

Grades 5-8 (AS), 9-12 (AS)

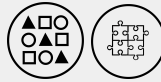
Duration: 40 min

Tools: one 9 pcs Set / pair or group

Pair / Group work

Keywords: Combinatorics, Proof, Impossible

609 - Hexagon Count



MATHS / COMBINATORICS



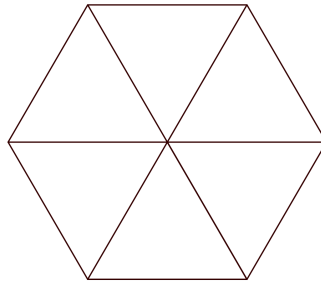
LOGIFACES
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DESCRIPTION

In Levels 1-3 students' task is to leave out the given 3 blocks and arrange the remaining blocks into a hexagon shape (as shown in the diagram) with a continuous surface.



The triples to leave out are:

LEVEL 1 blocks 222, 133, 123 or 222, 133, 132

LEVEL 2 blocks 222, 113, 112 or 222, 123, 132 or 222, 122, 123 or 222, 122, 132

LEVEL 3 blocks 222, 122, 112

An additional question for the Level 1-3 tasks is to find the number of different possibilities to arrange the remaining blocks into hexagons. (Two hexagons are different, if they cannot be moved into each other.)

LEVEL 4 Students' task is to find 6 blocks that form a hexagon shape. How many combinations are there? (Two hexagons are different, if they cannot be moved into each other.)

Which blocks can be left out?

SOLUTIONS / EXAMPLES

LEVEL 1 There is only one way to arrange the blocks into a hexagon.

LEVEL 2 There are two ways to arrange the blocks into a hexagon.

LEVEL 3 The remaining blocks cannot form a hexagon. After a number of failed attempts the students can be asked to prove the impossibility of forming a hexagon from those blocks. See Case 3 below for a possible proof.

LEVEL 4 The hexagon cannot contain the block 222, because there is only one other block with a vertical face 22.

CASE 1 The hexagon contains the block 122. In this case, the vertices in the middle must have length 1, because there is no other block with a vertical face 22. There are 6 hexagons that contain the block 122, the lengths of the outer vertical edges are listed in counterclockwise order:

223213 (blocks 222, 133, 123 are left out),

223123 (blocks 222, 133, 132 are left out),

223323 (blocks 222, 113, 112 are left out),

223233 (blocks 222, 113, 112 are left out),

223231 (blocks 222, 123, 132 are left out),

221323 (blocks 222, 123, 132 are left out).

CASE 2 The hexagon does not contain the block 122 but contains the block 112. If the vertical edges in the middle have length 1, then there are 4 hexagons:

123233 (blocks 222, 122, 132 are left out),

332321 (blocks 222, 122, 123 are left out),

123323 (blocks 222, 122, 132 are left out),

323321 (blocks 222, 122, 123 are left out).

The vertical edge in the middle cannot have length 2, because there are only 5 blocks that have a vertical edge of length 2 in this case.

CASE 3 The hexagon does not contain the blocks 122 and 112. The vertical edge in the middle can not have length 2, because only 4 blocks have a vertical edge of length 2. The vertical edge in the middle cannot have length 1 or 3, because only one block has a vertical face 11 or 33, respectively.

In summary, there are 10 different hexagons.

PRIOR KNOWLEDGE

None

RECOMMENDATIONS / COMMENTS

Finding all the cases given 6 blocks is an easier problem, and can be used to differentiate.

Finding a proof of the impossibility of arranging 6 given blocks into a hexagon is an advanced problem.

Finding all the cases is a difficult Combinatorics problem.