Mathematics Education Innovation

Lesson 14: Surface area of prisms

Goals

- Calculate the surface area of a prism, and explain (in writing) the solution method.
- Comprehend that surface area and volume are two different attributes of three-dimensional objects and are measured in different units.
- Interpret different methods for calculating the surface area of a prism, and evaluate (orally and in writing) their usefulness.

Learning Targets

- I can find and use shortcuts when calculating the surface area of a prism.
- I can picture the net of a prism to help me calculate its surface area.

Lesson Narrative

In Year 7, students used nets made up of rectangles and triangles to find the surface area of three-dimensional shapes. In this lesson they find surface areas of prisms, and see that structure of a prism allows for shortcuts in adding up the areas of the faces. They see that if the prism is sitting on its base, then the vertical sides can be unfolded into a single rectangle whose height is the height of the prism and whose length is the perimeter of the base. The purpose of the lesson is not to come up with a formula for the surface area of a prism, but to help students see and make use of the structure of the prism to find surface area efficiently.

Building On

- Recognise area as an attribute of plane shapes and understand concepts of area measurement.
- Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
- Represent three-dimensional shapes using nets made up of rectangles and triangles, and use the nets to find the surface area of these shapes. Apply these techniques in the context of solving real-world and mathematical problems.

Addressing

- Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and prisms.

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Building Towards

- Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and prisms.

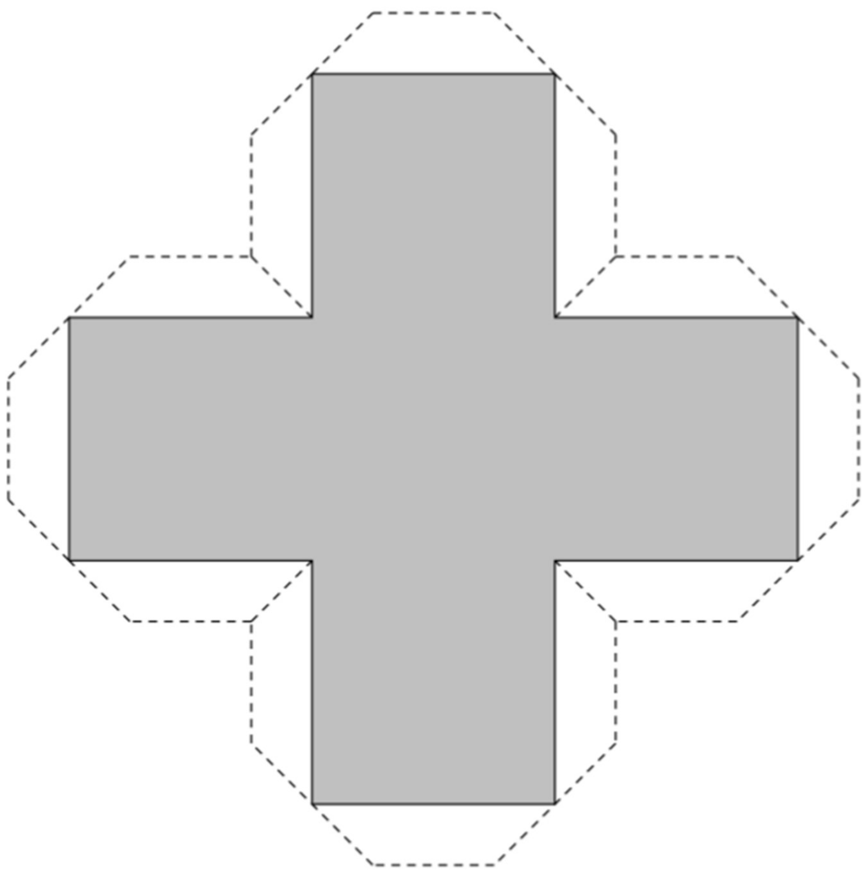
Instructional Routines

- Stronger and Clearer Each Time
- Discussion Supports

Required Materials

Materials assembled from the blackline master
Net (will need enlarging to the correct size)

2 of:



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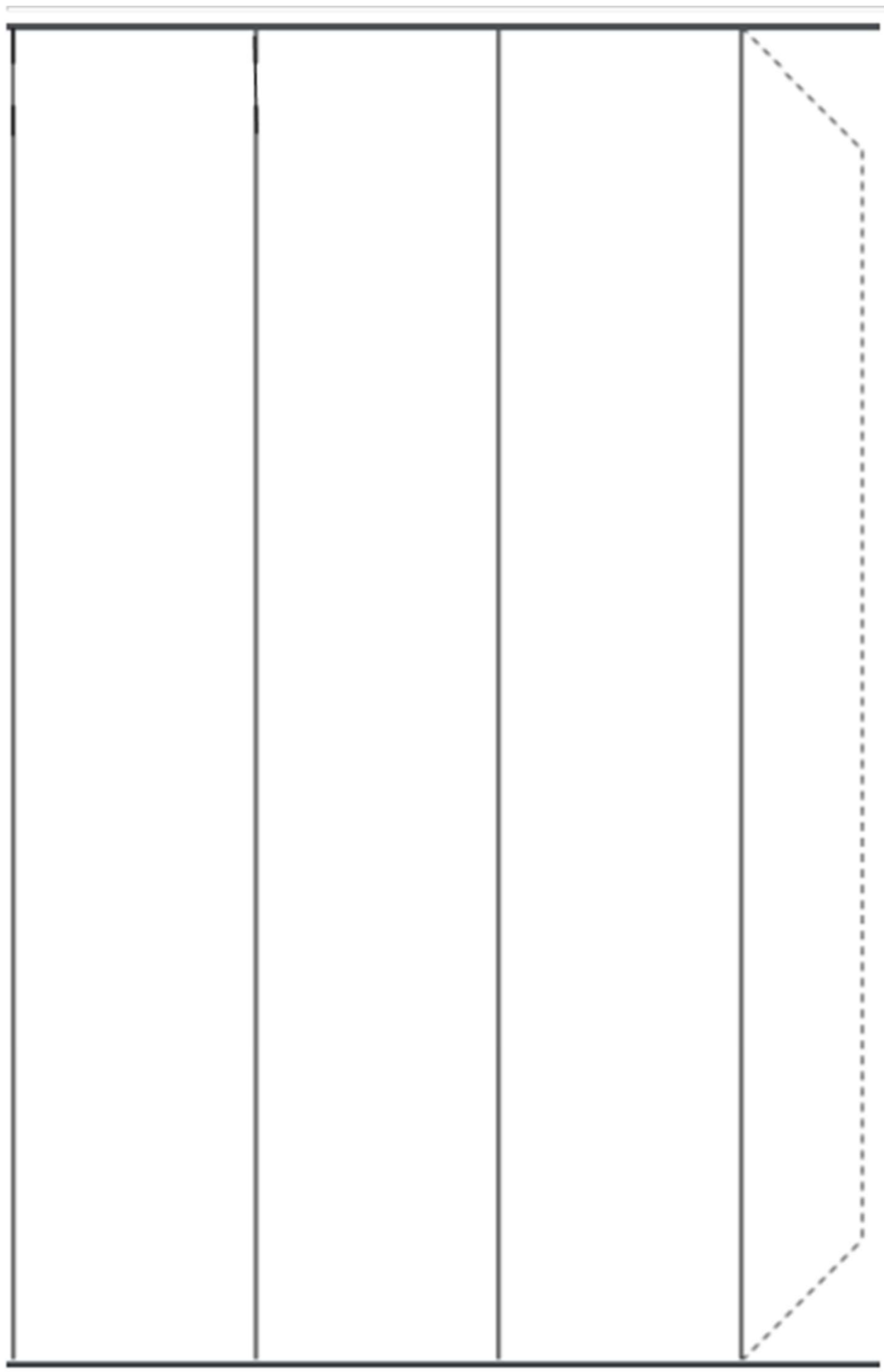
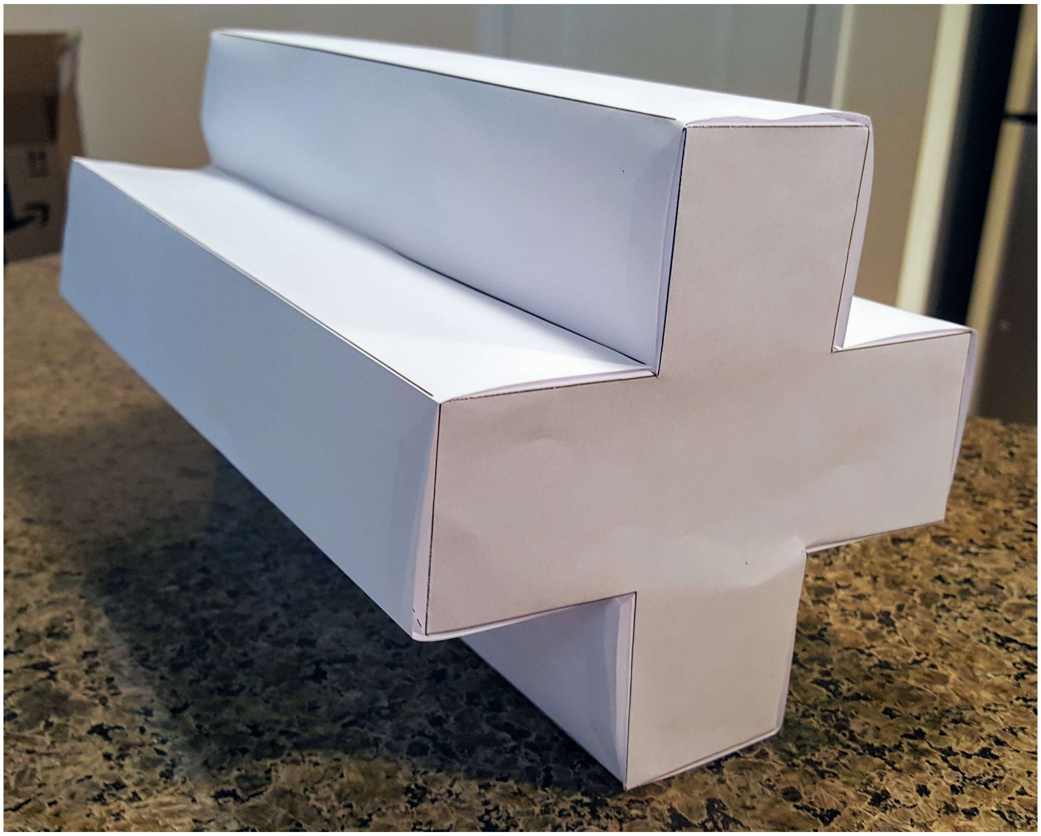


Required Preparation

Assemble the net from the blackline master to make a prism with a base in the shape of a plus sign. Make sure to print the blackline master at 100% scale so the dimensions are accurate. This prism will be used for both the warm-up and the following activity.



Student Learning Goals

Let's look at the surface area of prisms.

14.1 Multifaceted

Warm Up: 5 minutes

The purpose of this warm-up is for students to recognise important parts of solids in anticipation of computing volume and surface area. The shape used in the next activity is introduced in this warm-up as a way for students to start thinking about parts of solids and how we use them to compute surface area or volume.

Launch

Arrange students in groups of 2. Display the prism assembled from the blackline master for all to see. Give students 1 minute of quiet think time followed by time to discuss their ideas with a partner. Follow with a whole-class discussion.

Student Task Statement

Your teacher will show you a prism.

1. What are some things you could measure about the object?

2. What units would you use for these measurements?

Student Response

1. Answers vary. Sample responses: You could measure the length of each of the edges of the object. You could measure the volume of the object. You could find the area of the faces.

2. Answers vary. Sample responses: Lengths could be measured in inches or centimetres. Volume could be measured in cubic inches, cubic centimetres, or millilitres. Area could be measured in square inches or square centimetres.

Activity Synthesis

Select students to share their responses. Ask students to think about units that do not make sense to use for measurements (feet, miles, yards, etc). Invite students to share their explanations of why these units do not make sense to use.

14.2 So Many Faces

15 minutes

In this activity, students make sense of three different methods for calculating the surface area of a shape. Three different methods are described to students, and they are asked to determine which one they agree with (if any). They then think about generalising the methods to figure out if they would work for any prism. This activity connects to work they did with nets in a previous year and builds upon strategies students might have to calculate surface area.

As students work on the task, monitor for students who understand the different methods and can explain if any of them will work for any other prisms.

Note: It is not important for students to learn the term “lateral area.”

Instructional Routines

- Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Display the prism assembled previously in the warm-up for all to see. Ask students: “how might we find surface area of this prism?” Invite students to share their ideas. Give students 1 minute of quiet think to read Noah’s method for calculating surface area followed by time discuss whether they agree with Noah or not. Repeat this process for the remaining two methods. Once all three methods have been discussed give students 1–2 minutes of quiet work time to answer the rest of the questions in the task statement.

Representation: Internalise Comprehension. Demonstrate and encourage students to use colour coding and annotations to highlight connections between representations in a problem. For example, use different colours to represent each of Noah, Elena and Andre’s

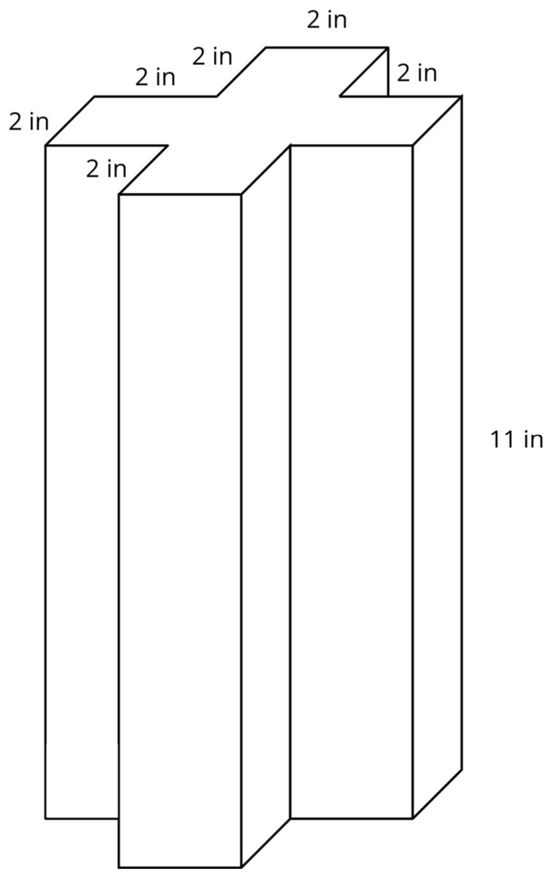
methods.
Supports accessibility for: Visual-spatial processing

Anticipated Misconceptions

Students may think that Andre’s method will not work for all prisms, because it will not work for solids that have a hole in their base and therefore more lateral area on the inside. Technically, these solids are not prisms, because their base is not a polygon. However, students could adapt Andre’s method to find the surface area of a solid composed of a prism and a hole.

Student Task Statement

Here is a picture of your teacher's prism:



Three students are trying to calculate the **surface area** of this prism.

- Noah says, “This is going to be a lot of work. We have to find the areas of 14 different faces and add them up.”
- Elena says, “It’s not so bad. All 12 rectangles are identical copies, so we can find the area for one of them, multiply that by 12 and then add on the areas of the 2 bases.”

- Andre says, "Wait, I see another way! Imagine unfolding the prism into a net. We can use 1 large rectangle instead of 12 smaller ones."

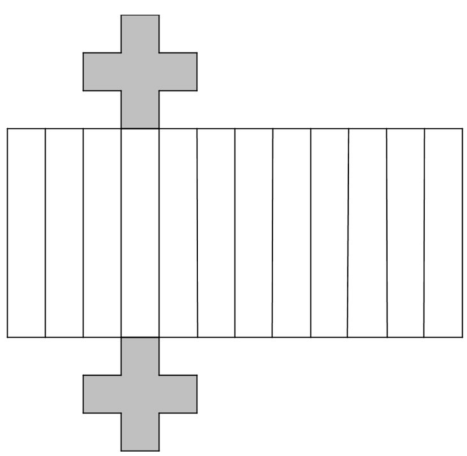
1. Do you agree with any of them? Explain your reasoning.
2. How big is the "1 large rectangle" Andre is talking about? Explain or show your reasoning. If you get stuck, consider drawing a net for the prism.
3. Will Noah's method always work for finding the surface area of any prism? Elena's method? Andre's method? Be prepared to explain your reasoning.
4. Which method do you prefer? Why?

Student Response

1. I agree with all three of them. Noah's method will work, but is not the most efficient. Elena's method is an improvement because we don't have to do the same calculation multiple times. Andre's method is more complicated to think about, but it should also work.
2. The height of the rectangle will be the same as the height of the prism, 11 inches. The length of the rectangle must wrap around the entire base, so it will be the same as the perimeter of the base, 24 inches.
3. Noah's method will always work for any prism. Elena's method only works when each line segment in the base is the same length, so it will not work for all shapes. Andre's method will work for all prisms because the long rectangle can fold around any base.
4. Answers vary. Sample response: I prefer Andre's method because it is not too difficult once you understand it and only needs two areas (the base and the long rectangle).

Activity Synthesis

Select previously identified students to share their reasoning. If not brought up in students' explanations, display the image for all to see and point out to students that the length of the "1 big rectangle" is equal to the perimeter of the base.



Students may have trouble generalising which method would work for any prism. Here are some guiding questions:

- "Which of the students' methods will work for finding the surface area of this particular prism?" (all 3)
- "Which of the students' methods will work for finding the surface area of any prism?" (Noah's and Andre's)
- "Which of the students' methods will work for finding the surface area of other three-dimensional shapes that are not prisms?" (only Noah's)

If not mentioned by students, be sure students understand:

- Noah's method will always work, but it can be inefficient if there are a lot of faces.
- Elena's method will not always work because the rectangles will not always be the same size, but we can notice that some shapes are the same and not have to work them all out individually.
- Andre's method does always work even if the rectangles have different widths. The length of the rectangle will be the same as the perimeter of the base and the width of the rectangle will be the height of the prism.
- Prisms can always be cut into three pieces: two bases and one rectangle whose length is the perimeter of a base and whose width is the height of the prism. This can be more efficient than the other methods because students only need to calculate two areas (since the two bases will be identical copies).
- This method only works for prisms. For other shapes, such as pyramids, Noah's method of finding all the faces individually or Elena's method of combining those faces into identical copy groups will work. Solids with holes, such as the triangular prism with a square hole, can use a variation on Elena's method: two congruent triangles with holes for the bases, one rectangle for the outside side faces, and another rectangle for the faces forming the hole.

Explain to students that they will have the opportunity in the next activity to practise using any of these strategies.

Writing, Listening, Conversing: Stronger and Clearer Each Time. Use this routine to help students improve a written responses to the question, "How big is the '1 large rectangle' Andre is talking about?" Give students time to meet with 2–3 partners, to share and get feedback on their responses.
Provide students with prompts for feedback that will help their partners strengthen their ideas and clarify their language (e.g., "Can you draw a picture to support your explanation?", "You should expand on….", "How does that match with Andre's thinking?", etc.). Invite students to go back and revise or refine their written explanation based on their peer feedback. These conversations will help students make sense of the different

methods for calculating the surface area of a shape.
Design Principle(s): Cultivate conversation; Optimise output (for explanation)

14.3 Revisiting a Pentagonal Prism

15 minutes

In this activity, students are presented with a shape that was used in a previous lesson to explore volume. Here, they explore its surface area and compare different methods from the previous task. Students work with a partner to share the task of investigating two methods to calculate the surface area.

As students work on the task, listen for students who find similarities and differences between the method they used and the one their partner used.

Instructional Routines

- Discussion Supports

Launch

Arrange students in groups of 2. Tell students that they might recognise this shape from a previous lesson, but today they are going to compare two different methods for calculating its surface area. Give students 1–2 minutes of quiet work time followed by time to swap their work with a partner to compare answers and methods. Follow with a whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge about the surface area of a pentagonal prism by calculating area of all the faces and then using the perimeter of the base. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing Speaking: Discussion Supports. Use this routine to support students as they describe their strategy for calculating the area of the prism using one of the two methods. Provide sentence frames for students to use such as: “First I _____. Then I _____, because . . .” or “I decomposed (or unfolded) _____ to find . . . .” Encourage students to consider what details are important to share, and to think about how they will explain their reasoning using mathematical language.
Design Principle(s): Optimise output (for explanation); Cultivate conversation

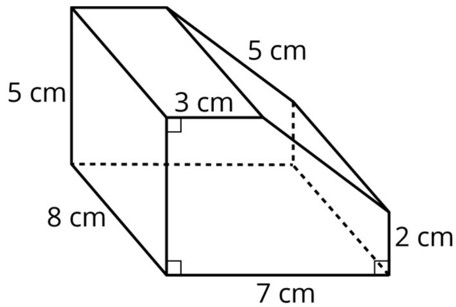
Student Task Statement

1. Between you and your partner, choose who will use each of these two methods to find the surface area of the prism.

    – Adding the areas of all the faces

    – Using the perimeter of the base.

2. Use your chosen method to calculate the surface area of the prism. Show your thinking. Organise it so it can be followed by others.



3. Swap papers with your partner, and check their work. Discuss your thinking. If you disagree, work to reach an agreement.

Student Response

1. No answer required.

2. The surface area is 234 cm^2. Explanations vary. Sample responses:

   - Adding the areas of all the faces: There are two pentagonal bases that can each be decomposed into two rectangles (3 cm by 5 cm, and 4 cm by 2 cm) and a right-angled triangle (base 4 cm and height 3 cm). Each pentagon has an area of 29 cm^2, since $15 + 8 + 6 = 29$. There are five rectangular faces, each with a side that is 8 cm. Their combined area is 176 cm^2, since $(3 \times 8) + (5 \times 8) + (7 \times 8) + (2 \times 8) + (5 \times 8) = 24 + 40 + 56 + 16 + 40 = 176$. The sum of the areas of the bases and the rectangles is 234 cm^2, since $2(29) + 176 = 58 + 176 = 234$.

   - Using the perimeter of the base: There are two pentagonal bases that have an area of 29 cm^2, since $15 + 8 + 6 = 29$. The perimeter of the pentagonal base is 22 cm, since $2 + 5 + 3 + 5 + 7 = 22$. All the rectangular faces, if unfolded, make a long rectangle that is 22 cm by 8 cm, so its area is 176 cm^2, $22 \times 8 = 176$. The sum of the areas of the two bases and the long rectangle is 234 cm^2, since $2(29) + 176 = 58 + 176 = 234$.

Are You Ready for More?

In a deck of cards, each card measures 6 cm by 9 cm.

1. When stacked, the deck is 2 cm tall, as shown in the first photo. Find the volume of this deck of cards.

2. Then the cards are fanned out, as shown in the second picture. The distance from the rightmost point on the bottom card to the rightmost point on the top card is now 7 cm instead of 2 cm. Find the volume of the new stack.

Student Response

1. 108 cm^3

2. 108 cm^3

Activity Synthesis

Select previously identified students to share the discussion they had with their partner. To highlight the difference between the two methods, ask:

- "How did you find the area of the base?"

- "How did you find any other areas you needed to solve the problem?"
- "How many different shapes did you need to calculate the area of when using the first method (calculating area of all the faces)?"
- "How many different shapes did you need to calculate the area of when using the second method (using perimeter of base)?"
- "Which method do you prefer for this problem? Why?"
- "Do you think you will prefer the same method for every problem? Why or why not?"
- "What would make you change methods?"
- "Do you need to know all of the measurements in the picture to solve for surface area?" (No, you just need to know the perimeter and area of the base and the height of the shape.)
- "Could you solve for volume with the measurements given in the picture? If so, are there any unnecessary measurements? If not, what else would you need to know?"

If not brought up in students' explanations, explain to students that the first method requires finding the area of 6 different shapes (there are 7 faces, but the two bases are the same). While the calculations using this method were simple, there were more pieces. The second method requires visualising the solid in a different way, but we only needed to find the area of two different pieces (the long rectangle and base).

Lesson Synthesis

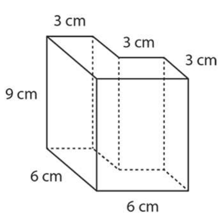
- "What is surface area?" (The total area of all the exposed faces of an object.)
- "What are some methods for calculating surface area of prisms?" (Find the area of each face and add them for the total. Find groups of faces that have the same area and save some computation. Find the area of the bases and add that to the area of a "long rectangle.")

14.4 Surface Area of a Hexagonal Prism

Cool Down: 5 minutes

Student Task Statement

Find the surface area of this prism. Show your reasoning. Organise it so it can be followed by others.



Student Response

The surface area is 270 cm². Possible strategy: The area of the base is 27 cm². The perimeter of the base is 24 cm, so the combined area of the sides is 216 cm², because $24 \times 9 = 216$. Therefore, the total surface area is 270 cm², because $27 \times 2 + 216 = 270$.

Student Lesson Summary

To find the surface area of a three-dimensional shape whose faces are made up of polygons, we can find the area of each face, and add them up!

Sometimes there are ways to simplify our work. For example, all of the faces of a cube with side length s are the same. We can find the area of one face, and multiply by 6. Since the area of one face of a cube is s^2, the surface area of a cube is $6s^2$.

We can use this technique to make it faster to find the surface area of any shape that has faces that are the same.

For prisms, there is another way. We can treat the prism as having three parts: two identical bases, and one long rectangle that has been taped along the edges of the bases. The rectangle has the same height as the prism, and its width is the perimeter of the base. To find the surface area, add the area of this rectangle to the areas of the two bases.

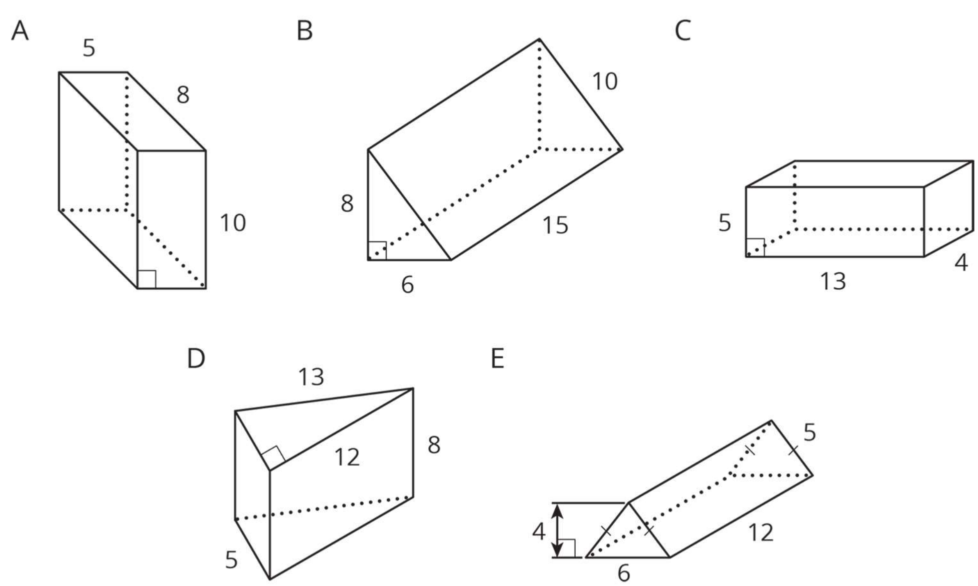
Glossary

- surface area

Lesson 14 Practice Problems

1. **Problem 1 Statement**

   Edge lengths are given in units. Find the surface area of each prism in square units.



Solution

a. 340

b. 408

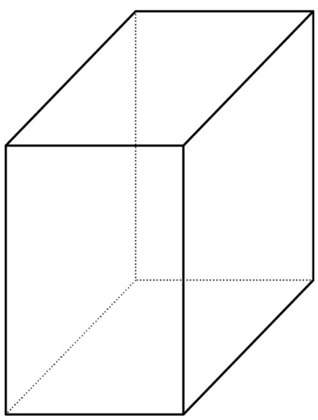
c. 274

d. 300

e. 216

2. **Problem 2 Statement**

Priya says, "No matter which way you slice this cuboid, the cross section will be a rectangle." Mai says, "I'm not so sure." Describe a slice that Mai might be thinking of.



Solution

If you keep your slices parallel to a set of faces, then the cross section does have to be a rectangle. But if you can slice in any direction, you can get a triangle. Imagine slicing off one small corner of the prism.

3. **Problem 3 Statement**

B is the intersection of line AC and line ED. Find each of the angles.

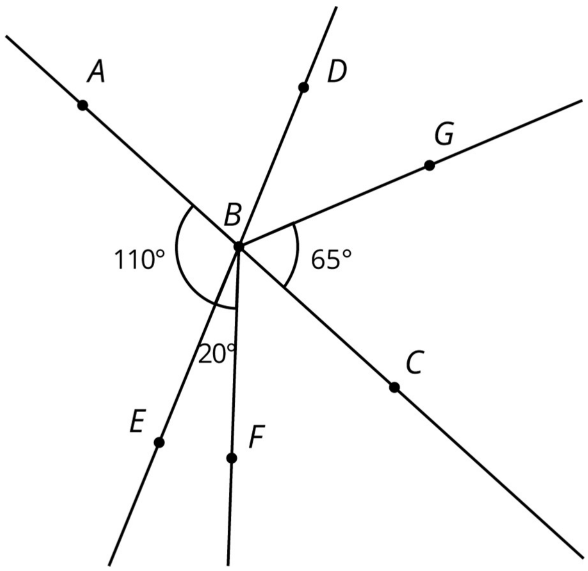
a. Angle ABF

b. Angle ABD

c. Angle EBC

d. Angle FBC

e. Angle DBG



Solution

a. 130 degrees (sum of angles ABE and EBF)

b. 70 degrees (supplementary with angle ABE)

c. 70 degrees (vertically opposite ABD)

d. 50 degrees (subtract angle EBF from angle EBC)

e. 45 degrees (subtract angles ABD and CBG from 180°)

4. **Problem 4 Statement**

Write each expression with fewer terms.

a. $12m - 4m$

b. $12m - 5k + m$

c. $9m + k - (3m - 2k)$

Solution

a. $8m$

b. $13m - 5k$

c. $6m + 3k$

5. **Problem 5 Statement**

a. Find 44% of 625 using the facts that 40% of 625 is 250 and 4% of 625 is 25.

b. What is 4.4% of 625?

c. What is 0.44% of 625?

Solution

a. 275 (Because 44% of a number equals 40% of the number plus an additional 4% of the number)

b. 27.5

c. 2.75