a divergência de um campo vetorial.

$$\operatorname{div} F = \nabla \cdot F = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$$

e o rotacional

$$\operatorname{rot} F = \nabla \cdot F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f & g & h \end{vmatrix}$$

o laplaciano ∇^2

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Resolução de exercícios: 15.1

4) O campo vetorial $F(x,y,z) = yz\,\vec{i} + xy^2\,\vec{j} + yz^2\,\vec{k}$ tem divergência _____ e rotacional _____.

$$\operatorname{div} F = \frac{\partial}{\partial x}(yz) + \frac{\partial}{\partial y}(xy^2) + \frac{\partial}{\partial z}(yz^2)$$

$$\operatorname{div} F = 0 + 2xy + 2yz = \boxed{2y(x+z)}$$

$$\operatorname{Rot} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xy^2 & yz^2 \end{vmatrix} \begin{matrix} i \\ \frac{\partial}{\partial x} \\ yz \end{matrix}$$

$$\therefore \left(\frac{\partial}{\partial z}(yz)j + \frac{\partial}{\partial y}(yz^2)i + \frac{\partial}{\partial x}(xy^2)k\right) - \left(\frac{\partial}{\partial x}(z^2)j + \frac{\partial}{\partial y}(yz)k + \frac{\partial}{\partial z}(xy^2)i\right)$$

credeal

/ /

$\therefore \left(y\vec{j} + z^2 i + y^2 k\right) - \left(0 + zk + 0\right) =$

$yj + z^2 i + y^2 k - zk$

$\langle z^2 i + yj\,(y^2 - z)k \rangle$

5) Esboce o campo vetorial desenhando alguns vetores representativos que não se intersectem.

5) $F(x,y) = 2i - \vec{j}$ p/ quaisquer pontos que pegar meu vetor é constante

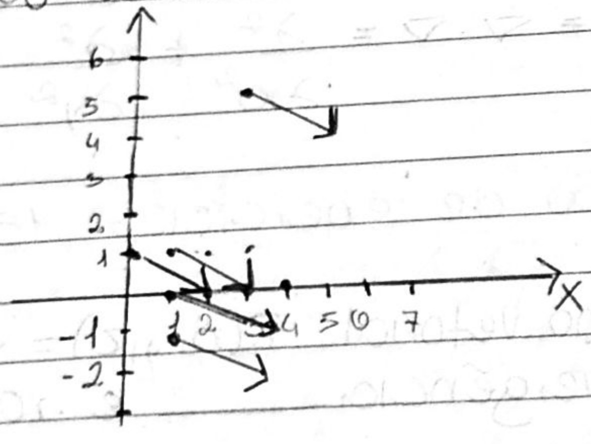
$\vec{F}(0,1) = 2i - \vec{j}$

$\vec{F}(1,1) = 2i - \vec{j}$

$\vec{F}(1,0) = 2i - \vec{j}$

$\vec{F}(1,-1) = 2i - \vec{j}$

$\vec{F}(3,5) = 2i - \vec{j}$

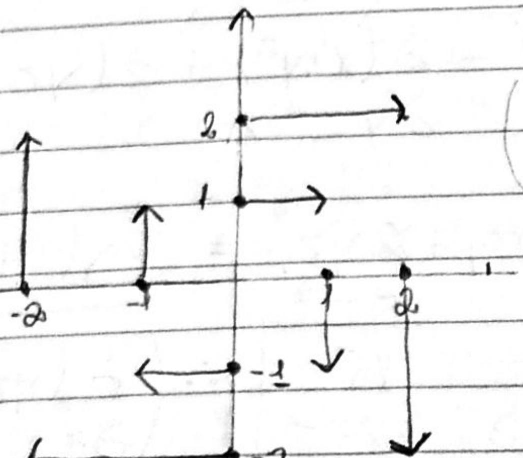


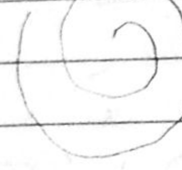
7) $F(x,y) = y\vec{i} - x\vec{j}$

$\vec{F}(0,1) = 1\vec{i}$

$\vec{F}(1,0) = -1\vec{j}$

$\vec{F}(0,-1) = -1\vec{i}$





$\vec{F}(-1,0) = 1\vec{j}$

$\vec{F}(0,2) = 2\vec{i}$

$\vec{F}(2,0) = -2\vec{j}$

$\vec{F}(0,-2) = -2\vec{i}$

$\vec{F}(-2,0) = 2\vec{j}$

17-22. Calcule div F e rot F.

17) $F(x,y,z) = x^2\vec{i} - 2\vec{j} + yz\vec{k}$.

$$\text{div} F = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(-2) + \frac{\partial}{\partial z}(yz)$$

$$\text{div} F = 2x + 0 + y = \boxed{2x+y}$$

$$\text{rot} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & -2 & yz \end{vmatrix} \left(\frac{\partial}{\partial z}(x^2) j + \frac{\partial}{\partial y}(yz) i + \frac{\partial}{\partial x}(-2) k\right) - \left(\frac{\partial}{\partial x}(yz) j + \frac{\partial}{\partial y}(x^2) k + \frac{\partial}{\partial z}(-2)\right.$$

$$= (0 + z\vec{i} + 0) - (0 + 0 + 0) = \boxed{z\vec{i}}$$

21) $F(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}(x\vec{i} + y\vec{j} + z\vec{k})$

$$F(x,y,z) = \left(\frac{x\vec{i}}{\sqrt{x^2+y^2+z^2}} + \frac{y\vec{j}}{\sqrt{x^2+y^2+z^2}} + \frac{z\vec{k}}{\sqrt{x^2+y^2+z^2}}\right)$$

$$\text{div} F = \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial z}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$\text{div} F = \frac{1}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}}(y^2+z^2) + (x^2+z^2) + (x^2+y^2)$$

3

$$\text{div} F = 3(2x^2+2y^2+2z^2) = 23(x^2+y^2+z^2)$$

$$\text{div} F = \boxed{\frac{2}{\sqrt{x^2+y^2+z^2}}}$$

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$$\begin{array}{ccc} \frac{x}{\sqrt{x^2+y^2+z^2}} & & \frac{z}{\sqrt{x^2+y^2+z^2}} \end{array}$$

$$\operatorname{rot} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{\sqrt{x^2+y^2+z^2}} & \frac{y}{\sqrt{x^2+y^2+z^2}} & \frac{z}{\sqrt{x^2+y^2+z^2}} \end{vmatrix}$$

$$\begin{array}{ccc} i & & k \end{array}$$

$$\left(\cancelto{0}{\frac{\partial}{\partial z}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right)} j + \cancelto{0}{\frac{\partial}{\partial y}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)} i + \cancelto{0}{\frac{\partial}{\partial x}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right)} k\right) -$$

$$\left(\cancelto{0}{\frac{\partial}{\partial x}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)} j + \cancelto{0}{\frac{\partial}{\partial y}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right)} k + \cancelto{0}{\frac{\partial}{\partial z}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right)} i\right)$$

$$\operatorname{rot} F = \boxed{0}$$