Lesson 8: Finding unknown side lengths

Goals

- Calculate unknown side lengths of a right-angled triangle by using Pythagoras' theorem, and explain (orally) the solution method.
- Label the "shorter sides" and "hypotenuse" on a diagram of a right-angled triangle.

Learning Targets

- If I know the lengths of two sides, I can find the length of the third side in a right-angled triangle.
- When I have a right-angled triangle, I can identify which side is the hypotenuse and which sides are the shorter sides.

Lesson Narrative

The purpose of this lesson is to use Pythagoras' theorem to find unknown side lengths of a right-angled triangle.

Addressing

- Apply Pythagoras' theorem to determine unknown side lengths in right-angled triangles in real-world and mathematical problems in two and three dimensions.

Building Towards

- Understand and apply Pythagoras' theorem.

Instructional Routines

- Clarify, Critique, Correct
- Discussion Supports
- Think Pair Share
- Which One Doesn't Belong?

Student Learning Goals

Let's find missing side lengths of right-angled triangles.

8.1 Which One Doesn't Belong: Equations

Warm Up: 5 minutes

The purpose of this warm-up is to prime students for solving equations that arise while using Pythagoras' theorem.

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the equations for all to see. Ask students to indicate when they have noticed one that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reason why a particular equation does not belong and together find at least one reason each question doesn't belong.

Student Task Statement

Which one doesn't belong?

$3^2 + b^2 = 5^2$

$b^2 = 5^2 - 3^2$

$3^2 + 5^2 = b^2$

$3^2 + 4^2 = 5^2$

Student Response

Answers vary. Sample responses:

$3^2 + b^2 = 5^2$: the only one where b is not isolated

$b^2 = 5^2 - 3^2$: the only one with one term on the left and two terms on the right, the only one with subtraction

$3^2 + 5^2 = b^2$: the only one that is not based on a 3-4-5 Pythagorean triple

$3^2 + 4^2 = 5^2$: the only one with all numbers

Activity Synthesis

Ask each group to share one reason why a particular equation does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given make sense.

8.2 Which One Is the Hypotenuse?

5 minutes

This activity helps students identify the hypotenuse in right-angled triangles in different orientations.

Instructional Routines

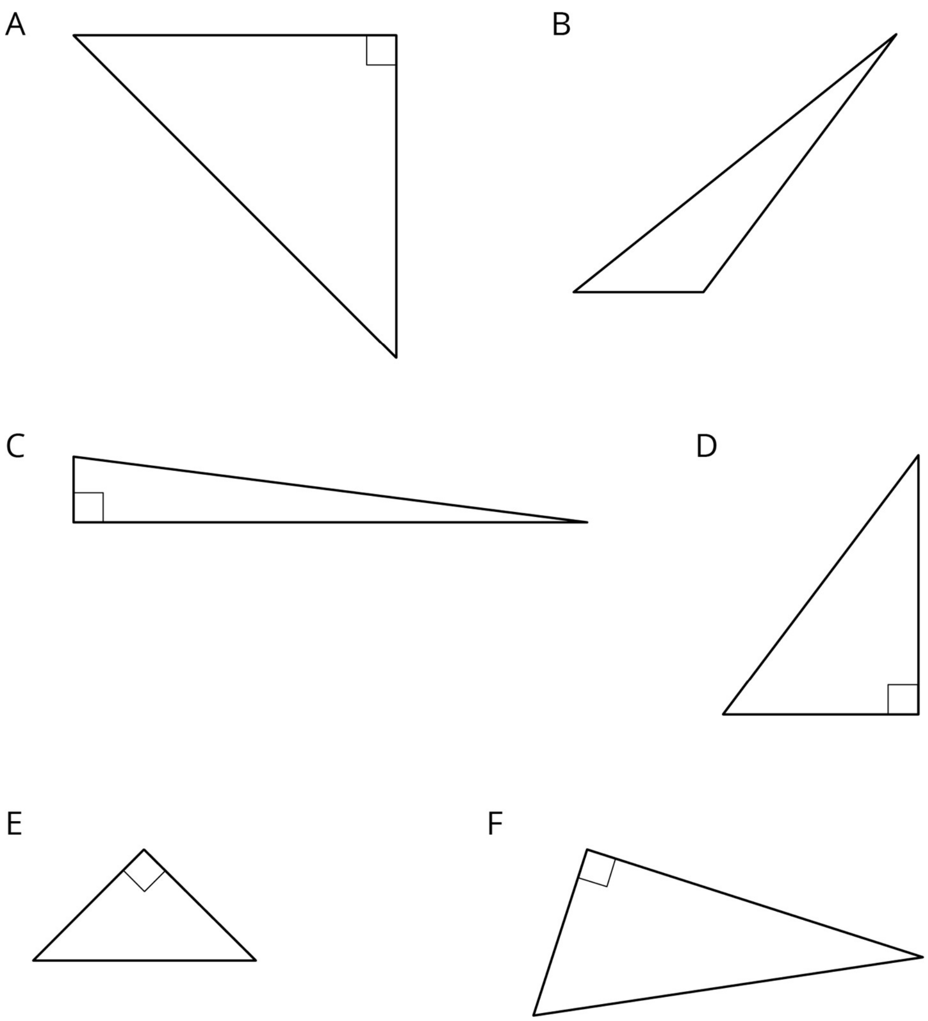
- Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time and then have them compare with a partner. Follow with a whole-class discussion.

Student Task Statement

Label all the hypotenuses with c.



Student Response

All triangles except Triangle B should have a c on the side opposite the right angle. Triangle B is not a right-angled triangle, therefore it does not have a hypotenuse.

Mathematics Education Innovation

Activity Synthesis

Ask students which triangles are right-angled triangles, and then ask them which side is the hypotenuse for each one. Ask, “In a right-angled triangle, does it matter which is a and which is b?” (No.)

Speaking, Listening: Discussion Supports. As students share, press for details in students’ reasoning by asking how they know the side they selected is the hypotenuse. Listen for and amplify the language students use to describe the important features of the hypotenuse (e.g., longest side of a right-angled triangle, side opposite the right angle). Then ask students to explain why Triangle B does not have a hypotenuse. This will support rich and inclusive discussion about strategies for identifying the hypotenuse of a right-angled triangle.
Design Principle(s): Support sense-making

8.3 Find the Missing Side Lengths

20 minutes

The purpose of this activity is to give students practice finding missing side lengths in a right-angled triangle using Pythagoras’ theorem.

Instructional Routines

- Clarify, Critique, Correct
- Think Pair Share

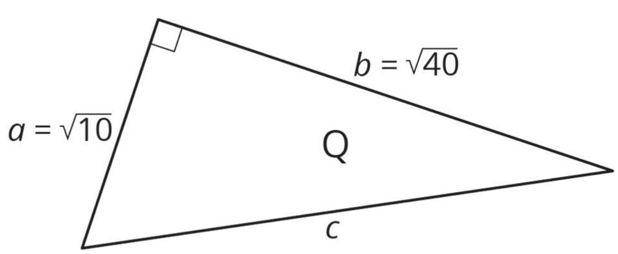
Launch

Arrange students in groups of 2. Give students 10 minutes of quiet work time and then have them compare with a partner. If partners disagree about any of their answers, ask them to explain their reasoning to one another until they reach agreement. Follow with a whole-class discussion.

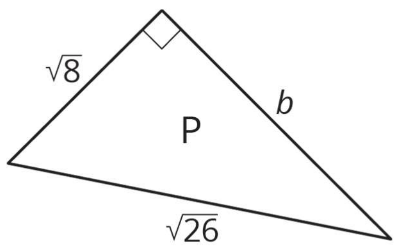
Representation: Internalise Comprehension. Activate or supply background knowledge by displaying Pythagoras’ theorem with a labelled diagram. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

1. Find c.



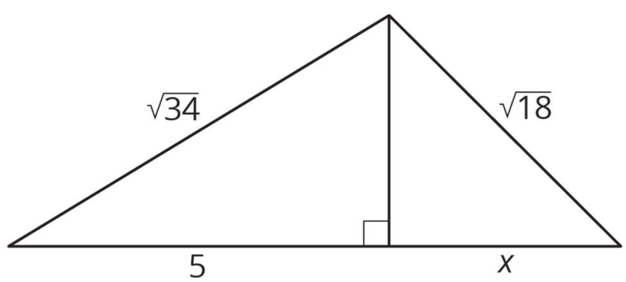
2. Find b.



3. A right-angled triangle has sides of length 2.4 cm and 6.5 cm. What is the length of the hypotenuse?

4. A right-angled triangle has a side of length $\frac{1}{4}$ and a hypotenuse of length $\frac{1}{3}$. What is the length of the other side?

5. Find the value of x in the figure.

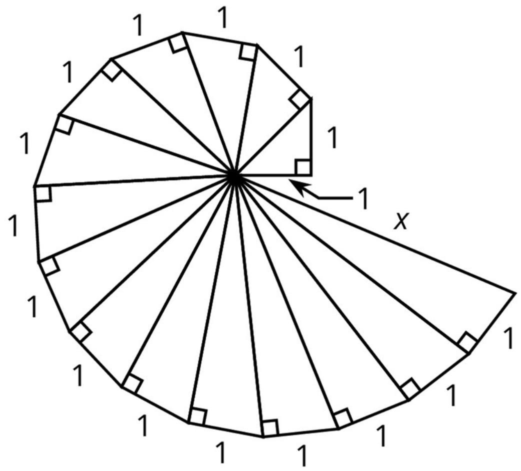


Student Response

1. $\sqrt{50}$
2. $\sqrt{18}$
3. $\sqrt{48.01}$
4. $\sqrt{\frac{7}{144}}$
5. $x = 3$

Are You Ready for More?

The spiral in the figure is made by starting with a right-angled triangle with both shorter sides measuring one unit each. Then a second right-angled triangle is built with one shorter side measuring one unit, and the other shorter side being the hypotenuse of the first triangle. A third right-angled triangle is built on the second triangle's hypotenuse, again with the other shorter side measuring one unit, and so on.



Find the length, x, of the hypotenuse of the last triangle constructed in the figure.

Student Response

We can repeatedly apply Pythagoras' theorem. The first hypotenuse equals $\sqrt{2}$, since $\left(\sqrt{2}\right)^2 = 1^2 + 1^2$. The second right-angled triangle has shorter sides 1 and $\sqrt{2}$, so has a hypotenuse of $\sqrt{3}$, since $\left(\sqrt{3}\right)^2 = \left(\sqrt{2}\right)^2 + 1^2$. This pattern continues, with the next hypotenuses having length $\sqrt{4}$, the $\sqrt{5}$, etc. By counting until the end, we find that the 15th and last hypotenuse has a length x equal to $\sqrt{16}$, so $x = 4$.

Activity Synthesis

Ask students to share how they found the missing side lengths. If students drew triangles for the two questions that did not have an image, display a few of these for all to see, noting any differences between them. For example, students may have drawn triangles with different orientations or labelled different sides as a and b.

For the last question, ask students to say what they did first to try and solve for x. For example, while many students may have found the length of the unknown altitude first and then used that value to find x, others may have set up the equation $34 - 5^2 = 18 - x^2$.

Point out that when you know two sides of a right-angled triangle, you can always find the third by using Pythagoras' identity $a^2 + b^2 = c^2$. Remind them that it is important to keep track of which side is the hypotenuse.

Reading, Writing, Speaking: Clarify, Critique, Correct. Before students share their method for the questions that did not have an image, present an incorrect solution based on a common error related to labelling the sides of a right-angled triangle. For the right-angled triangle with a side of length $\frac{1}{4}$ and a hypotenuse of length $\frac{1}{3}$, draw a right-angled triangle with the shorter sides labelled $\frac{1}{4}$ and $\frac{1}{3}$. Provide an incorrect explanation such as: "I know that $a = \frac{1}{4}$ and $b = \frac{1}{3}$, so when I use Pythagoras' Theorem, I get the equation $\left(\frac{1}{4}\right)^2 + \left(\frac{1}{3}\right)^2 = c^2$." Ask

students to identify the error, critique the reasoning, and revise the original statement. As students discuss in partners, listen for students who clarify the meaning of the hypotenuse and identify c as the length of the hypotenuse in Pythagoras' Theorem. This routine will engage students in meta-awareness as they critique and correct a common error when labelling the sides of a right-angled triangle.
Design Principles(s): Cultivate conversation; Maximise meta-awareness

Lesson Synthesis

The purpose of this discussion is to check that students understand Pythagoras' theorem and how it can be used to determine information about triangles. Ask students to draw a right-angled triangle and label 2 of the 3 sides. Tell them to swap triangles with another student, solve for the missing length, then swap back to check the other person's work. Select a few groups to share their triangles and, if possible, display them for all to see while sharing how they solved for the unknown length.

8.4 Could be the Hypotenuse, Could be a Shorter side

Cool Down: 5 minutes

Student Task Statement

A right-angled triangle has sides of length 3, 4, and x.

1. Find x if it is the hypotenuse.

2. Find x if it is one of the shorter sides.

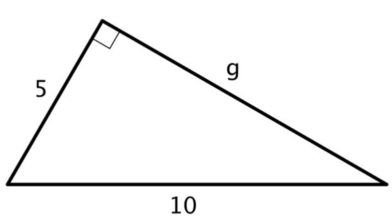
Student Response

1. $x = \sqrt{25}$ or $x = 5$

2. $x = \sqrt{7}$

Student Lesson Summary

There are many examples where the lengths of two shorter sides of a right-angled triangle are known and can be used to find the length of the hypotenuse with Pythagoras' theorem. Pythagoras' theorem can also be used if the length of the hypotenuse and one shorter side is known, and we want to find the length of the other shorter side. Here is a right-angled triangle, where one shorter side has a length of 5 units, the hypotenuse has a length of 10 units, and the length of the other shorter side is represented by g.



Start with $a^2 + b^2 = c^2$, make substitutions, and solve for the unknown value. Remember that c represents the hypotenuse: the side opposite the right angle. For this triangle, the hypotenuse is 10.

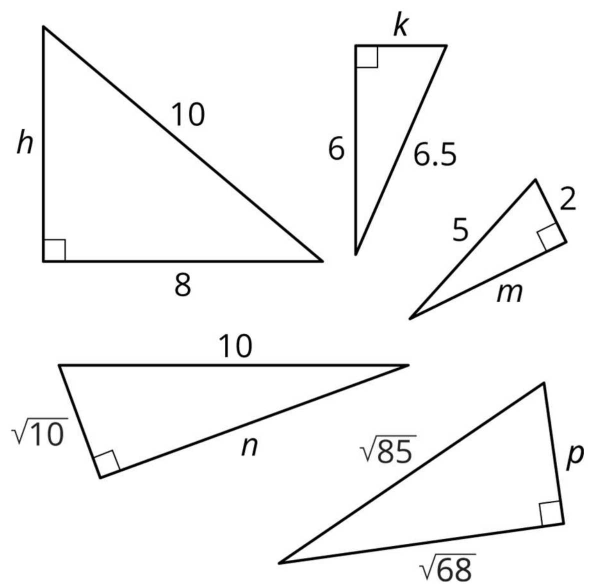
$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
5^2 + g^2 &= 10^2 \\
g^2 &= 10^2 - 5^2 \\
g^2 &= 100 - 25 \\
g^2 &= 75 \\
g &= \sqrt{75}
\end{aligned}
$$

Use estimation strategies to know that the length of the other shorter side is between 8 and 9 units, since 75 is between 64 and 81. A calculator with a square root function gives $\sqrt{75} \approx 8.66$.

Lesson 8 Practice Problems

1. **Problem 1 Statement**

   Find the exact value of each variable that represents a side length in a right-angled triangle.



Solution

$h = 6$ (because $100 - 64 = 36$ and $\sqrt{36} = 6$)

$k = 2.5$ (because $42.25 - 36 = 6.25$ and $\sqrt{6.25} = 2.5$)

$m = \sqrt{21}$ because $25 - 4 = 21$

$n = \sqrt{90}$ because $100 - 10 = 90$

$p = \sqrt{17}$ because $85 - 68 = 17$

2. **Problem 2 Statement**

A right-angled triangle has side lengths of a, b, and c units. The longest side has a length of c units. Complete each equation to show three relations among a, b, and c.

- $c^2 =$
- $a^2 =$
- $b^2 =$

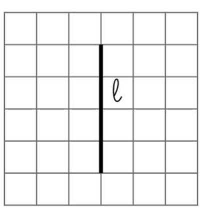
Solution

- $c^2 = a^2 + b^2$ or $c^2 = b^2 + a^2$
- $a^2 = c^2 - b^2$
- $b^2 = c^2 - a^2$

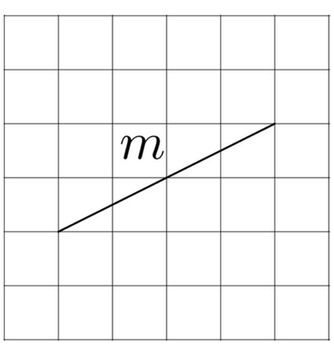
3. **Problem 3 Statement**

What is the exact length of each line segment? Explain or show your reasoning. (Each grid square represents 1 square unit.)

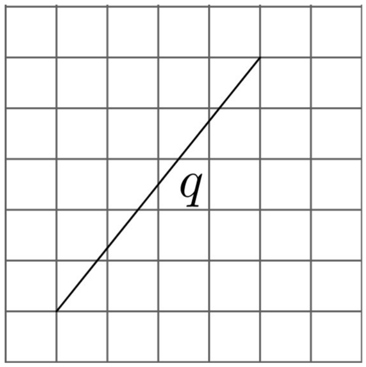
a.



b.



c.



Solution

a. 4 units. The segment is along the grid lines, so count the squares.

b. $\sqrt{20}$ because $4^2 + 2^2 = 20$

c. $\sqrt{41}$ because $4^2 + 5^2 = 41$

4. **Problem 4 Statement**

In 2015, there were roughly 1×10^6 high school football players and 2×10^3 professional football players in the United States. About how many times more high school football players are there? Explain how you know.

Solution

There are approximately 500 times more high school football players. $\frac{1 \times 10^6}{2 \times 10^3} = 0.5 \times 10^3 = 5 \times 10^2$

5. **Problem 5 Statement**

Evaluate:

a. $\left(\frac{1}{2}\right)^3$

b. $\left(\frac{1}{2}\right)^{-3}$

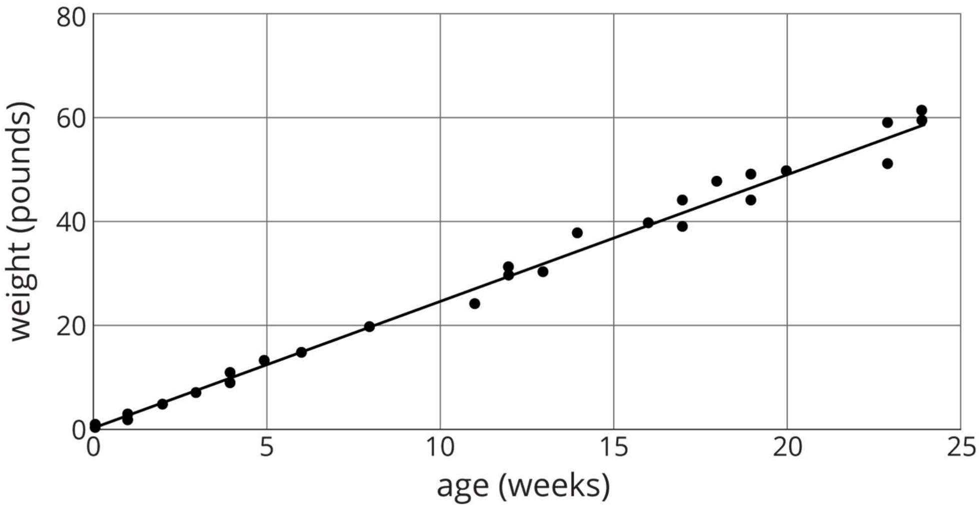
Solution

a. $\frac{1}{8}$

b. 8

6. **Problem 6 Statement**

Here is a scatter plot of weight vs. age for different Dobermans. The model, represented by $y = 2.45x + 1.22$, is graphed with the scatter plot. Here, x represents age in weeks, and y represents weight in pounds.



a. What does the gradient mean in this situation?

b. Based on this model, how heavy would you expect a newborn Doberman to be?

Solution

a. The gradient means that a Doberman can be expected to gain 2.45 pounds per week.

b. 1.22 pounds (the y-intercept of the function).