Lesson 8: How much for one?

Goals

- Calculate equivalent ratios between prices and quantities and present the solution method (using words and other representations).
- Calculate unit price and express it using the word "per" (orally and in writing).
- Understand the phrase "at this rate" indicates that equivalent ratios are involved.

Learning Targets

- I can choose and create diagrams to help me reason about prices.
- I can explain what the phrase "at this rate" means, using prices as an example.
- If I know the price of multiple things, I can find the price per thing.

Lesson Narrative

This lesson introduces students to the idea of **unit price**. Students use the word "per" to refer to the cost of one apple, one pound, one bottle, one ounce, etc., as in "£6 per pound" or "£1.50 per avocado." The phrase "at this rate" is used to indicate that the ratios of price to quantity are equivalent. (For example, "Pizza costs £1.25 per slice. At this rate, how much for 6 slices?") They find unit prices in different situations, and notice that unit prices are useful in computing prices for other amounts.

Students choose whether to draw double number lines or other representations to support their reasoning. They continue to use precision in stating the units that go with the numbers in a ratio in both verbal statements and diagrams.

Note that students are not expected to use or understand the term "unit rate" in this lesson.

Building On

- Number and Operations in Base Ten

Addressing

- Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- Co-Craft Questions
- Discussion Supports

- Number Talk
- Think Pair Share

Required Materials

Rulers
Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Student Learning Goals

Let's use ratios to describe how much things cost.

8.1 Number Talk: Remainders in Division

Warm Up: 10 minutes

This number talk encourages students to think about the numbers in a computation problem and rely on what they know about structure, patterns, and division with remainders to mentally solve a problem.

Only one problem is presented to allow students to share a variety of strategies for division. Notice how students handle a remainder in a problem, which may depend on their prior experiences with division. Students may write it as "r_" and struggle with either fraction or decimal notation. In the next lesson, when students begin finding unit price, they will need to be able to interpret the remainder in either decimal or fraction form.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display the problem for all to see. Give students 2 minutes of quiet think time and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Anticipated Misconceptions

Students may struggle to interpret the remainder as a decimal or fraction and may instead write *r 6*.

Student Task Statement

Find the quotient mentally.

$246 \div 12$

Student Response

- $246 \div 12 = 20.5$ or $246 \div 12 = 20\frac{1}{2}$

Possible strategies:

- Multiplying up: $(12 \times 20) + (12 \times \frac{1}{2})$
- Partial quotients: $(240 \div 12) + (6 \div 12)$

Activity Synthesis

Invite students to share their strategies. Record and display student explanations for all to see. Ask students to explain if or how the dividend or divisor impacted their choice of strategy and how they decided to write their remainder. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone solve the problem the same way but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

At the end of discussion, if time permits, ask a few students to share a story problem or context that $246 \div 12 = 20.5$ would represent.

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

8.2 Grocery Shopping

10 minutes (there is a digital version of this activity)

This activity continues the work on ratios involving one unit of something. Students determine the prices of grocery items and learn to use the term **unit price** to describe cost per unit. To determine unit prices, students may:

- Divide the cost by the number of items

- Use discrete diagrams
- Use a double number line

As students work, monitor for students using different methods.

If students choose to draw a double number line diagram, remind them to label each number line and to circle the ratio where they find the answer.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Co-Craft Questions
- Think Pair Share

Launch

Frame the task in shopping terms. Say that when most of us go shopping, we often see prices for multiple items or units (e.g., 2 bottles for £3, or £1.99 for three pounds, etc.). But sometimes we want to know how much it costs to buy just one item or one unit of something. Tell students they will explore the use of "per" in the context of shopping. Ask students to solve the problems involving "price for one" using any method, and to be ready to explain their reasoning. Provide access to rulers in case students choose to draw double number lines. Give students a few minutes of quiet think time. Pause after the first question, and if any students say the answer is 2, point out that is avocados per pound rather than pounds per avocado.

If students have digital access, they can use an applet to explore the problem and justify their reasoning before discussing with a partner. Allow students 1–2 minutes of work time and then demonstrate how to use the applet with the first problem.

Representation: Internalise Comprehension. Activate background knowledge about finding unit prices. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Writing and Conversing: Co-Craft Questions. In this routine, students are given a context or situation, often in the form of a problem stem with or without numerical values. Students develop mathematical questions that can be asked about the situation. A typical prompt is: "What mathematical questions could you ask about this situation?" The purpose of this routine is to allow students to make sense of a context before feeling pressure to produce answers, and to develop students' awareness of the language used in mathematics problems.
Design Principle(s): Maximise meta-awareness

How It Happens:

1. After students complete the first question, bring the class together and present only the stem:

   Twelve large bottles of water cost £9.

   Do not allow students to see the follow-up questions for this situation.

   Ask students, “What mathematical questions could you ask about this situation?”

2. Give students 1 minute of individual time to jot some notes, and then 3 minutes to share ideas with a partner.

   As pairs discuss, support students in using conversation skills to generate and refine their questions collaboratively by seeking clarity, referring to students’ written notes, and revoicing oral responses as necessary. Listen for students’ use of ‘per’ as they talk.

   If using the applet, have pairs use the applet together. Check that students correctly identify and enter the quantities (water bottles and pounds) so their sense-making with the tick marks is grounded in a clear enough interpretation of the situation.

3. Ask each pair of students to contribute one written question to a poster, the whiteboard, or digital projection. Call on 2–3 pairs of students to present their question to the whole class, and invite the class to make comparisons among the questions shared and their own questions.

   Listen for questions intended to ask about the unit price for a single water bottle, and listen for their use of ‘per’. Revoice student ideas with an emphasis on the use of ‘per’ wherever it serves to clarify a question.

4. Reveal the follow-up questions for this situation and give students a couple of minutes to compare these three questions to their own and those of their classmates. Identify similarities and differences.

   - How many bottles can you buy for £3?
   - What is the cost per bottle of water?
   - How much would 7 bottles of water cost?

5. Invite students to choose one question to answer (from the class or from the curriculum), and then have students move on to the following problems.

Anticipated Misconceptions

Some students may have difficulty with the answers not being integers. Either fractions or decimals are acceptable. Fractions provide the most direct route, but decimals are common for working with pounds and pence. Also, students may use the larger numbers as the dividend, simply because they are larger. Encourage students to check the reasonableness of their answers.

Student Task Statement

Answer each question and explain or show your reasoning. If you get stuck, consider drawing a double number line diagram.

1. Eight avocados cost £4.

    a. How much do 16 avocados cost?

    b. How much do 20 avocados cost?

    c. How much do 9 avocados cost?

2. Twelve large bottles of water cost £9.

    a. How many bottles can you buy for £3?

    b. What is the cost per bottle of water?

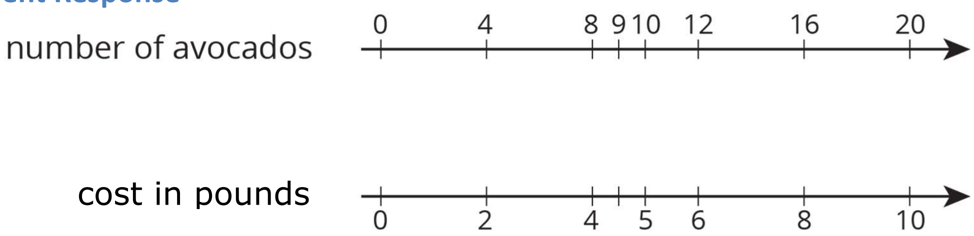
    c. How much would 7 bottles of water cost?

3. A 10-pound sack of flour costs £8.

   a. How much does 40 pounds of flour cost?

   b. What is the cost per pound of flour?

Student Response



1.

   a. 16 avocados cost £8, because $4 \times 2 = 8$ and $8 \times 2 = 16$.

   b. 20 avocados cost £10.

   c. 9 avocados cost £4.50.

2.

   a. You can buy 4 bottles for £3, because $9 \times \frac{1}{3} = 3$ and $12 \times \frac{1}{3} = 4$.

   b. The cost per bottle is £0.75, because $9 \div 12 = 0.75$.

   c. The cost for 7 bottles is £5.25, because $(0.75) \times 7 = 5.25$.

3.

   a. 40 pounds costs £32, because $10 \times 4 = 40$ and $8 \times 4 = 32$.

   b. The cost per pound is £0.80, because $8 \div 10 = 0.8$.

Are You Ready for More?

It is commonly thought that buying larger packages or containers, sometimes called *buying in bulk*, is a great way to save money. For example, a 6-pack of soda might cost £3 while a 12-pack of the same brand costs £5.

Find 3 different cases where it is not true that buying in bulk saves money. You may use the internet or go to a local supermarket and take photographs of the cases you find. Make sure the products are the same brand. For each example that you find, give the quantity or size of each, and describe how you know that the larger size is not a better deal.

Student Response

Answers vary.

Activity Synthesis

Select students who used unique methods to share their reasoning, as listed in the narrative. If no one used double number lines, represent one of the statements with a double number line diagram and display it for all to see. Although double number lines are not required in the task, their use in the context of problem situations helps students see their merits and illustrates how they might be used in other problems, especially as students transition from unit prices to constant speed and other contexts. Draw connections between the double number line strategy and the dividing by the numbers of items strategy.

Tell students that each "cost per one" unit being sold—avocado, pound, or bottle—is an example of a **unit price**. Ask them to name as many kinds of unit prices they can and to think of a situation where they might be used, starting with the list from the task:

- Cost per avocado
- Cost per pound
- Cost per bottle

Other possibilities include cost per litre, cost per ounce, cost per jelly bean, and so on.

8.3 More Shopping

15 minutes (there is a digital version of this activity)

In this task, students practice finding unit prices, using different reasoning strategies, and articulating their reasoning. They also learn about the term "at this rate."

As students work, observe their work and then assign one problem for each group to own and present to the class. (The problems can each be assigned to more than one group). Have them work together to create a visual display of their problem and its solution.

Instructional Routines

- Group Presentations

Launch

Arrange students in groups of 3–4. Provide tools for creating a visual display and access to rulers. Explain that they will work together to solve some shopping problems, run their work by you, and prepare to present an assigned problem to the class. Tell students that they can use double number lines if they wish.

Display the problem and read it aloud: Pizza costs £1.25 per slice. At this rate, how much will 6 slices cost?

Ask students what they think “at this rate” means in the question. Ensure they understand that “at this rate” means we know that equivalent ratios are involved:

- The ratio of cost to number of slices is £1.25 to 1. That is, pizza costs £1.25 per slice.
- The ratio of cost to number of slices is *something* to 6. That is, pizza costs *something* for 6 slices.

The *something* is the thing we are trying to figure out, and “at this rate” tells us that the two ratios in this situation are equivalent. Another way to understand “at this rate” in this context is “at this price per unit” and that the price per unit is the same no matter how many items or units are purchased.

Discuss any expectations for the group presentation. For example, each group member might be assigned a specific role for the presentation.

If students have digital access, they can use an applet to explore the problems and justify their reasoning before preparing their presentations.

Anticipated Misconceptions

The first and third questions involve using decimals to represent pence. If the decimal point is forgotten, remind students that the cost of the bracelet is less than one pound, and the cost of the crisps is in between one and two pounds.

Watch for students working in pence instead of pounds for the bracelets. They may come up with an answer of 275 pence. For these students, writing 25 pence as £0.25 should help, or consider reminding them of the avocados from a previous activity, which had a unit price of £0.50.

Student Task Statement

1. Four bags of crisps cost £6.
    a. What is the cost per bag?
    b. At this rate, how much will 7 bags of crisps cost?
2. At a used book sale, 5 books cost £15.
    a. What is the cost per book?
    b. At this rate, how many books can you buy for £21?
3. Neon bracelets cost £1 for 4.
    a. What is the cost per bracelet?
    b. At this rate, how much will 11 neon bracelets cost?

Pause here so your teacher can review your work.

4. Your teacher will assign you one of the problems. Create a visual display that shows your solution to the problem. Be prepared to share your solution with the class.

Student Response

a. The cost per bag is £1.50.

b. Seven bags cost £10.50.

a. The cost per book is £3.

b. You can buy 7 books for £21.

a. The cost per bracelet is 25 pence.

b. Eleven bracelets cost £2.75.

Activity Synthesis

Invite each group to present its assigned problem. After each group presents, highlight the group's strategy, accurate uses of the terms "at this rate" and "per," and the ways in which a double number line might have been used when working with unit price.

Lesson Synthesis

The main ideas to develop in this lesson are techniques for finding a **unit price**, and the things that can be done once a unit price is known.

Discuss with students the methods they use to find a unit price. The likely answers are:

- Division: if 2 bags of rice cost £3, then 1 bag costs $3 \div 2 = 1.50$.
- Double number line: adding tick marks to a double number line signifying 1 bag can determine the cost per bag. Briefly discuss with students the meaning of the word *per* (for each).

Discuss with students the things they can do once they know a unit price. Specifically, they can directly compute any cost when the number of items is known by multiplying the unit price by the number of items. You may want to point out to students that by multiplying, they are finding part of an equivalent ratio. For example, the ratio "£30 for 20 bags" is equivalent to the ratio "£3 for 2 bags."

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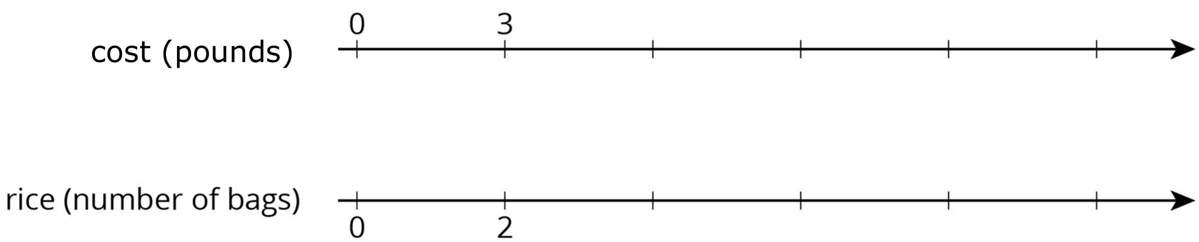
8.4 Unit Price of Rice

Cool Down: 5 minutes

Student Task Statement

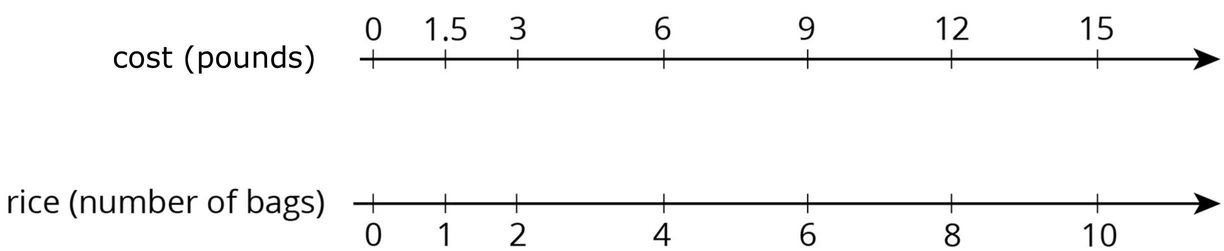
Here is a double number line showing that it costs £3 to buy 2 bags of rice:



1. At this rate, how many bags of rice can you buy with £12?
2. Find the cost per bag.
3. How much do 20 bags of rice cost?

Student Response

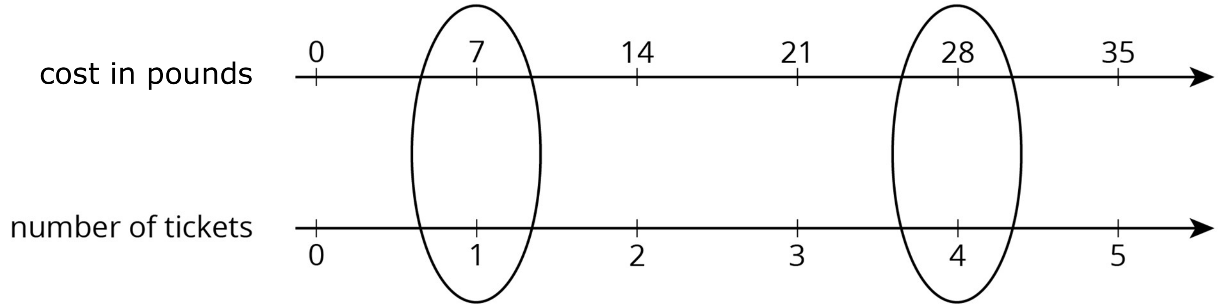
1. 8 bags cost £12.
2. The cost per bag is £1.50.
3. 20 bags cost £30. Multiply by the price for one bag or use an equivalent ratio.



Student Lesson Summary

The **unit price** is the price of 1 thing—for example, the price of 1 ticket, 1 slice of pizza, or 1 kilogram of peaches.

If 4 movie tickets cost £28, then the unit price would be the cost *per* ticket. We can create a double number line to find the unit price.



This double number line shows that the cost for 1 ticket is £7. We can also find the unit price by dividing, $28 \div 4 = 7$, or by multiplying, $28 \times \frac{1}{4} = 7$.

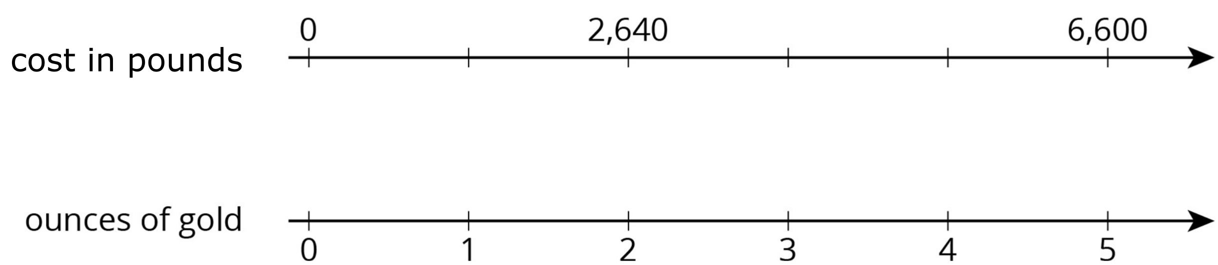
Glossary

- unit price

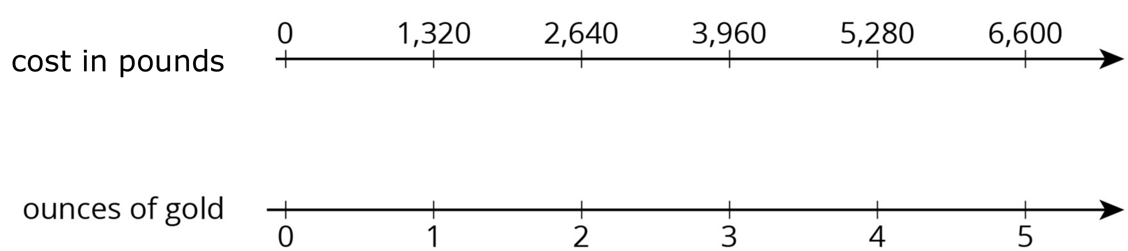
Lesson 8 Practice Problems

Problem 1 Statement

In 2016, the cost of 2 ounces of pure gold was £2 640. Complete the double number line to show the cost for 1, 3, and 4 ounces of gold.

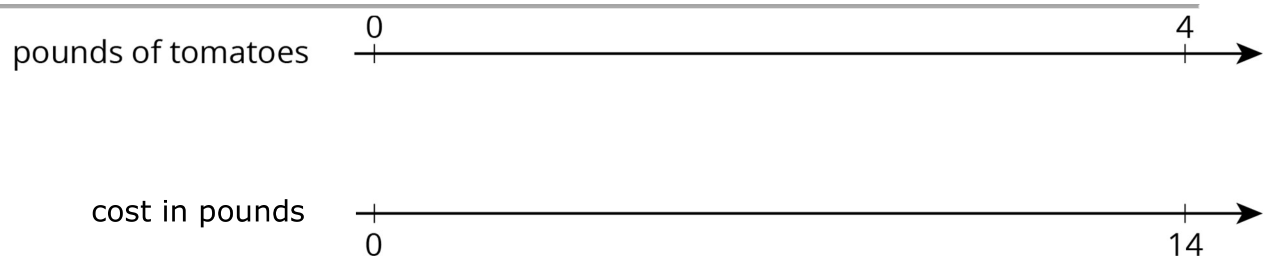


Solution

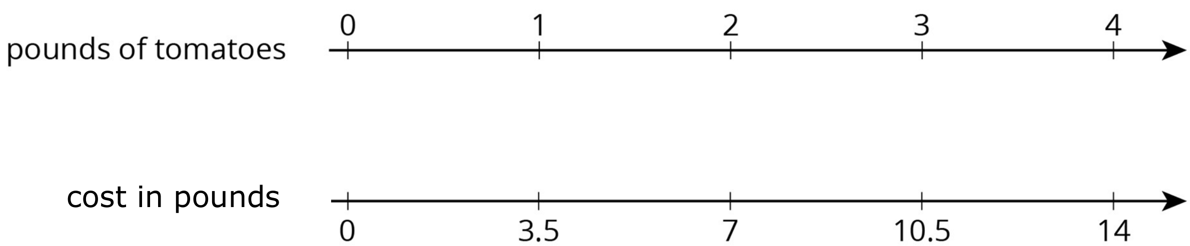


Problem 2 Statement

The double number line shows that 4 pounds of tomatoes cost £14. Draw tick marks and write labels to show the prices of 1, 2, and 3 pounds of tomatoes.



Solution



Problem 3 Statement

4 movie tickets cost £48. At this rate, what is the cost of:

a. 5 movie tickets?

b. 11 movie tickets?

Solution

a. £60 (1 ticket costs £12 because $48 \div 4 = 12$. 5 tickets cost £60 because $5 \times 12 = 60$.)

b. £132 (because $11 \times 12 = 132$)

Problem 4 Statement

Priya bought these items at the grocery store. Find each unit price.

a. 12 eggs for £3. How much is the cost per egg?

b. 3 pounds of peanuts for £7.50. How much is the cost per pound?

c. 4 rolls of toilet paper for £2. How much is the cost per roll?

d. 10 apples for £3.50. How much is the cost per apple?

Solution

a. 25 pence or £0.25

b. £2.50

c. 50 pence or £0.50

d. 35 pence or £0.35

Problem 5 Statement

Clare made a smoothie with 1 cup of yogurt, 3 tablespoons of peanut butter, 2 teaspoons of chocolate syrup, and 2 cups of crushed ice.

a. Kiran tried to double this recipe. He used 2 cups of yogurt, 6 tablespoons of peanut butter, 5 teaspoons of chocolate syrup, and 4 cups of crushed ice. He didn’t think it tasted right. Describe how the flavour of Kiran's recipe compares to Clare's recipe.

b. How should Kiran change the quantities that he used so that his smoothie tastes just like Clare's?

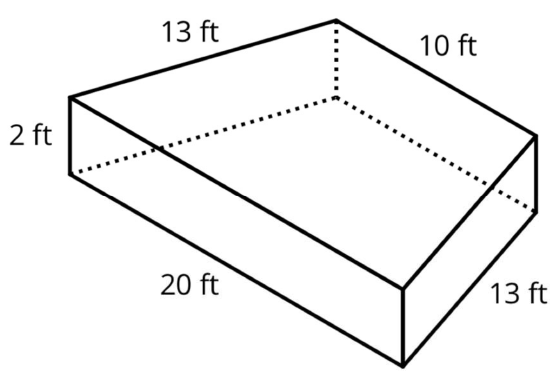
Solution

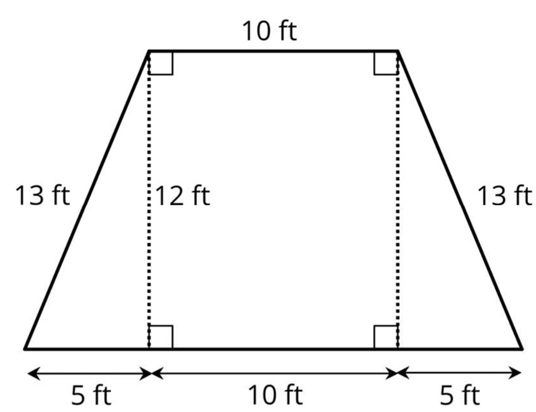
a. Kiran’s smoothie would be more chocolatey than Clare’s. All ingredients are doubled, but there is an extra teaspoon of chocolate syrup in his smoothie.

b. Answers vary. Sample response: he should use 4 teaspoons of chocolate syrup instead of 5.

Problem 6 Statement

A drama club is building a wooden stage in the shape of a trapezium-shaped prism. The height of the stage is 2 feet. Some measurements of the stage are shown here.





What is the area of all the faces of the stage, excluding the bottom? Show your reasoning. If you get stuck, consider drawing a net of the prism.

Solution

292 square feet. Sample reasoning: The trapezium-shaped face is 180 square feet since $(12 \times 10) + 2(\frac{1}{2} \times 12 \times 5) = 120 + 60 = 180$. The side faces are $2(13 \times 2) + (10 \times 2) + (20 \times 2)$ or 112 square feet.

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