Lesson 8: Calculating products of decimals

Goals

- Draw and label a diagram to check the answer to a decimal multiplication problem.
- Interpret a description (in written language) of a real-world situation involving multiplication of decimals, and write a multiplication problem to represent it.
- Use an algorithm to calculate the product of two decimals, and explain (orally) the solution method.

Learning Targets

- I can describe and apply a method for multiplying decimals.
- I know how to use a product of whole numbers to find a product of decimals.

Lesson Narrative

In this culminating lesson on multiplication, students continue to use the structure of base-ten numbers to make sense of calculations and consolidate their understanding of the themes from the previous lessons. They see that multiplication of decimals can be accomplished by:

- thinking of the decimals as products of whole numbers and fractions;
- writing the non-zero digits of the factors as whole numbers, multiplying them, and moving the digits in the product;
- representing the multiplication with an area diagram and finding partial products; and
- using a multiplication algorithm, the steps of which can be explained with the reasonings above.

Building On

- Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Building Towards

- Apply and extend previous understandings of arithmetic to algebraic expressions.

Instructional Routines

- Clarify, Critique, Correct

- Discussion Supports
- Number Talk
- Think Pair Share

Required Preparation

Some students might find it helpful to use graph paper to help them align the digits for vertical calculations (long multiplication). Consider having graph paper accessible for the activity: Calculating Products of Decimals.

Student Learning Goals

Let's multiply decimals.

8.1 Number Talk: Twenty Times a Number

Warm Up: 5 minutes

The purpose of this number talk is to have students see structure related to the distributive property in preparation for the problems using area diagrams they will solve in the lesson.

Instructional Routines

- Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Representation: Internalise Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organisation

Student Task Statement

Evaluate mentally.

20×5

$20 \times (0.8)$

$20 \times (0.04)$

$20 \times (5.84)$

Student Response

- 100

- 16
- 0.8
- 116.8

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to _____’s strategy?”
- “Do you agree or disagree? Why?”

Speaking: Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimise output (for explanation)

8.2 Calculating Products of Decimals

25 minutes

Students deepen and reinforce the ideas developed in previous activities: using area diagrams to find partial products, relating these partial products to the numbers in the algorithm, and using multiplication of whole numbers to find the product of decimals. By calculating products of decimals using column methods with whole numbers, students extend their understanding of multiplication to include multiplication of any pair of decimals.

Finally, with a range of methods for multiplying decimals in hand, students choose a method to solve a contextual problem. The application invites students to decide what mathematical operations to perform based on the context and then using the context to understand how to deal with the result of the complex calculations.

Instructional Routines

- Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Ask students to discuss and agree on each step before moving on to the next step. Give partners 8–10 minutes to complete the first three questions and follow with a brief whole-class discussion.

Ask students to explain the first question using fractions. If not brought up by students, highlight the idea that $(2.5) \times (1.2)$ is equivalent to $25 \times (0.1) \times 12 \times (0.1)$, which is the same as $25 \times 12 \times (0.01)$ (and also $25 \times 12 \times \frac{1}{100}$). The example shows that we can treat the non-zero digits of the factors as whole numbers, use the algorithm to multiply them, and then multiply the product by some power of 0.1 or $\frac{1}{10}$ (or divide by some power of 10) and move the digits accordingly.

Give students 2–3 minutes of quiet work time on the last question. Follow with a whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge. Review the algorithm to multiply whole numbers.
Supports accessibility for: Memory; Conceptual processing Speaking, Listening: Clarify, Critique, Correct. Present an incorrect response that reflects a possible misunderstanding about place value. Consider using this statement to open the discussion: "Since 46 times 9 is 414, then the product of 4.6 and 0.9 is 41.4." Ask pairs to clarify and critique by asking, "What questions do you have about this statement?" or "What errors do you notice?" Invite pairs to offer a correct response that includes a clear and correct mathematical explanation. This will help students explain how to find the product of numbers involving decimals.
Design Principle(s): Optimise output; Maximise meta-awareness

Anticipated Misconceptions

Students may not recall how to use the algorithm to multiply whole numbers. Consider reviewing the process prior to the activity.

Students may think that when calculating products, the decimal points need to line up. They may even write extra zeros at the end of a factor so there are the same amount of decimal places in each factor. Although this will not affect the answer, it is more efficient to align both factors to the right. If extra zeros are written at the end of a factor, there will be extra zeros accumulated in the calculation, and this can lead to careless errors.

Student Task Statement

1. A common way to find a product of decimals is to calculate a product of whole numbers, then consider place value.

$$
\begin{array}{rrrr}
 & & 2 & 5 \\
\times & & 1 & 2 \\
\hline
 & & 5 & 0 \\
+ & 2 & 5 & 0 \\
\hline
 & 3 & 0 & 0
\end{array}
$$

$$25 \times 12 = 300$$

$$(2.5) \times (1.2) = 3.00$$

Here is an example for (2.5) × (1.2).

Use what you know about decimals and place value to explain why the decimal point of the product is placed where it is.

2. Use the method shown in the first question to calculate each product.

   a. (4.6) × (0.9)

   b. (16.5) × (0.7)

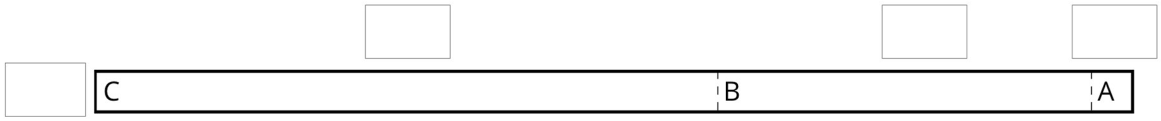
3. Use area diagrams to check your earlier calculations. For each problem:

   – Decompose each number into its base-ten units and write them in the boxes on each side of the rectangle.

   – Write the area of each lettered region in the diagram. Then find the area of the entire rectangle. Show your reasoning.

   a. (4.6) × (0.9)

   b. (16.5) × (0.7)



4. About how many centimetres are in 6.25 inches if 1 inch is about 2.5 centimetres? Show your reasoning.

Student Response

1. Answers vary. Sample responses: 2.5 is 0.1 times 25, and 1.2 is 0.1 times 12, so the product of 25 and 12 needs to be multiplied by $(0.1) \times (0.1)$. This is 0.01, which moves the decimal point two decimal places to the left. The number of decimal places in the product is the sum of the number of decimal places in the factors.

2.

   a. 4.14

   b. 11.55

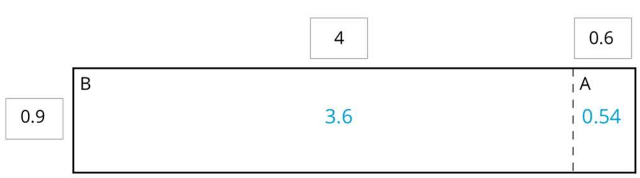
$$\begin{array}{r} 4\ 6 \\ \times\qquad 9 \\ \hline 4\ 1\ 4 \end{array} \qquad\qquad \begin{array}{r} 1\ 6\ 5 \\ \times\qquad 7 \\ \hline 1\ 1\ 5\ 5 \end{array}$$

$46 \times 9 = 414$ $\qquad$ $165 \times 7 = 1\,155$

$(4.6) \times (0.9) = 4.14$ $\qquad$ $(16.5) \times (0.7) = 11.55$

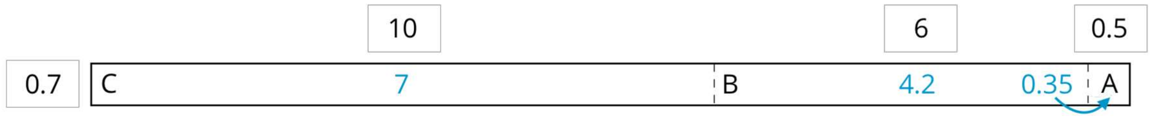
3.

   a.



- A: $(0.6) \times (0.9) = 0.54$, B: $4 \times (0.9) = 3.6$
- $0.54 + 3.6 = 4.14$, so $(4.6) \times (0.9) = 4.14$

   b.



- A: $(0.5) \times (0.7) = 0.35$, B: $6 \times (0.7) = 4.2$, C: $10 \times (0.7) = 7$
- $0.35 + 4.2 + 7 = 11.55$, so $(16.5) \times (0.7) = 11.55$

4. About 15.625 centimetres. $(6.25) \times (2.5) = 15.625$.

Activity Synthesis

Most of the discussions will have occurred in groups, but debrief as a class to tie a few ideas together. Ask a few students to share how they vertically calculated the products in the last several questions or to display the solutions for all to see. Discuss questions like:

- How did you know how to label the lengths of A, B, and C on the 16.5 by 0.7 rectangle? (The three digits in the number represent 10, 6, and 0.5, so the longest side is 10, the medium-length side is 6, and the shortest side is 0.5.)
- Which method—drawing an area diagram or using vertical calculations—do you prefer in finding products such as $(16.5) \times (0.7)$? Why? (Drawing an area diagram, because the visual representation helps us break up the calculation into smaller, more manageable pieces: $10 \times (0.7)$, $6 \times (0.7)$, and $(0.5) \times (0.7)$. Vertical calculation (long multiplication), because it is quicker to just multiply whole numbers and then consider place value.)
- How did you know where to place the digits in the last problem? (For part a, since 4.6 is 46 tenths and 0.9 is 9 tenths, we can compute 46×9 and then multiply the product by $\left(\frac{1}{10} \times \frac{1}{10}\right)$ or by $\frac{1}{100}$ to find $(4.6) \times (0.9)$.)

8.3 Practising Multiplication of Decimals

Optional: 15 minutes

This optional activity is an opportunity to practise the methods in this lesson to calculate products of decimals, and students have an opportunity to practise multiplying decimals in a real-world context. Students can choose to use area diagrams to help organise their work and support their reasoning. However, the goal of the activity is to have students practise using the multiplication algorithm on decimals.

Instructional Routines

- Discussion Supports
- Think Pair Share

Launch

Give students quiet think time to complete the activity and then time to share their explanation with a partner. Follow with whole-class discussion.

Representation: Internalise Comprehension. Activate or supply background knowledge. Provide students with access to a blank area diagram to support information processing. *Supports accessibility for: Visual-spatial processing; Organisation*

Student Task Statement

1. Calculate each product. Show your reasoning. If you get stuck, consider drawing an area diagram to help.

    a. $(5.6) \times (1.8)$

    b. $(0.008) \times (7.2)$

2. A rectangular playground is 18.2 metres by 12.75 metres.

    a. Find its area in square metres. Show your reasoning.

    b. If 1 metre is approximately 3.28 feet, what are the approximate side lengths of the playground in feet? Show your reasoning.

Student Response

1.

    a. 10.08. Sample reasoning:

    - $56 \times 18 = 1\,008$. 56 is 10 times 5.6 and 18 is 10 times 1.8, so 1 008 needs to be divided by 100 $1\,008 \div 100 = 10.08$
    - $56 \times 18 = 1\,008$. Each factor has 1 decimal place, so the product will have 2 decimal places (since $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$). Moving the digits of 1 008 two places to the right gives 10.08.

    b. 0.0576. Sample reasoning: $\frac{8}{1000} \times \frac{72}{10} = \frac{576}{10\,000}$

2.

    a. 232.05 square metres. $(18.2) \times (12.75) = 232.05$

    b. 59.696 feet and 41.82 feet. $(18.2) \times (3.28) = 59.696$ and $(12.75) \times (3.28) = 41.82$.

Are You Ready for More?

1. Write the following expressions as decimals.

    a. $1 - 0.1$

b. $1 - 0.1 + 10 - 0.01$

c. $1 - 0.1 + 10 - 0.01 + 100 - 0.001$

2. Describe the decimal that results as this process continues.

3. What would happen to the decimal if all of the addition and subtraction symbols became multiplication symbols? Explain your reasoning.

Student Response

1.

   a. 0.9

   b. 10.89

   c. 110.889

2. The decimal would consist of 0.8 sandwiched between 1's on the left and 8's on the right, ending with a 9 in the smallest decimal place.

3. The decimal would be equal to just the last number in each expression.

Activity Synthesis

Select students to share their strategies, being sure to highlight approaches using area diagrams and vertical calculations. Record the representations or strategies students share and display them for all to see.

Representing, Conversing: Discussion Supports. Use this routine help students prepare to share their strategies with the class. Give students time to discuss how they calculated each product for the first problem with a partner. Consider using this prompt to begin the conversation: "How did you use the multiplication algorithm to calculate the product of these two decimals? Provide a step-by-step explanation." The listener should press for details by asking clarifying questions such as, "Why did you do that first?" and "Could you explain that a different way?" Allow each student an opportunity as the speaker and listener. This will help students build confidence in explaining the process of multiplying decimals before students are selected in the whole-class discussion.
Design Principle(s): Support sense-making; Cultivate conversation

Lesson Synthesis

We have learned several ways to calculate products of decimals—by using fractions, multiplying non-zero digits of the decimals, using area diagrams and finding partial products, and long multiplication.

- How can working in fraction form help us find the product of two decimals?

- How can the product of two whole numbers (e.g., 48 and 19) help us find the product of two decimals with the same digits (e.g., 0.048 and 1.9)?
- How can we decompose decimal factors so they can be multiplied efficiently?

8.4 Calculate This!

Cool Down: 5 minutes

Student Task Statement

Calculate $(1.6) \times (0.215)$. Show your reasoning.

Student Response

$$\begin{array}{rr} & 215 \\ \times & 16 \\ \hline & 1290 \\ + & 215 \\ \hline & 3440 \end{array}$$

$215 \times 16 = 3440$

$(0.215) \times (1.6) = 0.344$

Student Lesson Summary

We can use 84×43 and what we know about place value to find $(8.4) \times (4.3)$.

Since 8.4 is 84 tenths and 4.3 is 43 tenths, then:

$$(8.4) \times (4.3) = \frac{84}{10} \times \frac{43}{10} = \frac{84 \times 43}{100}$$

That means we can compute 84×43 and then divide by 100 to find $(8.4) \times (4.3)$.

$84 \times 43 = 3612 \qquad (8.4) \times (4.3) = 36.12$

Using fractions such as $\frac{1}{10}, \frac{1}{100}$, and $\frac{1}{1000}$ allows us to find the product of two decimals using the following steps:

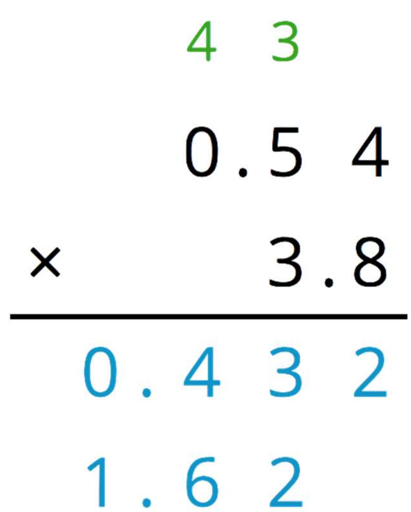
- Write each decimal factor as a product of a whole number and a fraction.
- Multiply the whole numbers.
- Multiply the fractions.
- Multiply the products of the whole numbers and fractions.

We know multiplying by fractions such as $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$ is the same as dividing by 10, 100, and 1 000, respectively. This means we can move the digits in the whole-number product to the right the appropriate number of spaces to correctly assign place value.

Lesson 8 Practice Problems

1. **Problem 1 Statement**

   Here are an unfinished calculation of (0.54) × (3.8) and a 0.54 by 3.8 rectangle.

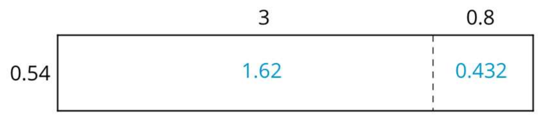


   a. Which part of the rectangle has an area of 0.432? Which part of the rectangle has an area of 1.62? Show your reasoning.

   b. What is (0.54) × (3.8)?

Solution

a. 0.432 is the area of the 0.8 by 0.54 rectangle because $(0.8) \times (0.54) = 0.432$. 1.62 is the area of the 3 by 0.54 rectangle because $3 \times (0.54) = 1.62$.



b. 2.16 $(0.54 + 1.62 = 2.052)$

2. **Problem 2 Statement**

Explain how the product of 3 and 65 could be used to find $(0.03) \times (0.65)$.

Solution

Answers vary. Sample response: We can use short multiplication to find 3 times 65, which equals 195. Because 0.03 is 3 hundredths and 0.65 is 65 hundredths, 195 will need to be multiplied by $(0.01) \times (0.01)$ or 0.0001. Multiplying by 0.0001 moves the digits 4 places to the right, so the product is 0.0195.

3. **Problem 3 Statement**

Use long multiplication to find each product.

a. $(5.4) \times (2.4)$

b. $(1.67) \times (3.5)$

Solution

a. 12.96

b. 5.845

4. **Problem 4 Statement**

A pound of blueberries costs £3.98 and a pound of clementines costs £2.49. What is the combined cost of 0.6 pound of blueberries and 1.8 pounds of clementines? Round your answer to the nearest cent.

Solution

£6.87. Sample reasoning: $(3.98) \times (0.6) = 2.388$, or about £2.39. $(2.49) \times (1.8) = 4.482$, or about £4.48. The combined cost is $2.39 + 4.48$ or 6.87.

5. **Problem 5 Statement**

Which has a greater value: $7.4 - 0.0022$ or $7.39 - 0.0012$? Show your reasoning.

Solution

$7.4 - 0.0022$ has a greater value. $7.4 - 0.0022 = 7.3978$ and $7.39 - 0.0012 = 7.3888$.

6. **Problem 6 Statement**

Andre is planting saplings (baby trees). It takes him 30 minutes to plant 3 saplings. If each sapling takes the same amount of time to plant, how long will it take Andre to plant 14 saplings? If you get stuck, consider using the table.

number of saplings	time in minutes
3	30
1	
14	

Solution

140 minutes (or equivalent). Possible strategy:

number of saplings	time in minutes
3	30
1	10
14	140