Lesson 5: Decimal points in products

Goals

- Generalise (orally and in writing) that the number of decimal places in a product is related to the number of decimal places in the factors.
- Justify (orally) the product of two decimals, which each have only one non-zero digit, by multiplying equivalent fractions that have a power of ten in the denominator.

Learning Targets

- I can use place value and fractions to reason about multiplication of decimals.

Lesson Narrative

In earlier years, students have multiplied base-ten numbers up to hundredths (either by multiplying two decimals to tenths or by multiplying a whole number and a decimal to hundredths). Here, students use what they know about fractions and place value to calculate products of decimals beyond the hundredths. They express each decimal as a product of a whole number and a fraction, and then they use the commutative and associative properties to compute the product. For example, they see that $(0.6) \times (0.5)$ can be viewed as $6 \times (0.1) \times 5 \times (0.1)$ and thus as $\left(6 \times \frac{1}{10}\right) \times \left(5 \times \frac{1}{10}\right)$. Multiplying the whole numbers and the fractions gives them $30 \times \frac{1}{100}$ and then 0.3.

Through repeated reasoning, students see how the number of decimal places in the factors can help them understand the value of the digits in the product.

Building On

- Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the digits when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
- Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Compute fluently with multi-digit numbers and find common factors and multiples.

Instructional Routines

- Collect and Display
- Clarify, Critique, Correct

Student Learning Goals

Let's look at products that are decimals.

5.1 Multiplying by 10

Warm Up: 5 minutes

In this warm-up, students use the structure of a set of multiplication equations to see the relationship between two numbers that differ by a factor of a power of 10. Students evaluate the expression $x \times 10$ by considering the effect of multiplication by 10.

Launch

Ask students to answer the following questions without writing anything and to be prepared to explain their reasoning. Follow with whole-class discussion.

Student Task Statement

1. In which equation is the value of x the largest?

   $x \times 10 = 810$

   $x \times 10 = 81$

   $x \times 10 = 8.1$

   $x \times 10 = 0.81$

2. How many times the size of 0.81 is 810?

Student Response

1. The x has the largest value in the first equation. Sample explanations:
   - When multiplied by 10, the x in the first equation has the largest product.
   - Each x is one tenth of the product, and the largest product is 810.
2. 810 is 1 000 times the size of 0.81. Sample explanations:
   - Multiplying 0.81 by 10 moves each digit one space to the left (increases the place value of each digit).
   - 810 is 10 times 81, and 81 is 100 times 0.81, so 810 must be 1 000 times 0.81.

Activity Synthesis

Ask students to share what they noticed about the first four equations. Record student explanations that connect multiplying a number by 10 with increasing the place value of each digit.

Discuss how students could use their observations from the first question to multiply 0.81 by a number to get 810.

5.2 Fractionally Speaking: Powers of Ten

15 minutes

In KS2, students recognise that multiplying a number by $\frac{1}{10}$ is the same as dividing the number by 10, and multiplying by $\frac{1}{100}$ is the same as dividing by 100. In this lesson, students will recognise and use the fact that multiplying by 0.1, 0.01, and 0.001 is equivalent to multiplying by $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$, respectively. In all cases, the essential point to understand is that in the base-ten system, the value of each place is $\frac{1}{10}$ the value of the place immediately to its left. Writing the decimals 0.1, 0.01, and 0.001 in fraction form will help students recognise how the position of the decimal point in the product is affected by the positions of the decimal points in the factors.

The structure of the base-ten system can serve as a guide for calculating products of decimals, and the goal of this lesson is to begin to uncover that structure.

Students might need to see decimal and fraction names. If so, display a place-value chart for reference.

Instructional Routines

- Collect and Display

Launch

Arrange students in groups of 2. Ask one student in each group to complete the questions for Partner A, and have the other take the questions for Partner B. Then ask them to discuss their responses, answer the second question together, and pause for a brief class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge about multiplying a number by $\frac{1}{10}$ or $\frac{1}{100}$. Remind students that multiplying a number by $\frac{1}{10}$ is the same as dividing by 10 and multiplying a number by $\frac{1}{100}$ is the same as dividing by 100.
Supports accessibility for: Memory; Conceptual processing Representing: Collect and Display. Use this routine while students are working through the first two questions. As students work, circulate and listen for the connections students make between the problems. Write the students' words and phrases on a visual display and update it throughout the remainder of the lesson. Listen for language like "the same," reciprocal, and inverse operation. Remind students to borrow language from the display as needed.
Design Principle(s): Optimise output (for explanation); Maximise meta-awareness

Anticipated Misconceptions

Students may readily see that $36 \times (0.1)$ but be unsure about what to do when the factor being multiplied by the decimal 0.1 and 0.01 is also a decimal (e.g., $(24.5) \times (0.1)$). Encourage them to try writing both decimals (e.g., 24.5 and 0.1) as fractions, multiply, and convert the resulting fraction back to a decimal.

Student Task Statement

Work with a partner. One person solves the problems labelled "Partner A" and the other person solves those labelled "Partner B." Then compare your results.

1. Find each product or quotient. Be prepared to explain your reasoning.

   Partner A

   a. $250 \times \frac{1}{10}$

   b. $250 \times \frac{1}{100}$

   c. $48 \div 10$

   d. $48 \div 100$

   Partner B

   a. $250 \div 10$

   b. $250 \div 100$

   c. $48 \times \frac{1}{10}$

   d. $48 \times \frac{1}{100}$

2. Use your work in the previous problems to find $720 \times (0.1)$ and $720 \times (0.01)$. Explain your reasoning.

   Pause here for a class discussion.

3. Find each product. Show your reasoning.

   a. $36 \times (0.1)$

   b. $(24.5) \times (0.1)$

   c. $(1.8) \times (0.1)$

   d. $54 \times (0.01)$

   e. $(9.2) \times (0.01)$

4. Jada says: "If you multiply a number by 0.001, the decimal point of the number moves three places to the left." Do you agree with her? Explain your reasoning.

Student Response

1. For both Partner A and Partner B:

   a. 25

   b. 2.5

   c. 4.8

   d. 0.48

2. $720 \times (0.1) = 72$ and $720 \times (0.01) = 7.2$. Sample reasoning:

   – 0.1 is equal to $\frac{1}{10}$, so $720 \times (0.1) = 720 \times \frac{1}{10}$, which is equal to $720 \div 10$ or 72.

   – 0.01 is equal to $\frac{1}{100}$, so $720 \times (0.01) = 720 \times \frac{1}{100}$, which is equal to $720 \div 100$ or 7.2.

3.

   a. 3.6. Sample reasoning: $36 \times (0.1)$ means 36 groups of 1 tenth or $36 \times \frac{1}{10}$, which equals 3.6.

   b. 2.45. Sample reasoning: $(24.5) \times \frac{1}{10} = 24.5 \div 10$, which is 2.45.

   c. 0.18. Sample reasoning: $(1.8) \times \frac{1}{10} = 1.8 \div 10$, which is 0.18.

   d. 0.54. Sample reasoning: $54 \times (0.01)$ means 54 groups of 1 hundredth or 54 hundredths.

   e. 0.092. Sample reasoning: $9.2 \div 100 = 0.092$.

4. I disagree. Sample reasoning: The decimal point does not move. Multiplying a number by a thousandth is the same as multiplying it by $\frac{1}{1000}$, which is the same as dividing it by 1 000. Dividing by 1 000 decreases the value of each digit causing each digit to move three spaces to the right.

Activity Synthesis

The purpose of this discussion is to highlight the placement of the decimal point in a product. Consider asking some of the following questions:

- "How does the size of a product compare to the size of the factor when the factor is multiplied by 0.1? How does the placement of each digit change?" (Multiplying by

0.1 makes the product ten times smaller than the factor. Each digit moves to the left one place.)

- "How does the size of a product compare to the size of the factor when the factor is multiplied by 0.01?" (Multiplying by 0.01 makes the factor one hundred times smaller. Each digit moves to the left two places.)
- "Can you predict the outcome of multiplying 750 by 0.1 or 0.01 without calculating? If so, how?" (Multiplying 750 by 0.1 would produce 75. Multiplying 750 by 0.01 would produce 7.5.)

5.3 Fractionally Speaking: Multiples of Powers of Ten

15 minutes

In this activity, students continue to think about products of decimals using fractions. They use what they know about $\frac{1}{10}$ and $\frac{1}{100}$, as well as the commutative and associative properties, to identify and write multiplication expressions that could help them find the product of two decimals.

While students may be able to start by calculating the value of each decimal product, the goal is for them to look for and use the structure of equivalent expressions, and later generalise the process to multiply any two decimals.

As students work, listen for the different ways students decide on which expressions are equivalent to $(0.6) \times (0.5)$. Identify a few students or groups with differing approaches so they can share later.

Instructional Routines

- Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Give groups 3–4 minutes to work on the first two questions, and then pause for a whole-class discussion. Discuss:

- Why is $(0.6) \times (0.5)$ equivalent to $6 \times \frac{1}{10} \times 5 \times \frac{1}{10}$? (0.6 is 6 tenths, which is the same as $6 \times \frac{1}{10}$, and 0.5 is 5 tenths, or $5 \times \frac{1}{10}$)
- Why is the expression $6 \times \frac{1}{10} \times 5 \times \frac{1}{10}$ equivalent to $6 \times 5 \times \frac{1}{10} \times \frac{1}{10}$? (We can 'switch' the places of 5 and $\frac{1}{10}$ in the multiplication and not change the product. This follows the commutative property of operations.)
- How did you find the value of $30 \times \frac{1}{100}$? (Multiplying by $\frac{1}{100}$ means dividing by 100, which means moving each digit 2 places to the left, so the result is 0.30 or 0.3.

Action and Expression: Develop Expression and Communication. Maintain a display of important terms and vocabulary. During the launch take time to review terms that students will need to access for this activity. Invite students to suggest language or diagrams to include that will support their understanding of commutative and associative properties of operations.
Supports accessibility for: Memory; Language

Anticipated Misconceptions

If students try to use a vertical calculation to find the products, ask them to instead do so by thinking of the decimals as fractions and about any patterns they observed.

Student Task Statement

1. Select **all** expressions that are equivalent to $(0.6) \times (0.5)$. Be prepared to explain your reasoning.

   a. $6 \times (0.1) \times 5 \times (0.1)$

   b. $6 \times (0.01) \times 5 \times (0.1)$

   c. $6 \times \frac{1}{10} \times 5 \times \frac{1}{10}$

   d. $6 \times \frac{1}{1\,000} \times 5 \times \frac{1}{100}$

   e. $6 \times (0.001) \times 5 \times (0.01)$

   f. $6 \times 5 \times \frac{1}{10} \times \frac{1}{10}$

   g. $\frac{6}{10} \times \frac{5}{10}$

2. Find the value of $(0.6) \times (0.5)$. Show your reasoning.

3. Find the value of each product by writing and reasoning with an equivalent expression with fractions.

   a. $(0.3) \times (0.02)$

   b. $(0.7) \times (0.05)$

Student Response

1. A, because $6 \times (0.1) \times 5 \times (0.1) = (0.6) \times (0.5)$
   C, because $6 \times \frac{1}{10} \times 5 \times \frac{1}{10} = \frac{6}{10} \times \frac{5}{10} = 0.6 \times 0.5$
   F, because $6 \times 5 \times \frac{1}{10} \times \frac{1}{10} = 6 \times \frac{1}{10} \times 5 \times \frac{1}{10}$
   G, because $\frac{6}{10} \times \frac{5}{10} = (0.6) \times (0.5)$

2. 0.3. Sample reasoning: $(0.6) \times (0.5) = \frac{6}{10} \times \frac{5}{10}$, which is $\frac{30}{100}$ or 0.3.

3. Expressions vary.

   a. 0.006. Sample reasoning: $\frac{3}{10} \times \frac{2}{100} = \frac{6}{1000}$, which is 0.006.

   b. 0.035. Sample reasoning: $(0.7) \times (0.05) = \frac{7}{10} \times \frac{5}{100}$, which is $\frac{35}{1000}$ or 0.035.

Are You Ready for More?

Ancient Romans used the letter I for 1, V for 5, X for 10, L for 50, C for 100, D for 500, and M for 1 000. Write a problem involving merchants at an agora, an open-air market, that uses multiplication of numbers written with Roman numerals.

Student Response

Answers vary.

Activity Synthesis

Select several students to share their responses and reasonings for the last question.

To conclude, ask students to consider how writing $6 \times 5 \times \frac{1}{100}$ might be a favorable way to find 0.6×0.5. Students may respond that using whole numbers and fractions makes multiplication simpler; even if there is division, it is division by a power of 10. In future lessons, students will apply this reasoning to find products of more elaborate decimals, such as $(0.24) \times (0.011)$.

Writing, Speaking: Clarify, Critique, Correct. Present an incorrect justification for one of the expressions from the first problem that was not equivalent (such as B or E). For example, "The expression $6 \times (0.01) \times 5 \times (0.1)$ is equivalent to 0.6×0.5 because the same whole numbers are used and where you put the zeros in a decimal doesn't matter." Ask students, "Is this reasoning correct? Why or why not?" Give students 1–2 minutes to write a brief explanation about why the expressions are not equivalent. Student responses should show attention to the place values of the digits. For students who need additional support, provide a sentence frame, such as "_____ is (equivalent/not equivalent) because _____." This will help students produce clearer justifications that demonstrate their reasoning about what equivalence means.
Design Principle(s): Optimise output (for justification); Cultivate conversation

Lesson Synthesis

We can use our understanding of fractions and place value in calculating the product of two decimals. Writing decimals in fraction form can help us determine the number of decimal places the product will have and place each digit in the product so that it has the correct value.

- What are some ways to find $(0.4) \times (0.0007)$? (We can think of 0.4 as $\frac{4}{10}$ and 0.0007 as $\frac{7}{10\,000}$, multiply the fractions to get $\frac{28}{100\,000}$, and write the product as the decimal 0.00028. Or we can write 0.4 as $4 \times \frac{1}{10}$ and 0.0007 as $7 \times \frac{1}{10\,000}$, multiply the whole numbers and the fractions, and again convert the fractional product into a decimal.)

- How might we tell which product will have a greater number of places to the right of the decimal point: $(0.03) \times (0.001)$ or $(0.3) \times (0.0001)$? (If we write the decimals as fractions and multiply them, we can see that both products equal $\frac{3}{100\,000}$ or 0.00003, so they would have the same number of places to the right of the decimal point.)

5.4 Placing Decimal Points in Products

Cool Down: 5 minutes

Student Task Statement

1. Use what you know about decimals or fractions to explain why $(0.2) \times (0.002) = 0.0004$.

2. A rectangular plot of land is 0.4 kilometre long and 0.07 kilometre wide. What is its area in square kilometres? Show your reasoning.

Student Response

1. Answers vary. Sample response: 0.2 is $\frac{2}{10}$, and 0.002 is $\frac{2}{1000}$. Multiplying the two we have: $\frac{2}{10} \times \frac{2}{1000} = \frac{4}{10\,000}$, which is 0.0004.

2. 0.028 square kilometres, because $(0.4) \times (0.07) = 0.028$

Student Lesson Summary

We can use fractions like $\frac{1}{10}$ and $\frac{1}{100}$ to reason about the location of the digits in a product of two decimals.

Let's take $24 \times (0.1)$ as an example. There are several ways to find the product:

- We can interpret it as 24 groups of 1 tenth (or 24 tenths), which is 2.4.

- We can think of it as $24 \times \frac{1}{10}$, which is equal to $\frac{24}{10}$ (and also equal to 2.4).

- Multiplying by $\frac{1}{10}$ has the same result as dividing by 10, so we can also think of the product as $24 \div 10$, which is equal to 2.4.

Similarly, we can think of $(0.7) \times (0.09)$ as 7 tenths times 9 hundredths, and write:

$\left(7 \times \frac{1}{10}\right) \times \left(9 \times \frac{1}{100}\right)$

We can rearrange whole numbers and fractions using the commutative property of multiplication:

$(7 \times 9) \times \left(\frac{1}{10} \times \frac{1}{100}\right)$

This tells us that $(0.7) \times (0.09) = 0.063$.

$63 \times \frac{1}{1\,000} = \frac{63}{1\,000}$

Here is another example: To find $(1.5) \times (0.43)$, we can think of 1.5 as 15 tenths and 0.43 as 43 hundredths. We can write the tenths and hundredths as fractions and rearrange the factors. $\left(15 \times \frac{1}{10}\right) \times \left(43 \times \frac{1}{100}\right) = 15 \times 43 \times \frac{1}{1000}$

Multiplying 15 and 43 gives us 645, and multiplying $\frac{1}{10}$ and $\frac{1}{100}$ gives us $\frac{1}{1000}$. So $(1.5) \times (0.43)$ is $645 \times \frac{1}{1000}$, which is 0.645.

Lesson 5 Practice Problems

1. **Problem 1 Statement**

   a. Find the product of each number and $\frac{1}{100}$.

      122.1

      11.8

      1 350.1

      1.704

   b. What happens to each digit of the original number when you multiply the original number by $\frac{1}{100}$? Why do you think that is? Explain your reasoning.

   Solution

   a. 1.221

   b. 0.118

   c. 13.501

   d. 0.01704

e. Answers vary. Sample response: Each digit moves 2 places to the right. Multiplying a decimal number by $\frac{1}{100}$ means dividing by 100, which moves each digit 2 places to the right.

2. **Problem 2 Statement**

Which expression has the same value as $(0.06) \times (0.154)$? Select **all** that apply.

a. $6 \times \frac{1}{100} \times 154 \times \frac{1}{1000}$

b. $6 \times 154 \times \frac{1}{100\,000}$

c. $6 \times (0.1) \times 154 \times (0.01)$

d. $6 \times 154 \times (0.00001)$

e. 0.00924

Solution ["A", "B", "D", "E"]

3. **Problem 3 Statement**

Calculate the value of each expression by writing the decimal factors as fractions, then writing their product as a decimal. Show your reasoning.

a. $(0.01) \times (0.02)$

b. $(0.3) \times (0.2)$

c. $(1.2) \times 5$

d. $(0.9) \times (1.1)$

e. $(1.5) \times 2$

Solution

a. 0.0002 because $0.01 = \frac{1}{100}$ and $0.02 = \frac{2}{100}$, so the product is $\frac{2}{10\,000}$

b. 0.06 because $0.3 = \frac{3}{10}$ and $0.2 = \frac{2}{10}$, so the product is $\frac{6}{100}$

c. 6 because $12 \times 5 = 60$ and 1.2 is one tenth of 12

d. 0.99 because $0.9 = \frac{9}{10}$ and $1.1 = \frac{11}{10}$, so the product is $\frac{99}{100}$

e. 3 because $1.5 = \frac{3}{2}$ and twice this is 3

4. **Problem 4 Statement**

Write three numerical expressions that are equivalent to $(0.0004) \times (0.005)$.

Solution

Answers vary. Possible responses:

- $4 \times (0.0001) \times 5 \times (0.001)$
- $4 \times 5 \times (0.0001) \times (0.001)$
- $\frac{4}{10\,000} \times \frac{5}{1\,000}$
- $\frac{1}{10\,000} \times 4 \times \frac{1}{1\,000} \times 5$

5. **Problem 5 Statement**

Calculate each sum.

a. $33.1 + 1.95$

a. $1.075 + 27.105$

a. $0.401 + 9.28$

Solution

a. 35.05

b. 28.18

c. 9.681

6. **Problem 6 Statement**

Calculate each difference. Show your reasoning.

a. $13.2 - 1.78$

a. $23.11 - 0.376$

a. $0.9 - 0.245$

Solution

a. 11.42

b. 22.734

c. 0.655 Sample reasoning:

a.

$$\begin{array}{r} \scriptstyle 9 \\ \scriptstyle 8\;\cancel{10}\;10 \\ 0.\cancel{9}\,\cancel{0}\,\cancel{0} \\ -\;0.2\;4\;5 \\ \hline 0.6\;5\;5 \end{array}$$

b.

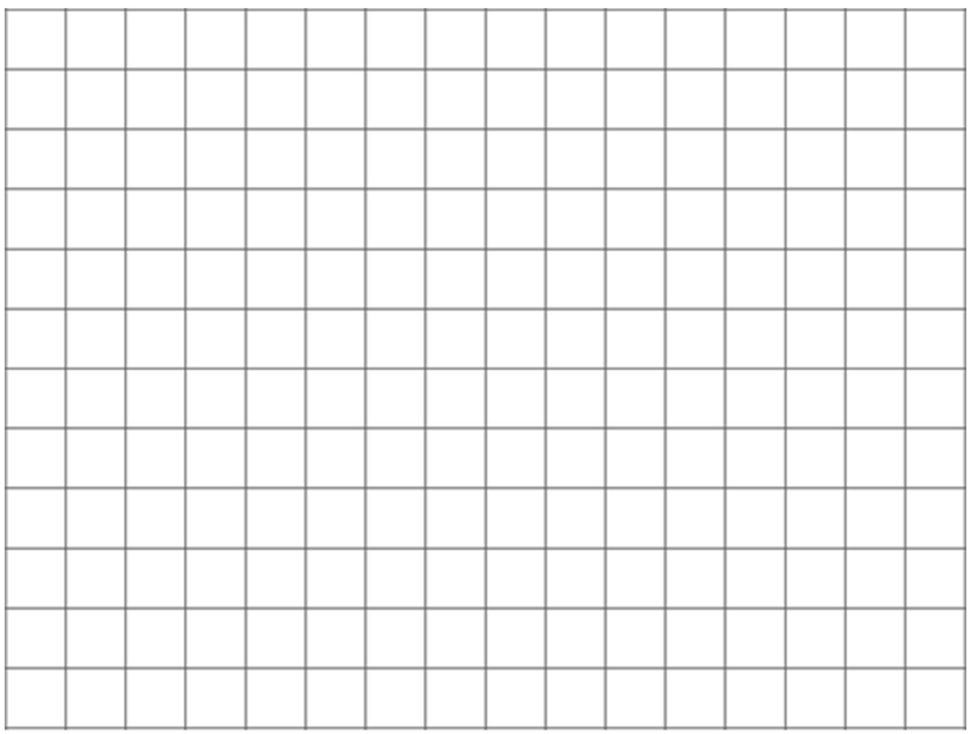
$$\begin{array}{r} \scriptstyle 11 \\ \scriptstyle 2\;\cancel{1} \\ 1\;\cancel{3}.\cancel{2}\;0 \\ -\;1.7\;8 \\ \hline 1\;1.4\;2 \end{array}$$

c.

$$\begin{array}{r} \scriptstyle 2\;10\;10\;10 \\ 2\;\cancel{3}.\cancel{1}\;\cancel{1}\;0 \\ -\;0.3\;7\;6 \\ \hline 2\;2.7\;3\;4 \end{array}$$

7. **Problem 7 Statement**

On the grid, draw a quadrilateral *that is not a rectangle* that has an area of 18 square units. Show how you know the area is 18 square units.



Solution

Answers vary. Sample responses: