Lesson 3: Exploring circumference

Goals

- Comprehend the word "pi" and the symbol π to refer to the constant of proportionality between the diameter and circumference of a circle.
- Create and describe (in writing) graphs that show measurements of circles.
- Generalise that the relationship between diameter and circumference is proportional and that the constant of proportionality is a little more than 3.

Learning Targets

- I can describe the relationship between circumference and diameter of any circle.
- I can explain what π means.

Lesson Narrative

In this lesson, students discover that there is a proportional relationship between the diameter and circumference of a circle. They use their knowledge from the previous unit on proportionality to estimate the constant of proportionality. Then they use the constant to compute the diameter given the circumference (and vice versa) for different circles. We define π as the value of the constant and discuss various commonly used approximations. In the next lesson, students use various approximations for pi to do computations. Also, relating the circumference to the radius is saved for the next lesson.

Determining that the relationship between the circumference and diameter of circles is proportional is an example of looking for and making use of structure.

Building On

- Measure and estimate lengths in standard units.

Addressing

- Know the formulae for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- Recognise and represent proportional relationships between quantities.
- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate grid and observing whether the graph is a straight line through the origin.

Building Towards

- Know the formulae for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Instructional Routines

- Compare and Connect
- Notice and Wonder
- Think Pair Share

Required Materials

Cylindrical household items
Empty toilet paper roll
Measuring tapes

Required Preparation

Household items: collect circular or cylindrical objects of different sizes, with diameters from 3 cm to 25 cm. Each group needs 3 items of relatively different sizes. Examples include food cans, hockey pucks, paper towel tubes, paper plates, CD's. Record the diameter and circumference of the objects for your reference during student work time.

The empty toilet paper roll is for optional use during the warm-up as a demonstration tool.

You will need one measuring tape per group of 2--4 students. Alternatively, you could use rulers and string.

Student Learning Goals

Let's explore the circumference of circles.

3.1 Which Is Greater?

Warm Up: 5 minutes

The purpose of this warm-up is to help students visualise circumference as a linear measurement, in preparation for examining the relationship between diameter and circumference in the next activity. Some students may be able to imagine unrolling the tube into a rectangle in order to compare its length and width. Other students may benefit from hands on experience with a real toilet paper tube.

Instructional Routines

- Think Pair Share

Launch

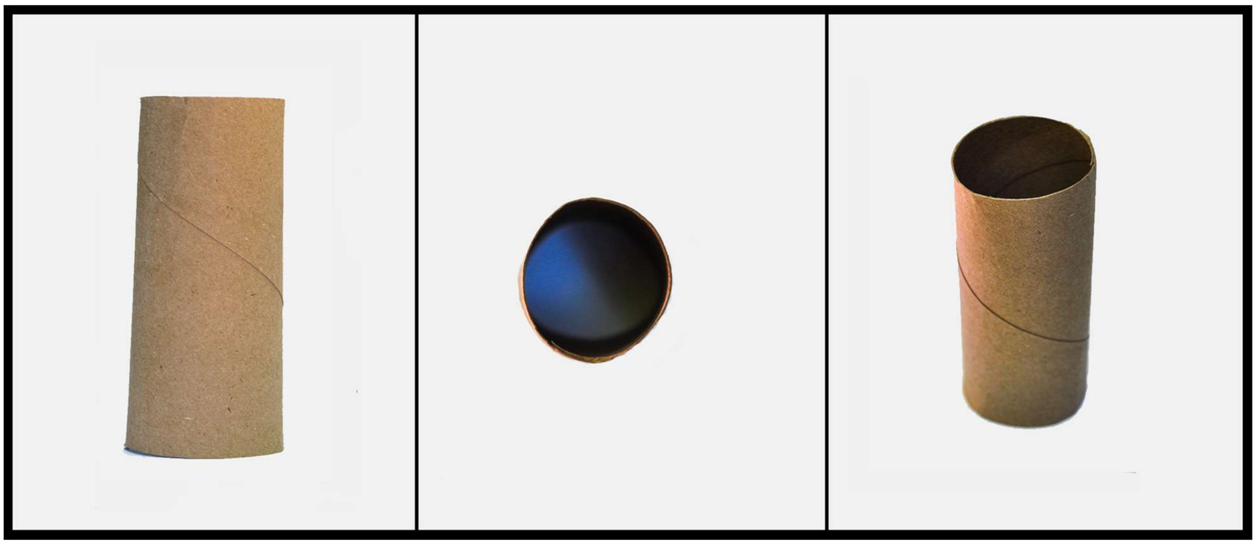
Arrange students in groups of 2. Display the image for all to see. Ask students to indicate when they have reasoning to support their response. Give students 1 minute of quiet think time and then time to share their thinking with their group.

Anticipated Misconceptions

Students may not understand what is meant by the height of the tube because it can sit in two different ways. Point these students to the first picture of the tube, and ask them to identify the height as the tube is sitting in that picture.

Student Task Statement

Clare wonders if the height of the toilet paper tube or the distance around the tube is greater. What information would she need in order to solve the problem? How could she find this out?



Student Response

Clare needs to measure the length of the tube and the distance around. To find the distance around she could measure the tube with a flexible measuring tape, or cut and flatten the tube.

Activity Synthesis

Poll students on which length they think is greater. Consider re-displaying the image for reference while students are explaining what they saw. To involve more students in the conversation, consider asking some of the following questions:

- What was important to you when making your decision?
- Did anyone think about the measurements in a different way?
- Do you agree or disagree? Why?

- What information do you think would help you make a better decision? If an actual toilet paper tube is available, demonstrate measuring the tube by unrolling the tube to measure the circumference and height.

It turns out that for a standard toilet paper tube, the distance around is greater than the distance from top to bottom.

3.2 Measuring Circumference and Diameter

25 minutes (there is a digital version of this activity)

In this activity, students measure the diameter and circumference of different circular objects and plot the data on a coordinate grid, recalling the structure of the first activity in this unit where they measured different parts of squares. Students use a graph in order to conjecture an important relationship between the circumference of a circle and its diameter. They notice that the two quantities appear to be proportional to each other. Based on the graph, they estimate that the constant of proportionality is close to 3 (a table of values shows that it is a little bigger than 3). This is their first estimate of pi.

This activity provides good, level-appropriate evidence that there is a constant of proportionality between the circumference of a circle and its diameter. Students will investigate this relationship further in school, using polygons inscribed in a circle for example.

To measure the circumference, students can use a flexible measuring tape or a piece of string wrapped around the object and then measure with a ruler. As students measure, encourage them to be as precise as possible. Even so, the best precision we can expect for the proportionality constant in this activity is “around 3” or possibly “a little bit bigger than 3.” This could be a good opportunity to talk about how many digits in the answer is reasonable. To get a good spread of points on the graph, it is important to use circles with a wide variety of diameters, from 3 cm to 25 cm. If there are points that deviate noticeably from the overall pattern, discuss how measurement error plays a factor.

As students work, monitor and select students who notice that the relationship between diameter and circumference appears to be proportional, and ask them to share during the whole-group discussion.

If students are using the digital version of the activity, they don’t necessarily need to measure physical objects, but we recommend they do so anyway.

Instructional Routines

- Compare and Connect
- Notice and Wonder

Launch

Arrange students in groups of 2–4. Distribute 3 circular objects and measuring tapes or string and rulers to each group. Especially if using string and rulers, it may be necessary to demonstrate the method for measuring the circumference.

Ask students to complete the first two questions in their group, and then gather additional information from two other groups (who measured *different* objects) for the third question.

If using the digital activity, students can work in groups of 2–4. They only need the applet to generate data for their investigation.

Action and Expression: Internalise Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organisation and problem solving. Check in with students after they have measured their first circular object to ensure they have a viable method for measuring circumference.
Supports accessibility for: Organisation; Attention

Anticipated Misconceptions

Students may try to measure the diameter without going across the widest part of the circle, or may struggle with measuring around the circumference. Mentally check that their measurements divide to get approximately 3 or compare with your own prepared table of data and prompt them to re-measure when their measurements are off by too much. If the circular object has a rim or lip, this could help students keep the measuring tape in place while measuring the circumference.

If students are struggling to see the proportional relationship, remind them of recent examples where they have seen similar graphs of proportional relationships. Ask them to estimate additional diameter-circumference pairs that would fit the pattern shown in the graph. Based on their graphs, do the values of the circumferences seem to relate to those of the diameters in a particular way? What seems to be that relationship?

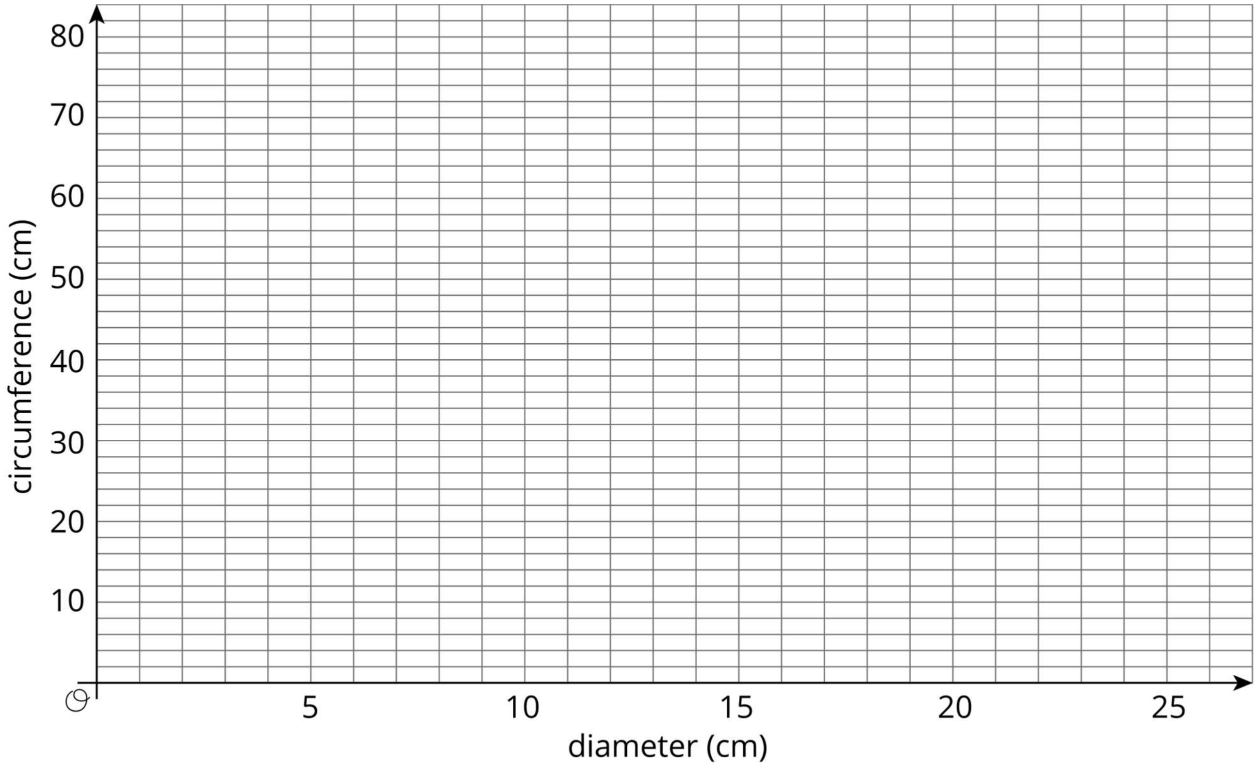
Student Task Statement

Your teacher will give you several circular objects.

1. Measure the diameter and the circumference of the circle in each object to the nearest tenth of a centimetre. Record your measurements in the table.

object	diameter (cm)	circumference (cm)

2. Plot the diameter and circumference values from the table on the coordinate grid. What do you notice?



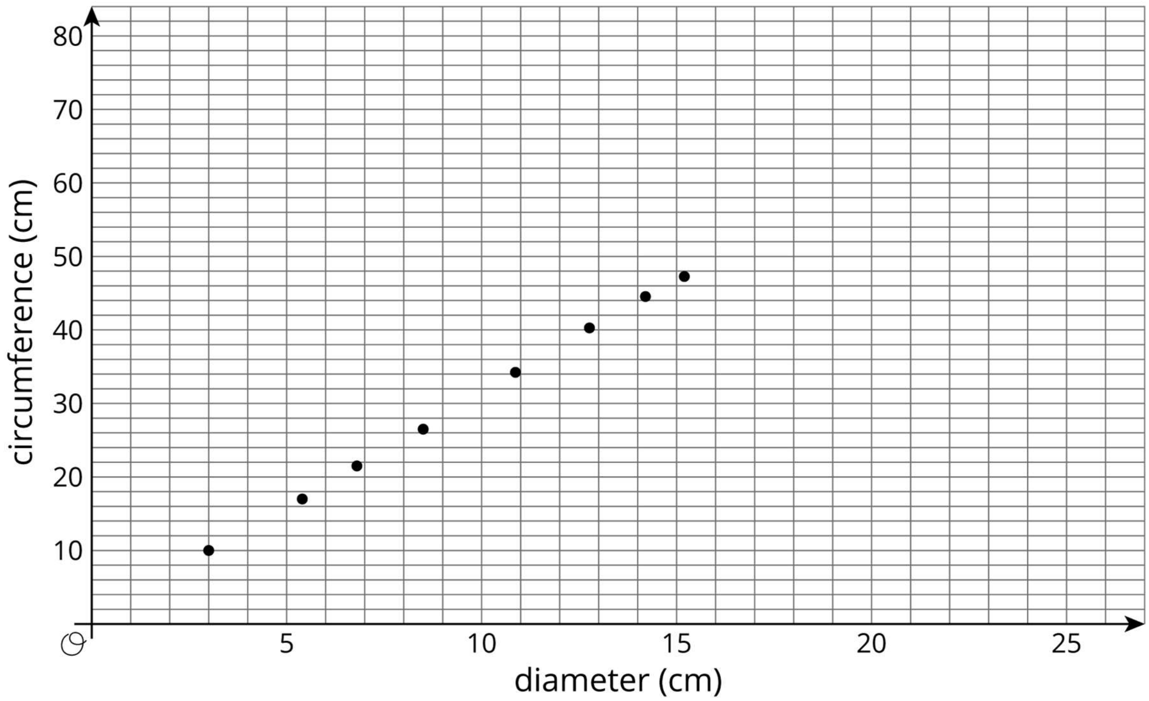
3. Plot the points from two other groups on the same coordinate grid. Do you see the same pattern that you noticed earlier?

Student Response

1. Answers vary. Sample response:

object	diameter (cm)	circumference (cm)
soup can	6.8	21.5
tomato paste can	5.4	17
tuna can	8.5	26.5

2. Answers vary. Sample response: Three of the points on the following graph.



The points look like they are close to lying on a straight line through $(0,0)$.

3. Answers vary. Sample response: Six more points on the previous graph.

Activity Synthesis

Display a graph for all to see, and plot some of the students' measurements for diameter and circumference. In cases where the same object was measured by multiple groups, include only one measurement per object. Ask students to share what they notice and what they wonder about the graph.

- Students may notice that the measurements appear to lie on a line (or are close to lying on a line) that goes through $(0,0)$. If students do not mention a proportional relationship, make this explicit.
- Students may wonder why some points are not on the line or what the constant of proportionality is.

Invite students to estimate the constant of proportionality. From the graph, it may be difficult to make a better estimate than about 3. Another strategy is to add a column to the table, and calculate the quotient of the circumference divided by the diameter for each row. For example,

object	diameter (cm)	circumference (cm)	circumference ÷ diameter
soup can	6.8	21.5	3.2
tomato paste can	5.4	17	3.1
tuna can	8.5	26.5	3.1

Ask students why these numbers might not be *exactly* the same (measurement error, rounding). Use the average of the quotients, rounded to one or two decimal places, to come up with a "working value" of the constant of proportionality: for the numbers in the sample table above, 3.1 would be an appropriate value. This class-generated proportionality constant will be used in the next activity, to help students understand how to compute circumference from diameter and vice versa. There's no need to mention *pi* or its usual approximations yet.

Time permitting, it could be worth discussing accuracy of measurements for circumference and diameter. Measuring the diameter to the nearest tenth of a centimetre can be done pretty reliably with a ruler. Measuring the circumference of a circle to the nearest tenth of a centimetre may or may not be reliable, depending on the method used. Wrapping a flexible measuring tape around the object is likely the most accurate method for measuring the circumference of a circle.

Representing, Conversing: Compare and Connect. After measuring their circular objects and recording the data, students should be prepared to share their written or digital graphs and tables. Ask students to tour each other's displays with their group, and say, "As you tour the room, look for graphs/tables with familiar patterns or relationships. Discuss with your group what you can conclude and select one person to record what you notice as you go." Circulate and amplify any conversation involving recurring patterns of "around 3" or "a little more than 3." Complete the task with a class discussion, asking each group share out what they found that was similar or relationships they saw in numbers or plotted points.
Design Principle(s): Cultivate conversation; Maximise meta-awareness

3.3 Calculating Circumference and Diameter

10 minutes

In this task, students use the constant of proportionality they estimated in the previous task to calculate circumferences of circles given the diameter and vice versa. The purpose is to reinforce the proportional relationship between circumference c and diameter d and use the formula $c = kd$, where k is about 3 or 3.1 (the value agreed upon in the previous task).

If there is not sufficient time to allow students to work through all of the calculations, consider dividing up the work, assigning one circle to each student or group of students. It is important to save enough time for the discussion about the meaning and value of π at the end of this activity.

Before students begin the task, a digital applet found at http://ggbm.at/H8UuD96V can be used to reinforce the constant of proportionality.

Launch

Ask students to label these measurements given in the table on the picture of the circles. Instruct students to use the constant of proportionality the class estimated in the previous activity. Give students quiet work time followed by whole-class discussion.

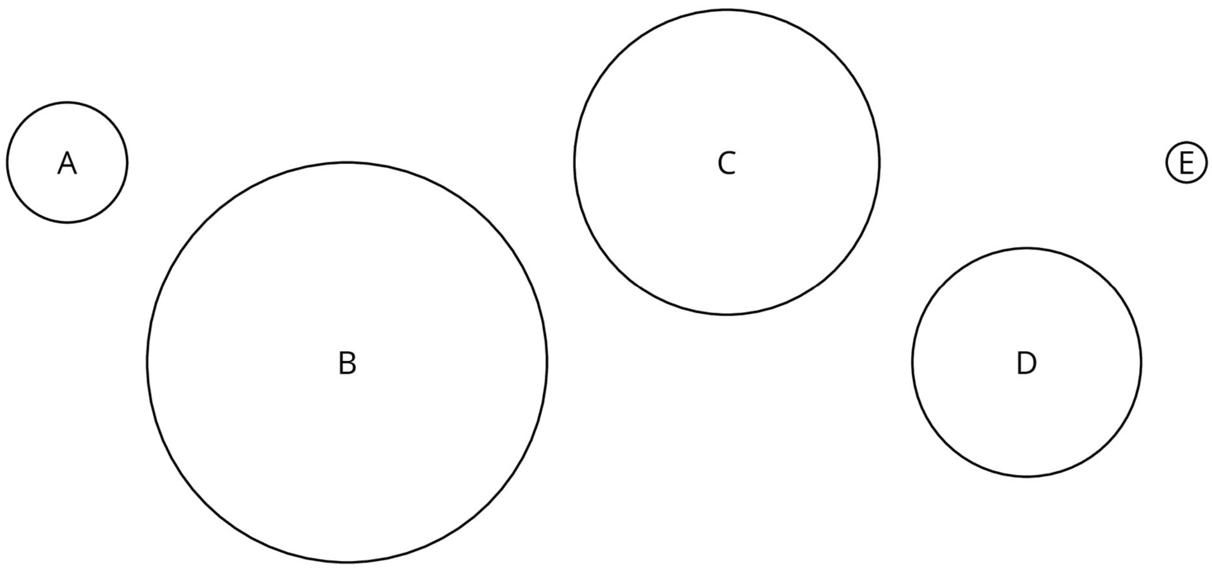
Anticipated Misconceptions

Some students may multiply the circumference by the constant of proportionality instead of dividing by it. Prompt them to consider whether the diameter can be longer than the circumference of a circle.

Some students may struggle to divide by 3.1 if that is the constant of proportionality decided on in the previous lesson. Ask these students if they could use an easier number as their constant, and allow them to divide by 3 instead. Then ask them how their answer would have changed if they divided by 3.1.

Student Task Statement

Here are five circles. One measurement for each circle is given in the table.



Use the constant of proportionality estimated in the previous activity to complete the table.

	diameter (cm)	circumference (cm)
circle A	3	
circle B	10	
circle C		24
circle D		18
circle E	1	

Student Response

Answers vary. Sample response using 3.1 for the constant of proportionality

	diameter (cm)	circumference (cm)
circle A	3	9.3
circle B	10	31
circle C	7.7	24
circle D	5.8	18
circle E	1	3.1

Are You Ready for More?

The circumference of Earth is approximately 40 000 km. If you made a circle of wire around the globe, that is only 10 metres (0.01 km) longer than the circumference of the globe, could a flea, a mouse, or even a person creep under it?

Student Response

Yes! Each metre added to the diameter of a circle adds about 3.1 metres to the circumference of the circle. So if the circumference of Earth is increased by 10 metres, this means that a little more than 3 metres have been added to the diameter. So there would be about 1.5 metres of distance between the rope and Earth, making it easy for a flea, mouse, or person to go under the rope!

Activity Synthesis

Display the table from the task statement for all to see. Ask students to share their answers and write these in the table, resolving any discrepancies.

Ask students if they noticed any connections between the tables and the graphs from the previous activity. Students will have used the equation $C = kd$, possibly without realising it, to fill in the missing information so it is important for them to make this connection. To reinforce this connection, add the points from the table they just completed to the graph from the previous activity. Point out that the constant of proportionality appears in the table as the unit rate in the row with diameter 1 cm.

Ask if students have heard of the number **pi** or seen the symbol π. Define π as the exact value of the constant of proportionality for this relationship. Explain that the exact value of π is a decimal with infinitely many digits and no repeating pattern, so an approximation is often used. Frequently used approximations for π include $\frac{22}{7}$, 3.14, and 3.14159, but none of these are exactly equal to π.

Lesson Synthesis

The main ideas are:

- Diameter and circumference are proportional to each other.
- We can find one from the other using the relationship $C = kd$, where k is the proportionality constant, which we have estimated as 3.1 (average from task "Measuring Circumference and Diameter"). For example, for a circle of diameter 4 cm, we have $3.1 \times 4 \approx 12.4$, so the circle has a circumference approximately equal to 12.4 cm.
- The exact constant of proportionality is called π. Frequently used approximations for π are $\frac{22}{7}$, 3.14, and 3.14159, but none of these are exactly π.

3.4 Identifying Circumference and Diameter

Cool Down: 5 minutes

Student Task Statement

Select **all** the pairs that could be reasonable approximations for the diameter and circumference of a circle. Explain your reasoning.

1. 5 metres and 22 metres.
2. 19 inches and 60 inches.
3. 33 centimetres and 80 centimetres.

Student Response

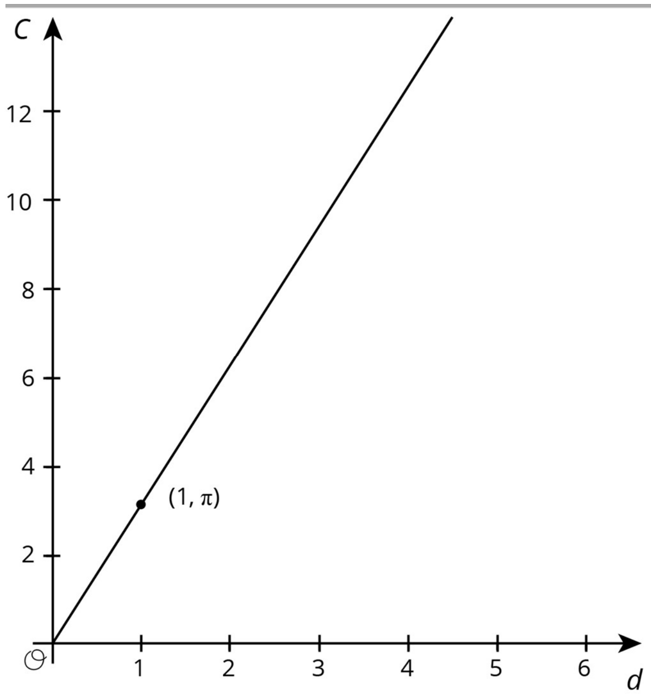
Only B could be the diameter and circumference of a circle, because the quotient is a little more than 3. The others are too far off to be correct.

1. Does not work, because $22 \div 5 > 4$.
2. Does work, because $60 \div 19 \approx 3.158$.
3. Does not work, because $80 \div 33 < 2.5$.

Student Lesson Summary

There is a proportional relationship between the diameter and circumference of any circle. That means that if we write C for circumference and d for diameter, we know that $C = kd$, where k is the constant of proportionality.

The exact value for the constant of proportionality is called **π**. Some frequently used approximations for π are $\frac{22}{7}$, 3.14, and 3.14159, but none of these is exactly π.



We can use this to estimate the circumference if we know the diameter, and vice versa. For example, using 3.1 as an approximation for π, if a circle has a diameter of 4 cm, then the circumference is about $(3.1) \times 4 = 12.4$ or 12.4 cm.

The relationship between the circumference and the diameter can be written as

$$C = \pi d$$

Glossary

- pi (π)

Lesson 3 Practice Problems

1. **Problem 1 Statement**

   Diego measured the diameter and circumference of several circular objects and recorded his measurements in the table.

object	diameter (cm)	circumference (cm)
£2 coin	3	10
flying disc	23	28
jar lid	8	25
flower pot	15	48

   One of his measurements is inaccurate. Which measurement is it? Explain how you know.

   Solution

The measurement for the flying disc is very inaccurate. It should be about 3 times the diameter (or a little more).

2. **Problem 2 Statement**

Complete the table. Use one of the approximate values for π discussed in class (for example 3.14, $\frac{22}{7}$, 3.1416). Explain or show your reasoning.

object	diameter	circumference
hula hoop	35 in	
circular pond		556 ft
magnifying glass	5.2 cm	
car tyre		71.6 in

Solution

object	diameter	circumference
hula hoop	35 in	110 in
circular pond	177 ft	556 ft
magnifying glass	5.2 cm	16 cm
car tyre	22.8 in	71.6 in

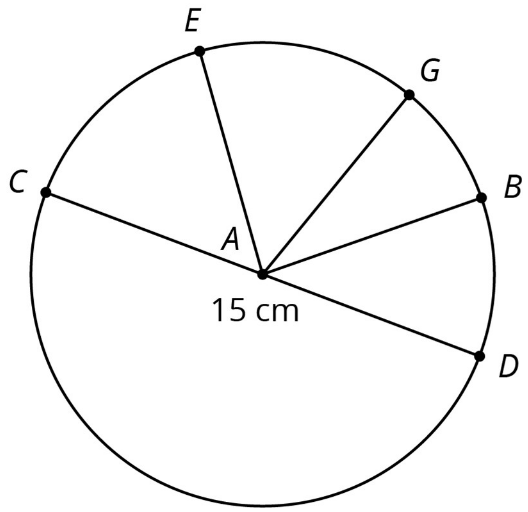
The constant of proportionality is about 3.14. The given diameters are multiplied by 3.14 to find the missing circumferences. The given circumferences are divided by 3.14 to find the missing diameters. Both the missing circumferences and the missing diameters have been rounded.

3. **Problem 3 Statement**

A is the centre of the circle, and the length of CD is 15 centimetres.

a. Name a segment that is a radius. How long is it?

b. Name a segment that is a diameter. How long is it?



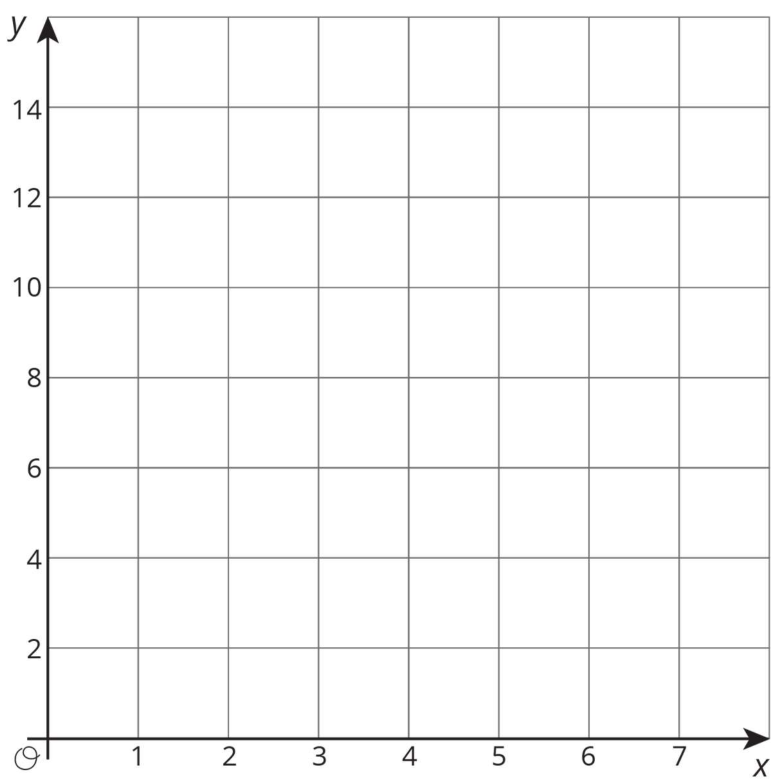
Solution

a. Answers vary. Sample responses: AC, AD, AB, AE, AG, 7.5 cm

b. CD, 15 cm

4. **Problem 4 Statement**

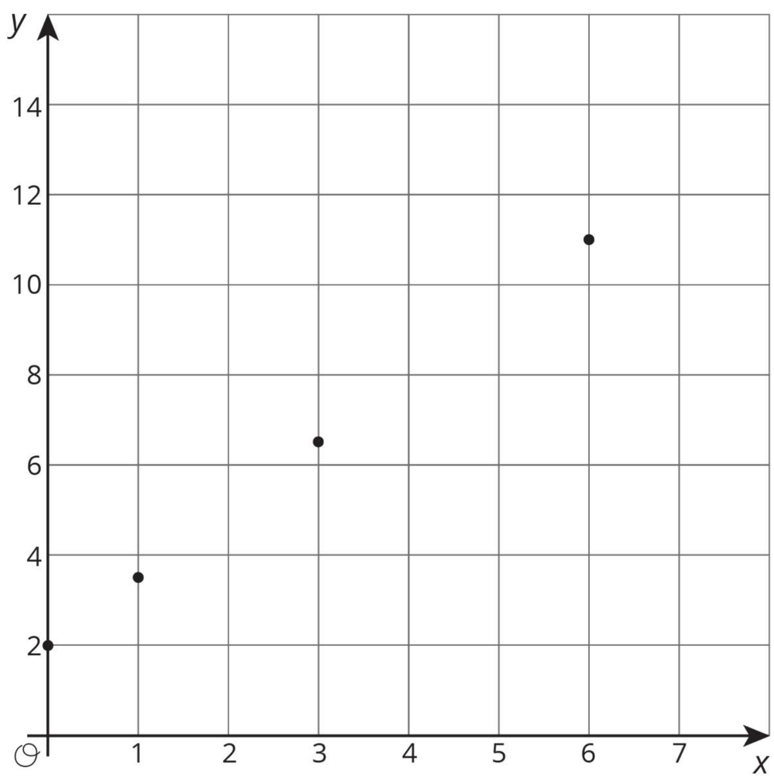
a. Consider the equation $y = 1.5x + 2$. Find four pairs of x and y values that make the equation true. Plot the points (x, y) on the coordinate grid.



b. Based on the graph, can this be a proportional relationship? Why or why not?

Solution

a. Answers vary. Sample response:



b. Answers vary. Sample response: No, this relationship could not be proportional because the graph does not go through (0,0).