Lesson 6: Distinguishing between two types of situations

Goals

- Categorise equations of the forms $px + q = r$ and $p(x + q) = r$, and describe (orally) the categories.
- Interpret a verbal description of a situation (in written language), and write an equation of the form $px + q = r$ or $p(x + q) = r$ to represent it.

Learning Targets

- I understand the similarities and differences between the two main types of equations we are studying in this unit.
- When I have a situation or a bar model, I can represent it with an equation.

Lesson Narrative

The purpose of this lesson is to distinguish equations of the form $px + q = r$ and $p(x + q) = r$. Corresponding bar models are used as tools in this work, along with situations that these equations can represent. First, students sort equations into categories of their choosing. The main categories to highlight distinguish between the two main types of equations being studied. Then, students consider two stories and corresponding diagrams and write equations to represent them. They use these representations to find an unknown value in the story.

Addressing

- Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

Building Towards

- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- Three Reads
- Discussion Supports
- Take Turns
- Think Pair Share

- Which One Doesn't Belong?

Required Materials

Pre-printed slips, cut from copies of the blackline master

$4(x+3)=9$	$4\times x+12=9$
$4+3x=9$	$9=12+4x$

$100=8(x+9)$	$8(9+x)=100$	$63+9x=100$
$9x+63=100$	$100=9x+63$	$9(7+x)=100$
$100=(x+7)\times 9$	$100=8x+72$	$72+8x=100$
$100=(x+9)\times 8$		

Required Preparation

Print and cut up copies of the blackline master ahead of time. You will need 1 set for every 2 students. If possible, copy each complete set on a different colour of paper, so that a stray slip can quickly be put back.

Student Learning Goals

Let's think about equations with and without brackets and the kinds of situations they describe.

6.1 Which One Doesn't Belong: Seeing Structure

Warm Up: 5 minutes

This warm-up prompts students to compare four equations. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the equations in comparison to one another. To allow all students to access the activity, each equation has one obvious reason it does not belong. Encourage students to find reasons based on the structure of the equation. During the discussion, listen for important ideas and terminology.

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the equations for all to see. Ask students to indicate when they have noticed one that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. In their groups, tell each student to share their reasoning about why a particular question does not belong, and together, find at least one reason each question doesn't belong.

Student Task Statement

Which equation doesn't belong?

$4(x + 3) = 9$

$4 \times x + 12 = 9$

$4 + 3x = 9$

$9 = 12 + 4x$

Student Response

1. $4(x + 3) = 9$ (only one with a side that is only the product of two expressions)
2. $4 \times x + 12 = 9$ (only one that uses the multiplication sign instead of next-to for multiplication)
3. $4 + 3x = 9$ (only one not equivalent to the others, possibly notice it's the only one with a positive solution)
4. $9 = 12 + 4x$ (only one with only a number on the left side)

Activity Synthesis

Ask each group to share one reason why a particular equation does not belong. Record and display the responses for all to see. After each response, ask the class whether they agree or disagree. Since there is no single correct answer to the question 'which one does not belong?', attend to students' explanations and ensure the reasons given are correct. During the discussion, ask students to explain the meaning of any terminology they use, such as *coefficient* or *solution*. Also, press students on unsubstantiated claims.

6.2 Card Sort: Categories of Equations

15 minutes

The goal of this activity is for students to notice the structure of equations. Any way of sorting is fine, but the discussion should land on explaining how equations involving an expression like $p(x + q)$ are different from ones that have an expression like $px + q$. Monitor for different ways groups choose to categorise the equations, but especially for categories that distinguish between these two types of expressions. As students work, encourage them to refine their descriptions of equations using more precise language and mathematical terms.

Instructional Routines

- Discussion Supports
- Take Turns

Launch

Arrange students in groups of 2. Tell them that in this activity, they will sort some cards into categories of their choosing. When they sort the equations, they should work with their partner to come up with categories, and then take turns sorting each equation into one of their categories, explaining why they are doing so. If necessary, demonstrate this protocol before students start working.

Distribute one set of cards to each group of students. Give students 5 minutes to work with their partner, followed by a whole-class discussion.

Student Task Statement

Your teacher will give you a set of cards that show equations. Sort the cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories. Then, sort the cards into 2 categories in a different way. Be prepared to explain the meaning of your new categories.

Student Response

Categories vary.

Activity Synthesis

Select groups of students to share their categories and how they sorted their equations. You can choose as many different types of categories as time allows, but ensure that one set of categories distinguishes between equations that involved expressions of the form $px + q$ vs $p(x + q)$. Attend to the language that students use to describe their categories and equations, giving them opportunities to describe their equations more precisely. Highlight the use of terms like *coefficient*, *sum*, *product*, *variable*, and *solution*.

Speaking: Discussion Supports. Use this routine to support whole-class discussion when students share the categories they created to sort their equations. After groups describe the set of categories that distinguish between equations that involved expressions of the form $px + q$ vs. $p(x + q)$, call on students to restate and/or revoice their peers' descriptions using mathematical terms (e.g., *coefficient*, *sum*, *product*, *variable*, and *solution*.). This will provide more students with an opportunity to produce language that describes the form and structure of the two equations.
Design Principle(s): Support sense-making; Maximise meta-awareness

6.3 Even More Situations, Diagrams, and Equations

15 minutes

This activity is an opportunity to put together the learning of the past several lessons: correspondences between bar models, equations, and stories, and using representations to reason about a solution. The focus of this activity is still contrasting the two main types of equations that students encounter in this unit.

Instructional Routines

- Three Reads
- Think Pair Share

Launch

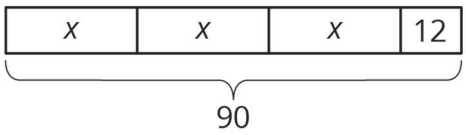
Keep students in the same groups. 5 minutes of quiet work time followed by sharing with a partner and a whole-class discussion.

Representation: Internalise Comprehension. Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems, and other text-based content.
Supports accessibility for: Language; Conceptual processing Reading, Representing: Three Reads. Use this routine with the first story to support students' understanding of the situation and how to represent it with a bar model or equation. Use the first read to orient students to the situation (Lin and volunteers are hanging flyers at school). Use the second read to identify the important quantities (number of volunteers, number of flyers each). After the third read, ask students to brainstorm how the situation can be represented in a bar model or equation. This will help students make connections between equations and diagrams and understand the language of a situation that will distinguish which form of

and equation would be an appropriate representation.
Design Principle(s): Support sense-making; Maximise meta-awareness

Student Task Statement

A



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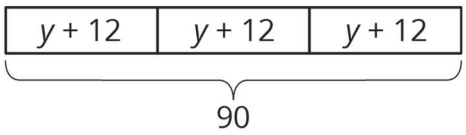


Story 1: Lin had 90 flyers to hang up around the school. She gave 12 flyers to each of three volunteers. Then she took the remaining flyers and divided them up equally between the three volunteers.

Story 2: Lin had 90 flyers to hang up around the school. After giving the same number of flyers to each of three volunteers, she had 12 left to hang up by herself.

1. Which diagram goes with which story? Be prepared to explain your reasoning.
2. In each diagram, what part of the story does the variable represent?
3. Write an equation corresponding to each story. If you get stuck, use the diagram.
4. Find the value of the variable in the story.

Student Response

1. Diagram A goes with story 2 and diagram B goes with story 1.
2. In diagram A, x represents the number of flyers she gave to each volunteer. In diagram B, y represents the remaining flyers she gave to each volunteer (after giving each of them 12 to start).
3. Story 1: $3(y + 12) = 90$ or equivalent. Story 2: $3x + 12 = 90$ or equivalent.
4. Story 1: $y = 18$. Story 2: $x = 26$.

Are You Ready for More?

A tutor is starting a business. In the first year, they start with 5 clients and charge £10 per week for an hour of tutoring with each client. For each year following, they double the number of clients and the number of hours each week. Each new client will be charged 150% of the charges of the clients from the previous year.

1. Organise the weekly earnings for each year in a table.
2. Assuming a full-time week is 40 hours per week, how many years will it take to reach full time and how many new clients will be taken on that year?
3. After reaching full time, what is the tutor's annual salary if they take 2 weeks of vacation?

4. Is there another business model you'd recommend for the tutor? Explain your reasoning.

Student Response

Year	Existing Charges (q)	Rate on New Clients (p)	New Clients (x)	Weekly Rate ($px + q$)
1	0	10	5	50
2	50	15	10	200
3	200	22.5	20	700
4	700	33.75	5	868.75

1. 4 years, 5 clients

2. £43 437.50

3. Answers vary. Sample response: The tutor could raise rates on existing clients by a little bit each year.

Activity Synthesis

For each story, select 1 or more groups to present the matching diagram, their equation, and their solution method. Possible questions to ask:

- "How were the diagrams alike? How were they different?" (They have the same numbers and a letter. One has 3 equal groups and an extra bit, the other just has 3 equal groups, but each group is a sum.)
- "How were the stories alike? How were they different?" (They were both about distributing 90 flyers. In one story, Lin makes a series of moves to each volunteer. In the other story, she gives each volunteer the same amount, but then there are some left over.)
- "What parts of the story made you think that one diagram represented it?"
- "Explain how you reasoned about the story, diagram, or equation to find the value of the variable."

Lesson Synthesis

Display the two equations from the last activity for all to see:

$3x + 12 = 90$

$3(y + 12) = 90$

Tell students, "These equations have lots of things in common. They each have a 3, a letter, a 12, a 90, an equals sign, multiplication, and addition. Explain how these equations are *different*." Ask students to think about this question quietly for a moment and share with a partner, then ask a few students to share with the whole class.

Highlight any responses that speak in general terms about the structure of the equations. For example, one equation is the sum of a product and a number and the other is the product of a number and a sum. Alternatively, if we evaluated one expression for a value of the variable, we would multiply it by 3 first and then add 12. For the other, we would add 12 first and then multiply by 3. One has three equal groups and an extra bit, and the other just has 3 equal groups, but the groups are each the result of adding 12 to an unknown.

6.4 After School Tutoring

Cool Down: 5 minutes

Student Task Statement

Write an equation for each story. Then, find the number of problems originally assigned by each teacher. If you get stuck, try drawing a diagram to represent the story.

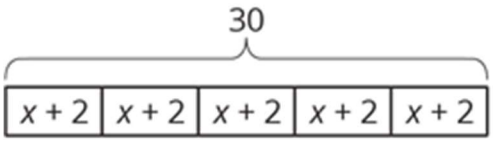
1. Five students came for after-school tutoring. Lin's teacher assigned each of them the same number of problems to complete. Then he assigned each student 2 more problems. 30 problems were assigned in all.

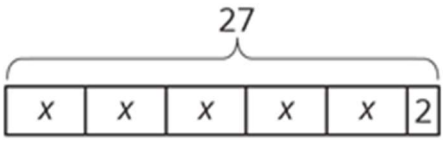
2. Five students came for after-school tutoring. Priya's teacher assigned each of them the same number of problems to complete. Then she assigned 2 more problems to one of the students. 27 problems were assigned in all.

Student Response

1. $5(x + 2) = 30$ (or equivalent), solution: 4

2. $5x + 2 = 27$ (or equivalent), solution: 5





Student Lesson Summary

In this unit, we encounter two main types of situations that can be represented with an equation. Here is an example of each type:

1. After adding 8 students to each of 6 same-sized teams, there were 72 students altogether.

2. After adding an 8-pound box of tennis rackets to a crate with 6 identical boxes of ping pong paddles, the crate weighed 72 pounds.

The first situation has all equal parts, since additions are made to *each* team. An equation that represents this situation is $6(x + 8) = 72$, where x represents the original number of students on each team. Eight students were added to each group, there are 6 groups, and there are a total of 72 students.

In the second situation, there are 6 equal parts added to one other part. An equation that represents this situation is $6x + 8 = 72$, where x represents the weight of a box of ping pong paddles, there are 6 boxes of ping pong paddles, there is an additional box that weighs 8 pounds, and the crate weighs 72 pounds altogether.

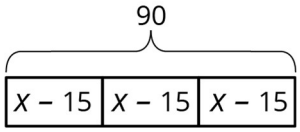
In the first situation, there were 6 equal groups, and 8 students added to each group. $6(x + 8) = 72$.

In the second situation, there were 6 equal groups, but 8 more pounds in addition to that. $6x + 8 = 72$.

Lesson 6 Practice Problems

1. **Problem 1 Statement**

   A school ordered 3 large boxes of board markers. After giving 15 markers to each of 3 teachers, there were 90 markers left. The diagram represents the situation. How many markers were originally in each box?

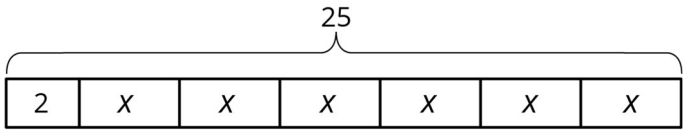


   Solution

   45

2. **Problem 2 Statement**

   The diagram can be represented by the equation $25 = 2 + 6x$. Explain where you can see the 6 in the diagram.



   Solution

   There are 6 equal parts labelled x.

3. **Problem 3 Statement**

   Match each equation to a story. (Two of the stories match the same equation.)

   A. $3(x + 5) = 17$

   B. $3x + 5 = 17$

C. $5(x + 3) = 17$

D. $5x + 3 = 17$

a. Jada's teacher fills a travel bag with 5 copies of a textbook. The weight of the bag and books is 17 pounds. The empty travel bag weighs 3 pounds. How much does each book weigh?

b. A piece of scenery for the school play is in the shape of a 5-foot-long rectangle. The designer decides to increase the length. There will be 3 identical rectangles with a total length of 17 feet. By how much did the designer increase the length of each rectangle?

c. Elena spends £17 and buys a £3 book and a bookmark for each of her 5 cousins. How much does each bookmark cost?

d. Noah packs up bags at the food pantry to deliver to families. He packs 5 bags that weigh a total of 17 pounds. Each bag contains 3 pounds of groceries and a packet of papers with health-related information. How much does each packet of papers weigh?

e. Andre has 3 times as many pencils as Noah and 5 pens. He has 17 pens and pencils all together. How many pencils does Noah have?

Solution

a. D

b. A

c. C

d. C

e. B

4. **Problem 4 Statement**

Elena walked 20 minutes more than Lin. Jada walked twice as long as Elena. Jada walked for 90 minutes. The equation $2(x + 20) = 90$ describes this situation. Match each expression with the statement in the story with the expression it represents.

A. x

B. $x + 20$

C. $2(x + 20)$

D. 90

1. The number of minutes that Jada walked

2. The number of minutes that Elena walked
3. The number of minutes that Lin walked

Solution

- A: 3
- B: 2
- C: 1
- D: 1