

## Lesson 18: Using common multiples and common factors

### Goals

- Choose to calculate the highest common factor or least common multiple to solve a problem about a real-world situation, and justify (orally) the choice.
- Present (orally, in writing, and using other representations) the solution method for a problem involving highest common factor or least common multiple.

### Learning Targets

- I can solve problems using common factors and multiples.

### Lesson Narrative

In this lesson, students apply what they have learned about factors and multiples to solve a variety of problems. In the first activity, students use what they have learned about common factors and common multiples to solve less structured problems in context. The two activities that follow are optional. The optional activity "More Factors and Multiples" allows students to explore common factors and common multiples of 3 whole numbers and present their findings. The optional activity "Factors and Multiples Bingo" allows students to practise finding multiples and factors.

### Addressing

- Find the highest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express  $36 + 8$  as  $4(9 + 2)$ .

### Instructional Routines

- Group Presentations
- Collect and Display
- Three Reads
- Discussion Supports

### Required Materials

**Mini-whiteboard pens (if desired)**

**Pre-printed slips, cut from copies of the blackline master**

Factors and Multiples Bingo All common factors of 24 and 36	Factors and Multiples Bingo All numbers with 4 or more factors	Factors and Multiples Bingo All even square numbers
Factors and Multiples Bingo All even multiples of 3	Factors and Multiples Bingo All numbers divisible by 8	Factors and Multiples Bingo All two-digit numbers with digits that sum to 6
Factors and Multiples Bingo All common multiples of 4 and 10	Factors and Multiples Bingo All common factors of 45 and 90	Factors and Multiples Bingo All common factors of 25 and 75
Factors and Multiples Bingo All odd square numbers	Factors and Multiples Bingo All prime numbers greater than 17	Factors and Multiples Bingo All prime numbers less than 19
Factors and Multiples Bingo All single-digit composite numbers	Factors and Multiples Bingo All powers of 10	Factors and Multiples Bingo All multiples of 15
Factors and Multiples Bingo All numbers with exactly 3 factors	Factors and Multiples Bingo All even multiples of 7	Factors and Multiples Bingo All cubic numbers
Factors and Multiples Bingo All odd multiples of 3	Factors and Multiples Bingo All factors of 100	Factors and Multiples Bingo All odd factors of 180

Factors and Multiples Bingo				Factors and Multiples Bingo			
24	20	33	15	4	21	30	16
7	4	14	84	8	14	15	42
99	10	36	11	9	19	32	1
12	26	0	75	22	46	39	25

Factors and Multiples Bingo				Factors and Multiples Bingo			
44	25	31	5	34	15	11	50
76	48	24	27	6	44	42	75
92	18	3	16	35	90	3	18
62	56	70	0	60	16	80	10
Factors and Multiples Bingo				Factors and Multiples Bingo			
4	19	13	54	3	14	1	0
65	42	12	7	66	24	12	5
33	9	23	28	36	60	37	48
66	26	84	18	6	56	63	10

Factors and Multiples Bingo				Factors and Multiples Bingo			
32	25	31	57	30	15	13	0
68	42	48	60	67	45	12	50
3	0	64	28	55	20	36	18
66	76	81	10	63	16	49	64
Factors and Multiples Bingo				Factors and Multiples Bingo			
4	15	21	0	4	18	21	5
16	42	52	17	26	24	48	71
30	45	35	1	45	19	32	8
6	26	8	4	61	16	36	10

Factors and Multiples Bingo				Factors and Multiples Bingo			
56	63	64	60	3	25	17	56
8	25	42	36	16	42	40	7
35	9	37	1	85	9	36	48
66	36	81	100	63	49	26	14

Factors and Multiples Bingo				Factors and Multiples Bingo			
75	24	81	33	3	60	0	23
22	63	14	92	20	26	4	52
90	49	8	3	18	45	46	42
26	12	70	5	63	76	34	25

Factors and Multiples Bingo				Factors and Multiples Bingo			
99	14	28	39	6	25	15	32
64	3	75	18	21	4	7	16
71	9	0	48	24	10	66	19
80	19	15	33	35	13	50	30

Factors and Multiples Bingo				Factors and Multiples Bingo			
6	44	84	0	37	1	31	17
66	19	30	32	4	27	44	16
9	1	54	7	65	18	35	55
62	36	15	12	8	56	11	42

---

### Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

#### Required Preparation

If using the optional activity, "More Factors and Multiples," provide access to tools for creating a visual display.

If using the optional "Factors and Multiples Bingo" activity, prepare one set of pre-printed slips and one set of pre-printed Bingo boards cut from the blackline master. Each group of 2 students will share 1 Bingo board. It is suggested that the Bingo boards are laminated and mini-whiteboard pens are used to cross off the numbers as this makes the resources reusable.

#### Student Learning Goals

Let's use common factors and common multiple to solve problems.

## 18.1 Keeping a Steady Beat

### Warm Up: 5 minutes

In this warm-up, students explore the concept of least common multiple using rhythm.

#### Launch

Tell the class they will be establishing a steady beat as a class. Tell half the class to clap on every other beat and tell the other half to say "yeah!" on every third beat. If time permits, repeat this activity for 3 and 4, 2 and 4, 4 and 6. Follow with a whole-class discussion.

#### Student Task Statement

Your teacher will give you instructions for playing a rhythm game. As you play the game, think about these questions:

- When will the two sounds happen at the same time?
- How does this game relate to common factors or common multiples?

#### Student Response

When claps happen every 2 beats and "yeah!"s happen every 3 beats, they happen at the same time after 6, 12, 18, and 24 beats. These are all common multiples of 2 and 3. The first time it happens is the least common multiple, which is after 6 beats.

---

## Activity Synthesis

Select 1-2 students to explain what they noticed about the activity and how it is related to least common multiples. Tell students that during the activities in this lesson, they will look for opportunities to solve problems using common factors and common multiples.

## 18.2 Factors and Multiples

### 20 minutes

In this activity, students work in pairs to solve problems that involve thinking about factors and multiples as well as the highest common factor and least common multiple. After solving the problems, they reflect on what type of mathematical work was part of each problem and then record this information into a table. Students begin to notice similarities in the types of problems that involve factors and in those that involve multiples. Students must make sense and persevere as they decide how the problems relate to common factors and common multiples.

### Instructional Routines

- Collect and Display

### Launch

Arrange students in groups of 2. Give students 15 minutes of work time followed by whole-class discussion.

*Action and Expression: Internalise Executive Functions.* To support development of organisational skills, check in with students within the first 2–3 minutes of work time. Look for students to find common factors and common multiples to solve problems involving combinations of quantities.

*Supports accessibility for: Memory; Organisation Writing, Speaking, Listening: Collect and Display.* While pairs are working, circulate and listen to student talk about the strategies they used to determine whether they are answering problems about the least common multiple or the highest common factor. Write down common phrases you hear students use such as “smallest,” “common,” “largest,” “show up twice,” etc. Write the students’ words on a visual display, and continue to use it as a reference throughout the lesson. This will help students use mathematical language as they make sense of how the problems relate to common factors and common multiples.

*Design Principle(s): Support sense-making*

### Student Task Statement

Work with your partner to solve the following problems.

1. **Party.** Elena is buying cups and plates for her party. Cups are sold in packs of 8 and plates are sold in packs of 6. She wants to have the same number of plates and cups.
  - a. Find a number of plates and cups that meets her requirement.

- b. How many packs of each supply will she need to buy to get that number?
  - c. Name two other quantities of plates and cups she could get to meet her requirement.
  
2. **Tiles.** A restaurant owner is replacing the restaurant’s bathroom floor with square tiles. The tiles will be laid side-by-side to cover the entire bathroom with no gaps, and none of the tiles can be cut. The floor is a rectangle that measures 24 feet by 18 feet.
  - a. What is the largest possible tile size she could use? Write the side length in feet. Explain how you know it’s the largest possible tile.
  - b. How many of these largest size tiles are needed?
  - c. Name more tile sizes that are whole number of feet that she could use to cover the bathroom floor. Write the side lengths (in feet) of the square tiles.
  
3. **Stickers.** To celebrate the first day of spring, Lin is putting stickers on some of the 100 lockers along one side of her middle school’s hallway. She puts a skateboard sticker on every 4th locker (starting with locker 4), and a kite sticker on every 5th locker (starting with locker 5).
  - a. Name three lockers that will get both stickers.
  - b. After Lin makes her way down the hall, will the 30th locker have no stickers, 1 sticker, or 2 stickers? Explain how you know.
  
4. **Kits.** The school nurse is assembling first-aid kits for the teachers. She has 75 bandages and 90 throat lozenges. All the kits must have the same number of each supply, and all supplies must be used.
  - a. What is the largest number of kits the nurse can make?
  - b. How many bandages and lozenges will be in each kit?
  
5. What kind of mathematical work was involved in each of the previous problems? Put a tick to show what the questions were about.

problem	finding multiples	finding least common multiple	finding factors	finding highest common factor
Party				
Tiles				
Stickers				
Kits				

**Student Response**

1.

- a. Answers vary. Sample response: Elena could have 24 cups and 24 plates.
- b. Answers vary. Sample response: Elena could get 3 packages of cups and 4 packages of plates to get 24 of each.
- c. Answers vary. Sample response: She could get 48 of each if she gets 6 packages of cups and 8 packages of plates. She could also get 72 of each if she gets 9 packages of cups and 12 packages of plates.

2.

- a. The largest square that will fit has a length of 6 ft.
- b. 12 tiles are needed to cover the floor.
- c. She could also use tiles of length 1, 2, or 3 ft.

3.

- a. Answers vary. Any 3 lockers from 20, 40, 60, 80, and 100 are valid responses. The least common multiple of 4 and 5 is 20. All common multiples of 4 and 5 are multiples of 20.
- b. Since 30 is not a multiple of 4, the 30th locker will not get a flower sticker. It will only get 1 sticker.

4.

- a. The greatest amount of kits that can be made is 15.
- b. Each kit will have 5 bandages and 6 lozenges.

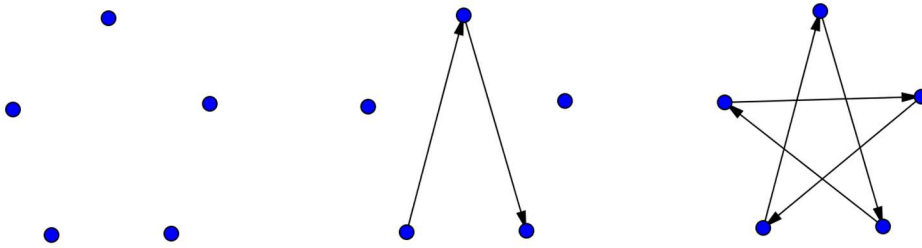
5.

problem	finding multiples	finding least common multiple	finding factors	finding highest common factor
Party	✓	✓		
Tiles			✓	✓
Stickers	✓	✓		
Kits			✓	✓

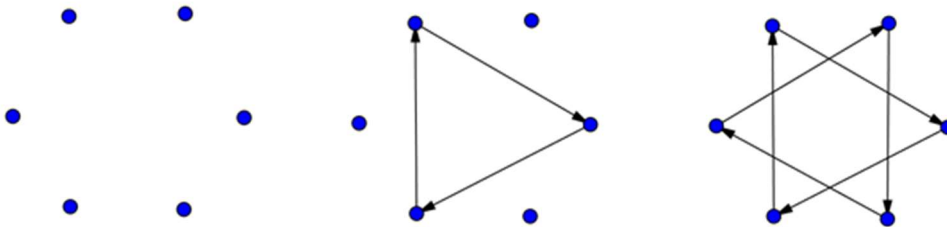
### Are You Ready for More?

You probably know how to draw a five-pointed star without lifting your pencil. One way to do this is to start with five dots arranged in a circle, then connect every second dot.



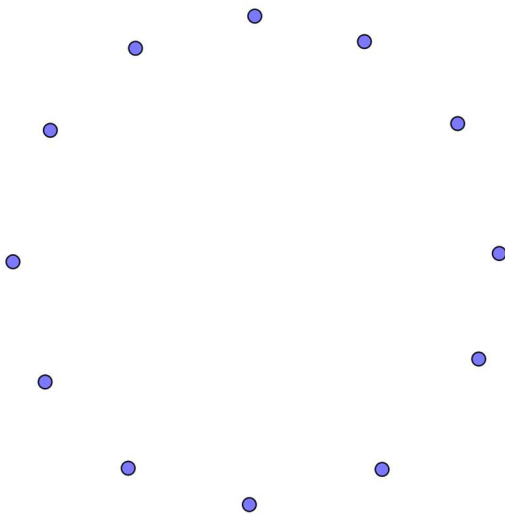


If you try the same thing with six dots arranged in a circle, you will have to lift your pencil. Once you make the first triangle, you'll have to find an empty dot and start the process over. Your six-pointed star has two pieces that are each drawn without lifting the pencil.

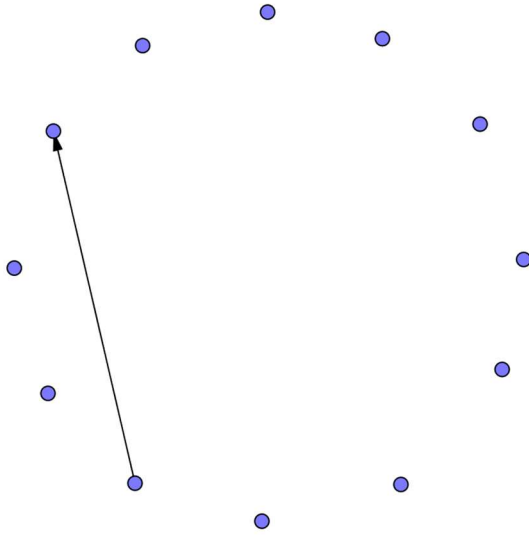


With twelve dots arranged in a circle, we can make some twelve-pointed stars.

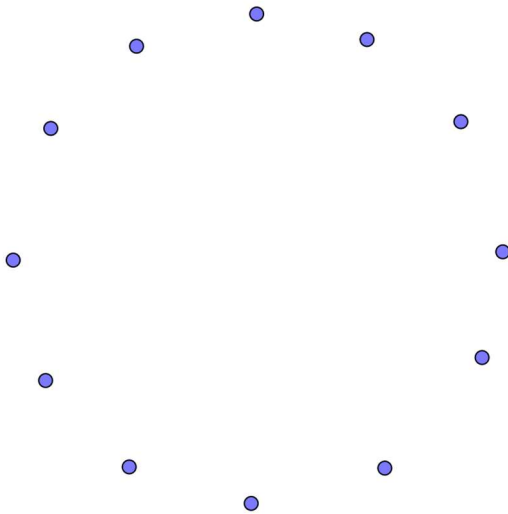
1. Start with one dot and connect every second dot, as if you were drawing a five-pointed star. Can you draw the twelve-pointed star without lifting your pencil? If not, how many pieces does the twelve-pointed star have?



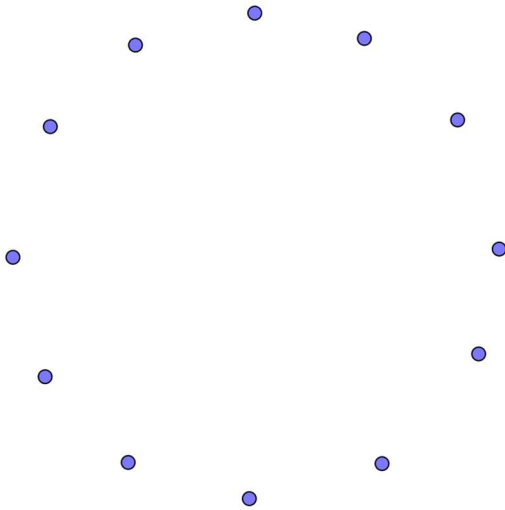
2. This time, connect every third dot. Can you draw this twelve-pointed star without lifting your pencil? If not, how many pieces do you get?



3. What do you think will happen if you connect every fourth dot? Try it. How many pieces do you get?



4. Do you think there is any way to draw a twelve-pointed star without lifting your pencil? Try it out.
-



5. Now investigate eight-pointed stars, nine-pointed stars, and ten-pointed stars. What patterns do you notice?

#### Student Response

1. No, you have to lift up your pencil. There are two pieces.
2. No, you have to lift up your pencil here, too. This time, there are three pieces.
3. No. There are four pieces.
4. Yes. If you connect every fifth dot, you can draw the star in one motion. This also works if you connect every seventh dot. (or every "first" or 11th dot).
5. Answers vary. You can only draw the star without picking up your pencil if the total number of dots and the "skip number" have no factors in common.

#### Activity Synthesis

The purpose of the discussion for students to express what kinds of problems have to do with least common multiples and what kinds have to do with highest common factor. For each problem, ask students to indicate whether they think the problem had to do with common multiples or common factors, and invite a few to share their reasoning. Select students to explain their reasoning about how they solved the problems as time allows.

### 18.3 More Factors and Multiples

#### Optional: 40 minutes

This activity is optional because it asks students to think about highest common factor and least common multiple for sets of 3 whole numbers, where the standards only call for students to analyse pairs of whole numbers. In this activity, students work in groups to

---

---

predict whether a problem involves common factors or common multiples. Groups then solve 1 assigned problem, create a visual display to represent their work, and prepare a brief presentation.

### Instructional Routines

- Group Presentations
- Three Reads

### Launch

Arrange students in groups of 4. Provide access to tools for creating a visual display. Tell students that they will first read through each problem and discuss whether its solution has to do with finding common factors or common multiples. Give students 5 minutes to discuss questions 1–5. When finished discussing questions 1–5, assign each group 1 of those questions to solve. Give students 10 minutes of work time to solve and 10 minutes to create their visual display and prepare a short presentation.

*Engagement: Internalise Self-Regulation.* Provide a project checklist that chunks the various steps of the project into a set of manageable tasks.

*Supports accessibility for: Organisation; Attention Reading: Three Reads.* Use this routine to support reading comprehension of this problem. In the first read, students read the problem with the goal of comprehending the situation (e.g., #1, We have the number of football games in a month). In the second read, ask students to analyse the mathematical structure of the story by naming quantities (e.g., #1, every 3rd day and every 4th day). In the third read, ask students to brainstorm possible mathematical solution strategies to answer the questions. Remind students to apply strategies from the previous activity, and encourage students to draw diagrams if necessary. This helps students connect the language in the situations to start identifying when to look for the least common multiple or the highest common factor.

*Design Principle(s): Support sense-making*

### Student Task Statement

Here are five more problems. Read and discuss each one with your group. *Without solving*, predict whether each problem involves finding common multiples or finding common factors. Circle one or more options to show your prediction.

1. **Football.** Diego and Andre are both in a summer football league. During the month of August, Diego has a game every 3rd day, starting August 3rd, and Andre has a game every 4th day, starting August 4th.
    - common multiples
    - least common multiple
    - common factors
    - highest common factor
-

- a. What is the first date that both boys will have a game?
    - b. How many of their games fall on the same date?
  2. **Performances.** During a performing arts festival, students from primary and secondary schools will be grouped together for various performances. There are 32 primary students and 40 secondary school students. The arts director wants identical groups for the performances, with students from both schools in each group. Each student can be a part of only one group.
    - common multiples
    - least common multiple
    - common factors
    - highest common factor
    - a. Name all possible groupings.
    - b. What is the largest number of groups that can be formed? How many primary school students and how many secondary school students will be in each group?
  3. **Lights.** There is a string of holiday lights with red, gold, and blue lights. The red lights are set to blink every 12 seconds, the gold lights are set to blink every 8 seconds, and the blue lights are set to blink every 6 seconds. The lights are on an automatic timer that starts each day at 7:00 p.m. and stops at midnight.
    - common multiples
    - least common multiple
    - common factors
    - highest common factor
    - a. After how much time will all 3 lights blink at the exact same time?
    - b. How many times total will this happen in one day?
  4. **Banners.** Noah has two pieces of cloth. He is making square banners for students to hold during the opening day game. One piece of cloth is 72 inches wide. The other is 90 inches wide. He wants to use all the cloth, and each square banner must be of equal width and as wide as possible.
    - common multiples
    - least common multiple
    - common factors
-

- highest common factor
  - a. How wide should he cut the banners?
  - b. How many banners can he cut?
5. **Dancers.** At Elena's dance recital her performance begins with a line of 48 dancers that perform in the dark with a black light that illuminates white clothing. All 48 dancers enter the stage in a straight line. Every 3rd dancer wears a white headband, every 5th dancer wears a white belt, and every 9th dancer wears a set of white gloves.
- common multiples
  - least common multiple
  - common factors
  - highest common factor
  - a. If Elena is the 30th dancer, what accessories will she wear?
  - b. Will any of the dancers wear all 3 accessories? If so, which one(s)?
  - c. How many of each accessory will the dance teacher need to order?
6. Your teacher will assign your group a problem. Work with your group to solve the problem. Show your reasoning. Pause here so your teacher can review your work.
7. Work with your group to create a visual display that includes a diagram, an equation, and a mathematical vocabulary word that would help to explain your mathematical thinking while solving the problem.
8. Prepare a short presentation in which all group members are involved. Your presentation should include: the problem (read aloud), your group's prediction of what mathematical concept the problem involved, and an explanation of each step of the solving process.

### Student Response

1. This problem involves multiples and least common multiple.
- a. August 12th. They will also have a game together on August 24th.
  - b. Two dates: August 12th and 24th.  $3 \times 8 = 24$  and  $4 \times 6 = 24$ . There are no more dates because the next multiple of 12 is 36 and there are only 31 days in August.
2. This problem involves factors and highest common factor.
- a. There could be 1 group with 32 primary school students and 40 secondary school students, 2 groups with 16 primary school students and 20 secondary
-

---

school students, 4 groups with 8 primary school students and 10 secondary school students, or 8 groups with 4 primary school students and 5 secondary school students.

- b. 8 groups, with 9 students in each group.
3. This problem involves multiples and least common multiple.
    - a. Every 24 seconds.
    - b. In 5 hours there are 300 minutes and 1 800 seconds.  $1800 \div 24 = 75$ . The 3 lights will blink together 75 times each day.
  4. This problem involves factors and highest common factor.
    - a. 18 inches wide.
    - b. 9 banners,  $72 \div 18 = 4$ ,  $90 \div 18 = 5$ , and  $4 + 5 = 9$
  5. This problem involves multiples and least common multiple.
    - a. Elena will wear a headband and belt. 30 is a multiple of 3 and 5 but not of 9.
    - b. The 45th dancer will have all 3 accessories because it is the least common multiple of 3, 5, and 9.
    - c. 16 headbands (there are 16 multiples of 3 up to 48 and  $48 \div 3 = 16$ ), 9 belts (there are 9 multiples of 5 up to 48), and 5 sets of gloves (there are 5 multiples of 9 up to 48).

### Activity Synthesis

Give each group the opportunity to briefly present their visual display and approach to their problem. If time allows, highlight the different ways in which students used diagrams, equations, and vocabulary to represent their work.

## 18.4 Factors and Multiples Bingo

### Optional: 20 minutes

This game provides an opportunity for students to review factors and multiples. There are two versions of the game:

- Version A, “10 Anywhere”: The teacher mixes the calling cards and randomly selects one at a time. Upon selecting the card, the teacher reads the statement out loud and records it on the board. Students tick all numbers that fit the statement. When the next card is called, all numbers that fit this next statement are ticked. If a number has already been ticked, it gets a second tick. (For example, let’s say the first card called is “common factors of 25 and 75” and the second card called is “odd multiples of 5.” The numbers 5 and 25 fit both statements, so these numbers would then be double ticked.)

---

The first group that gets 10 ticks anywhere on the board calls "bingo". The teacher listens for the first voice heard. This student will have the opportunity to prove that they have bingo by calling out each number covered on their board and referring to the corresponding statements listed on the board. If a student makes an error, the teacher says, "out" and the game proceeds.

- Version B, "4 in a Row": In this game, multiple ticks in one square are not allowed. Instead, there need to be 4 ticks in a row (horizontally, vertically, or diagonally). The same process of calling cards and waiting to hear a winner takes place. In between games, have students switch either bingo cards or partners.

### Instructional Routines

- Discussion Supports

### Launch

Arrange students in groups of 2. Distribute pens and 1 pre-cut Bingo board from blackline master to each group. Explain how both versions of Bingo are played. Play at least one round of each version with the class. Play as many rounds as time allows before whole-class discussion.

*Representation: Internalise Comprehension.* Begin with a physical demonstration of one example round of each version of the game to support connections between new situations involving bingo and prior understandings of factors and multiples.

*Supports accessibility for: Conceptual processing; Visual-spatial processing*

### Anticipated Misconceptions

Preview the calling cards and decide if students are familiar with all the vocabulary used in the statements. Some of the words may need to be reviewed ahead of time. While playing the game, if you notice students making errors in identifying numbers that fit a particular statement, stop and discuss the meaning of the statement's mathematical vocabulary.

### Student Task Statement

Your teacher will explain the directions for a bingo game. Here are some things to keep in mind:

- Share one bingo board and a pen with a partner.
- To play the game, your teacher will read statements aloud. You may help one another decide what numbers fit each statement, but speak only in a whisper. If the teacher hears anything above a whisper, you are out.
- The first person to call bingo needs to call out each number and identify the statement that it corresponds to. If there is an error in identifying statements, that player is out and the round continues.

Good luck, and have fun!



---

## Student Response

Answers vary.

## Activity Synthesis

After playing a few rounds, discuss:

- “Is this a game of luck, strategy, or a combination of both? Explain how you know.” (Luck is part of the game (if the teacher is selecting statements at random), but students must also be familiar with the mathematical vocabulary words and must also be accurate in identifying all the factors and multiples of various numbers.)
- “Are some numbers better to have on a game board than others? Explain how you know.” (Yes, the number zero is not a good number to have because there not many statements fit this value. Larger composite numbers are probably more likely to get covered, because they have many factors and could also be multiples of the smaller numbers that precede them.)

*Speaking: Discussion Supports.* If students need review on vocabulary words, use this to amplify mathematical uses of language to communicate about finding factors and multiples. Invite students to use these words when stating students’ ideas. Provide sentence frames for students to use such as: “To find the factors, I can \_\_\_.”, or “To find the multiples, I can \_\_\_.”.

*Design Principle(s): Support sense-making*

## Lesson Synthesis

In this lesson, students solved more challenging problems that involved least common multiple and highest common factor. Here are some questions for discussion:

- “What is highest common factor? How is it determined?” (The highest common factor of 2 whole numbers is the greatest number that divides both numbers without remainder. It’s possible to find the highest common factor by listing all factors for both number and choosing the largest that appears in both lists.)
- “What types of situations involve finding highest common factor?” (Situations that involve having to divide two different numbers into equal groups with no remainders.)
- “What is least common multiple? How is it determined?” (The least common multiple of 2 whole numbers is the least number that is a multiple of both numbers. It is possible to find the least common multiple by listing multiples of each number in order and choosing the first multiple that appears on both lists.)
- “What types of situations involve finding least common multiple?” (Situations that involve different numbers that need to be multiplied to make the same number.)

---

## 18.5 What Kind of Problem?

### Cool Down: 5 minutes

#### Student Task Statement

1. For each problem, tell whether finding the answer requires finding a highest common factor or a least common multiple. You do not need solve the problems.
  - a. Elena has 20 apples and 35 crackers for making snack bags. She wants to make as many snack bags as possible and wants each bag to have the same combination of apples and crackers. What is the largest number of snack bags she could make?
  - b. A string of holiday lights at a store have three colours that flash at different times. Red lights flash every fifth second. Blue lights flash every third seconds. Green light flashes every four seconds. The store owner turns on the lights. After how many seconds will all three lights flash at the same time for the first time?
  - c. A florist orders sunflowers every 6 days, starting from the sixth day of the year, and daisies every 4 days, starting from the fourth day of the year. When (or on which day) will she orders both kinds of flowers on the same day?
  - d. Noah has 12 yellow square cards and 18 green ones. All the cards are the same size. He would like to arrange the square cards into two rectangles—one of each colour. He wants both the yellow and green rectangles to have the same height and to be as tall as possible. What is the tallest possible height for the two rectangles?
2. Explain how you know which problem(s) involves finding the highest common factor.

#### Student Response

1.
    - a. Highest common factor
    - b. Least common multiple
    - c. Least common multiple
    - d. Highest common factor
  2. Answers vary. Sample responses:
    - A problem is about finding the highest common factor when it involves looking for the greatest number of equal sets that can be made from a certain number of items (with none left over), or looking for the greatest length of a rectangle given a certain number of square units.
-

- A problem is about finding the highest common factor when it involves finding the largest possible number that divides into two whole numbers without a remainder.

### Student Lesson Summary

If a problem requires dividing two whole numbers by the same whole number, solving it involves looking for a common factor. If it requires finding the *largest* number that can divide into the two whole numbers, we are looking for the *highest common factor*.

Suppose we have 12 bagels and 18 muffins and want to make bags so each bag has the same combination of bagels and muffins. The common factors of 12 and 18 tell us possible number of bags that can be made.

The common factors of 12 and 18 are 1, 2, 3, and 6. For these numbers of bags, here are the number of bagels and muffins per bag.

- 1 bag: 12 bagels and 18 muffins
- 2 bags: 6 bagels and 9 muffins
- 3 bags: 4 bagels and 6 muffins
- 6 bags: 2 bagels and 3 muffins

We can see that the largest number of bags that can be made, 6, is the highest common factor.

If a problem requires finding a number that is a multiple of two given numbers, solving it involves looking for a common multiple. If it requires finding the *first* instance the two numbers share a multiple, we are looking for the *least common multiple*.

Suppose forks are sold in boxes of 9 and spoons are sold in boxes of 15, and we want to buy an equal number of each. The multiples of 9 tell us how many forks we could buy, and the multiples of 15 tell us how many spoons we could buy, as shown here.

- Forks: 9, 18, 27, 36, 45, 54, 63, 72, 90...
- Spoons: 15, 30, 45, 60, 75, 90...

If we want as many forks as spoons, our options are 45, 90, 135, and so on, but the smallest number of utensils we could buy is 45, the least common multiple. This means buying 5 boxes of forks ( $5 \times 9 = 45$ ) and 3 boxes of spoons ( $3 \times 15 = 45$ ).

---

## Lesson 18 Practice Problems

### 1. Problem 1 Statement

Mai, Clare, and Noah are making signs to advertise the school dance. It takes Mai 6 minutes to complete a sign, it takes Clare 8 minutes to complete a sign, and it takes Noah 5 minutes to complete a sign. They keep working at the same rate for half an hour.

- Will Mai and Clare complete a sign at the same time? Explain your reasoning.
- Will Mai and Noah complete a sign at the same time? Explain your reasoning.
- Will Clare and Noah complete a sign at the same time? Explain your reasoning.
- Will all three students complete a sign at the same time? Explain your reasoning.

### Solution

- Answers vary. Sample response: Yes, they will both finish at 24 minutes, because 24 is a common multiple of 6 and 8.
- Answers vary. Sample response: Yes, they will both finish at 30 minutes, because 30 is a common multiple of 6 and 5.
- Answers vary. Sample response: No. The first common multiple of 8 and 5 is 40, and 40 minutes is longer than a half hour.
- Answers vary. Sample response: No. If Clare and Noah will not finish together, then all three won't, either.

### 2. Problem 2 Statement

Diego has 48 chocolate chip cookies, 64 vanilla cookies, and 100 raisin cookies for a bake sale. He wants to make bags that have all three cookie flavours and the same number of each flavour per bag.

- How many bags can he make without having any cookies left over?
- Find the another solution to this problem.

### Solution

(The two solutions could swap places.)

- 4 bags with 12 chocolate chip cookies, 16 vanilla cookies, and 25 raisin cookies.
- 2 bags with 24 chocolate chip cookies, 32 vanilla cookies, and 50 raisin cookies.

### 3. Problem 3 Statement

- a. Find the product of 12 and 8.
- b. Find the highest common factor of 12 and 8.
- c. Find the least common multiple of 12 and 8.
- d. Find the product of the highest common factor and the least common multiple of 12 and 8.
- e. What do you notice about the answers to question 1 and question 4?
- f. Choose 2 other numbers and repeat the previous steps. Do you get the same results?

#### Solution

- a.  $12 \times 8 = 96$
- b. 4 is the highest common factor.
- c. 24 is the least common multiple.
- d.  $4 \times 24 = 96$ .
- e. The answers are the same.
- f. Answers vary. Yes, the results are the same.

### 4. Problem 4 Statement

- a. Given the point (5.5,-7), name a second point so that the two points form a vertical line segment. What is the length of the line segment?
- b. Given the point (3,3.5), name a second point so that the two points form a horizontal line segment. What is the length of the line segment?

#### Solution

- a. Answers vary. Any answer that has an  $x$ -coordinate of 5.5 will form a vertical line segment. The length will be the distance between -7 and the other  $y$ -coordinate.
- b. Answers vary. Any answer that has a  $y$ -coordinate of 3.5 will form a horizontal line segment. The length will be the distance between 3 and the other  $x$ -coordinate.

### 5. Problem 5 Statement

Find the value of each expression mentally.

---

- a.  $\frac{1}{2} \times 37 - \frac{1}{2} \times 7$
- b.  $3.5 \times 40 + 3.5 \times 60$
- c.  $999 \times 5$

**Solution**

- a. 15
- b. 350
- c. 4995



© These materials were derived and adapted from Illustrative Mathematics's IM 6–8 Math™. IM 6–8 Math was originally developed by Open Up Resources and authored by Illustrative Mathematics®, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0) <https://creativecommons.org/licenses/by/4.0/>. OUR's 6–8 Math Curriculum is available at <https://openupresources.org/math-curriculum/>. Adaptations and updates to IM 6–8 Math™ are copyright 2019 by Illustrative Mathematics®, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). Further adaptations have been made by MEI.