, so (9,16) is *not* on the graph of $x + y = 15$. In general, if we have two lines in the coordinate plane,

- The coordinates of a point that is on both lines makes both equations true.
- The coordinates of a point on only one line makes only one equation true.
- The coordinates of a point on neither line make both equations false.

Lesson 10 Practice Problems

1. **Problem 1 Statement**

   a. Match the lines m and n to the statements they represent:



      i. A set of points where the coordinates of each point have a sum of 2

      ii. A set of points where the y-coordinate of each point is 10 less than its x-coordinate

   b. Match the labelled points on the graph to statements about their coordinates:

      i. Two numbers with a sum of 2

      ii. Two numbers where the y-coordinate is 10 less than the x-coordinate

      iii. Two numbers with a sum of 2 and where the y-coordinate is 10 less than the x-coordinate

Solution

i. n

ii. m

i. A, B, C

ii. D, B, E

iii. B

2. **Problem 2 Statement**

Here is an equation: $4x - 4 = 4x +$ __. What could you write in the blank so the equation would be true for:

a. No values of x

b. All values of x

c. One value of x

Solution

a. Answers vary. Sample response: 19. $4x - 4 = 4x + 19$ has no solutions.

b. Answers vary. Sample response: -4. $4x - 4 = 4x + -4$ is true for all values of x.

c. Answers vary. Sample response: $4x$. $4x - 4 = 4x + 4x$ has one solution ($x =$ -1).

3. **Problem 3 Statement**

Priya and Mai have agreed to go to the movies the weekend after they have earned the *same* amount of money for the *same* number of work hours.

Mai earns £7 per hour mowing her neighbours' lawns. She also earned £14 for hauling away bags of recyclables for some neighbours.

Priya babysits her neighbour's children. The table shows the amount of money m she earns in h hours.

h	m
1	£8.40
2	£16.80
4	£33.60

a. How many hours do they each have to work before they go to the movies?

b. How much will each of them have earned?

c. Explain where the solution can be seen in tables of values, graphs, and equations that represent Priya's and Mai's hourly earnings.

Solution

a. 10 hours

b. £84

c. Explanations vary. Sample response: In a table of values for each person, we would see the same entry for h and m in both tables. In the graph, the solution is found in the coordinates of the point (h, m) where the graphs of the two relationships intersect. In the equations, it is the value of h when we set the two expressions for m equal to each other: $8.4h = 7h + 14$.

4. **Problem 4 Statement**

For each equation, explain what you could do first to each side of the equation so that there would be no fractions. You do not have to solve the equations (unless you want more practice).

a. $\frac{3x-4}{8} = \frac{x+2}{3}$

b. $\frac{3(2-r)}{4} = \frac{3+r}{6}$

a. $\frac{4p+3}{8} = \frac{p+2}{4}$

b. $\frac{2(a-7)}{15} = \frac{a+4}{6}$

Solution

a. Explanations vary. Sample response: If you multiply each side by 24 (the least common multiple of 8 and 3), then the equation becomes $3(3x - 4) = 8(x + 2)$. (The solution is $x = 28$, for those who go the extra mile)

b. Explanations vary. Sample response: If you multiply each side by 12 (the least common multiple of 6 and 4), then the equation becomes $9(2 - r) = 2(3 + r)$. (The solution is $r = \frac{12}{11}$, for those who go the extra mile)

c. Explanations vary. Sample response: If you multiply each side by 8 (the least common multiple of 8 and 4), then the equation becomes $4p + 3 = 2(p + 2)$. (The solution is $p = \frac{1}{2}$, for those who go the extra mile)

d. Explanations vary. Sample response: If you multiply each side by 30 (the least common multiple of 6 and 15), then the equation becomes $4(a - 7) = 5(a + 4)$. (The solution is $a =$ -48, for those who go the extra mile)

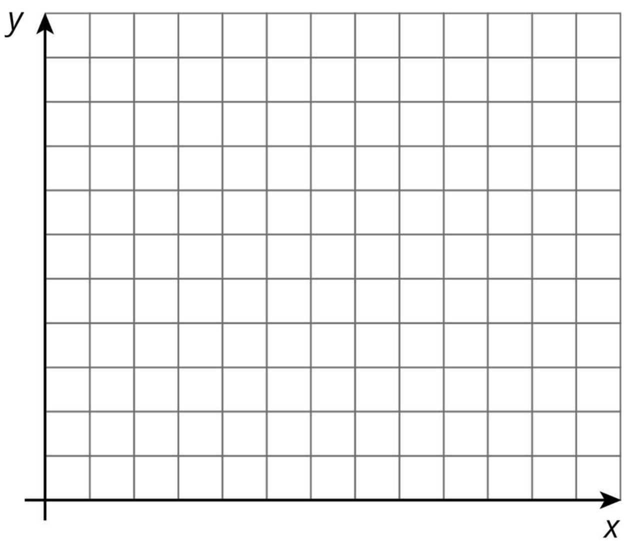
5. **Problem 5 Statement**

The owner of a new restaurant is ordering tables and chairs. He wants to have only tables for 2 and tables for 4. The total number of people that can be seated in the restaurant is 120.

a. Describe some possible combinations of 2-seat tables and 4-seat tables that will seat 120 customers. Explain how you found them.

b. Write an equation to represent the situation. What do the variables represent?

c. Create a graph to represent the situation.



d. What does the gradient tell us about the situation?

e. Interpret the x and y intercepts in the situation.

Solution

a. No 2-seat and 30 4-seat, 10 2-seat and 25 4-seat, 40 2-seat and 10 4-seat. Explanations vary. Sample response: I decided on a number for the 2-seat tables, then figured out how many people that would be (multiply number of tables by 2) and subtracted that from 120. Then I divided by 4 to get the number of 4-seat tables needed for the remaining people.

b. Answers vary. Sample response: $2x + 4y = 120$. x represents the number of 2-seat tables and y represents the number of 4-seat tables.

c. Graph is the line connecting (0,30) and (60,0).

d. Answers vary. Sample response. The gradient is $\frac{-1}{2}$. $\frac{-1}{2}$ tells us that for every one fewer 4-seat table we can use 2 2-seat tables.

e. The intercepts are (0,30) and (60,0). They tell us how many tables there will be if only 4-seat tables are used (30) or only 2-seat tables are used (60).