

## Lesson 3: Reasoning to find area

### Goals

- Compare and contrast (orally) different strategies for calculating the area of a polygon.
- Find the area of a polygon by decomposing, rearranging, subtracting or enclosing shapes, and explain (orally and in writing) the solution method.
- Include appropriate units (in spoken and written language) when stating the area of a polygon.

### Learning Targets

- I can use different reasoning strategies to find the area of shapes.

### Lesson Narrative

This lesson is the third of three lessons that use the following principles for reasoning about shapes to find area:

- If two shapes can be placed one on top of the other so that they match up exactly, then they have the same area.
- If a shape is composed from pieces that don't overlap, the sum of the areas of the pieces is the area of the shape. If a given shape is decomposed into pieces, then the area of the given shape is the sum of the areas of the pieces.

Following these principles, students can use several strategies to find the area of a shape. They can:

- Decompose it into shapes whose areas they can calculate.
- Decompose and rearrange it into shapes whose areas they can calculate.
- Consider it as a shape with one or more missing pieces, calculate the area of the shape, then subtract the areas of the missing pieces.
- Enclose it with a shape whose area they can calculate, consider the result as a region with missing pieces, and find its area using the previous strategy.

Use of these strategies involves looking for and making use of structure; explaining them involves constructing logical arguments. For now, rectangles are the only shapes whose areas students know how to calculate, but the strategies will become more powerful as students' repertoires grow. This lesson includes one shape for which the "enclosing" strategy is appropriate, however, that strategy is not the main focus of the lesson and is not included in the list of strategies at the end.

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### Building On

- Recognise area as additive. Find areas of rectilinear shapes by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

### Addressing

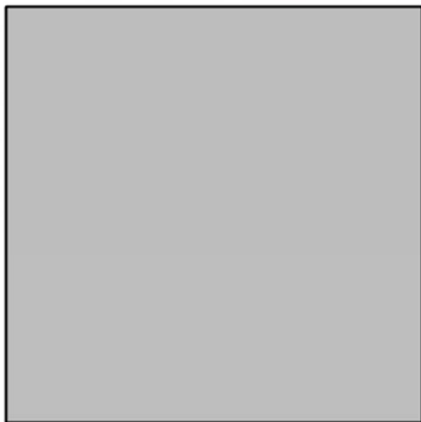
- Find the area of right-angled triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

### Instructional Routines

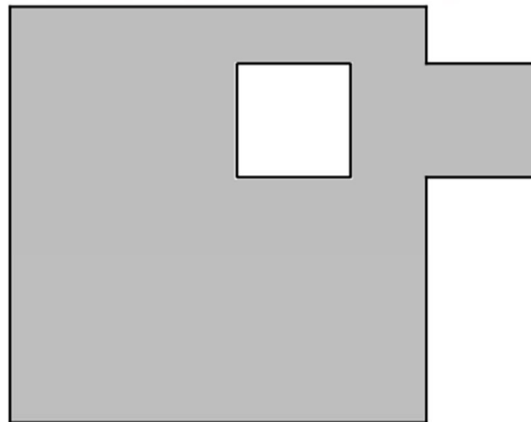
- Anticipate, Monitor, Select, Sequence, Connect
- Clarify, Critique, Correct
- Compare and Connect
- Think Pair Share

### Required Materials

#### Copies of blackline master



**A**



**B**

### Geometry toolkits

tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles.

### Required Preparation

Make sure students have access to items in their geometry toolkits: tracing paper, graph paper, coloured pencils, scissors, and an index card to use as a straightedge or to mark right angles.

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For the warm-up activity, prepare several copies of the pair of shapes on the blackline master, in case students propose cutting them out to compare the areas.

### Student Learning Goals

Let's decompose and rearrange shapes to find their areas.

## 3.1 Comparing Regions

### Warm Up: 5 minutes

This activity prompts students to use reasoning strategies from earlier lessons to compare the areas of two shapes. It is also an opportunity to use (or introduce) tracing paper as a way to illustrate *decomposing* and *rearranging* a shape.

As students work, look for students who are able to explain or show how they know that the areas are equal. Some students may simply look at the shapes and say, with no justification, that they have the same area. Urge them to think of a way to show that their conclusion is true.

### Launch

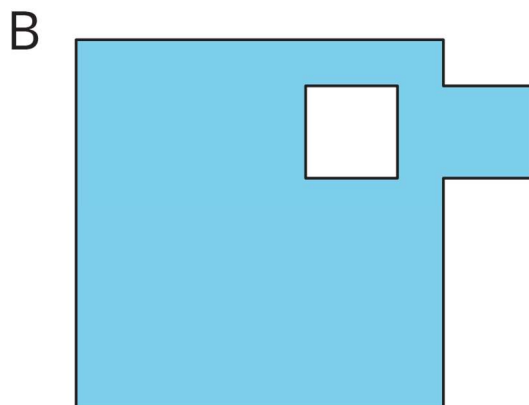
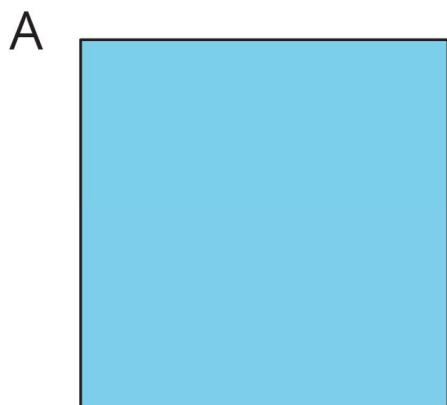
Give students access to their geometry toolkits, and allow for 2 minutes of quiet think time. Ask them to be ready to support their answer, and remind them to use the tools at their disposal. Have copies of the blackline master ready for students who propose cutting the shapes out for comparison or as a way to differentiate the activity.

### Anticipated Misconceptions

Students may interpret the area of shape B as the entire region inside the outer boundary including the unfilled square. Clarify that we want to compare the areas of only the shaded parts of shape B and shape A.

### Student Task Statement

Is the area of shape A greater than, less than, or equal to the area of the shaded region in shape B? Be prepared to explain your reasoning.



## Student Response

The areas are equal. Possible strategies:

- **Measuring:** Measure the side lengths of the small, unfilled rectangle and the small rectangle that are on the side of shape B. Both rectangles have the same side lengths, so their areas are equal. This means the rectangle on the side fills the whole in the middle. Measure the side lengths of the large shaded square in shape A and then in shape B. Both have the same side lengths, so their areas are equal.
- **Using scissors:** Cut off the little square on the side of shape B, and use it to fill the hole in the middle of shape B. Then you get a square that matches up exactly with shape A.
- **Using tracing paper:** Trace the boundary of the little square on the side of shape B, move the tracing paper over the unshaded hole, and you can see that the little shaded square is exactly the same size as the hole. If you moved that little shaded square to fill the unshaded hole, you would get a big shaded square. If you trace the boundary of that big shaded square and put the tracing paper over shape A, you can show that the boundary of that square matches up exactly with shape A.

## Activity Synthesis

Start the discussion by asking the students to indicate which of the three possible responses—area of shape A is greater, area of shape B is greater, or the areas are equal—they chose.

Select previously identified students to share their explanations. If no student mentioned using tracing paper, demonstrate the following.

- *Decomposing and rearranging shape B:* Place a piece of tracing paper over shape B. Draw the boundary of the small square, making a dotted auxiliary line to show its separation from the large square. Move the tracing paper so that the boundary of the small square matches up exactly with the boundary of the square-shaped hole in shape B. Draw the boundary of the large square. Explain that the small square matches up exactly with the hole, so we know the small, shaded square and the hole have equal area.
- *Matching the two shapes:* Move the tracing paper over shape A so that the boundary of the rearranged shape B matches up exactly with that of shape A. Say, “When two shapes that are overlaid one on top of another match up exactly, their areas are equal.”

Highlight the strategies and principles that are central to this unit. Tell students, “We just decomposed and rearranged shape B so that it matches up exactly with shape A. When two shapes that are overlaid one on top of another match up exactly, we can say that their areas are equal.”

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## 3.2 On the Grid

### 20 minutes

This activity gives students opportunities to find areas of regions using a variety of strategies. When working with a grid, students may start by counting squares, as they had done in earlier years. However, the shapes have been chosen to elicit the strategies listed in the Lesson Narrative.

- Shape A can be easily decomposed into rectangles.
- Shape B can be decomposed into rectangles. Or, more efficiently, it can be seen as a square with a missing piece, and the area of the inner unshaded square can be subtracted from the area of the larger square.
- Shape C can also be seen as having a missing piece, but subtracting the area of the unshaded shape does not work because the side lengths of the inner square are unknown. Instead, the shaded triangles can be decomposed and rearranged into rectangles.
- Shape D can be decomposed and rearranged into rectangles. It can also be viewed as the inner square of shape C.

As students work, identify students who use these strategies and can illustrate or explain them clearly. Ask them to share later. Look for at least two strategies being used for each shape (one strategy as shown in the Student Response and at least one other).

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Clarify, Critique, Correct
- Think Pair Share

### Launch

Tell students that they will find the areas of various shapes on a grid. To encourage students to use a more level-appropriate strategy for finding areas, show them a strategy from earlier years. As a class, find the area of shape A by counting the squares one by one aloud. Confirm that there are 24 square units, and then ask students to think about other ways to find the area of Shape A and other shapes besides counting each square.

Arrange students in groups of 2. Ask one partner to start with shapes A and C, and the other with B and D. Give students 4–5 minutes of quiet think time, and provide access to their geometry toolkits. Then, give them a few minutes to share their responses with their partner. Emphasise that as one partner explains, the other should listen carefully and see if they agree or disagree with the answer and explanations.

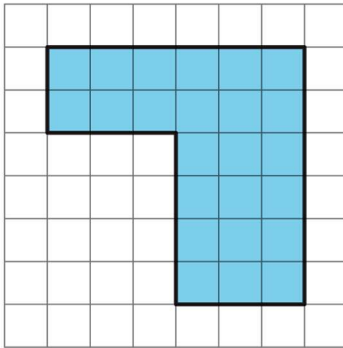
### Anticipated Misconceptions

Some students may count both complete and partial grid squares instead of looking for ways to decompose and rearrange larger shapes. Ask them if they can find a way to find the area by decomposing and rearranging larger pieces. The discussion at the end, during which everyone sees a variety of strategies, is especially important for these students.

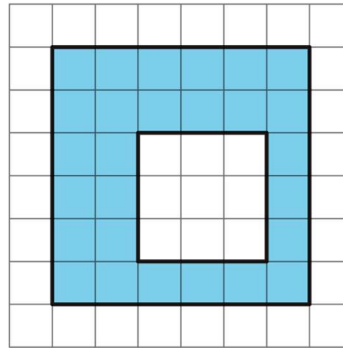
### Student Task Statement

Each grid square is 1 square unit. Find the area, in square units, of each shaded region without counting every square. Be prepared to explain your reasoning.

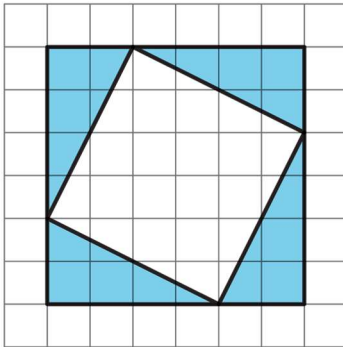
A



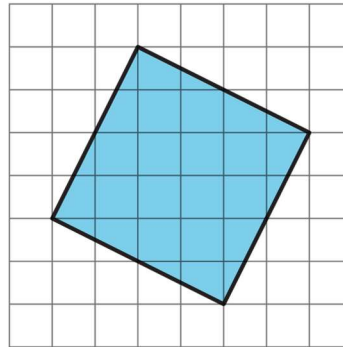
B



C



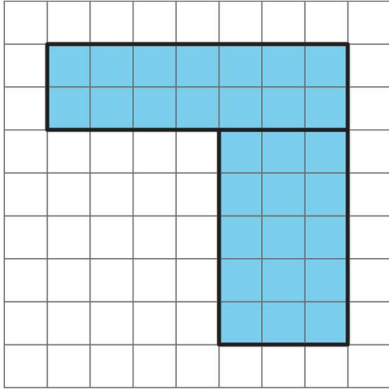
D



### Student Response

Strategies vary.

A: 24 square units. Sample strategy: Decompose the shape into rectangles. One way is shown here.  $(2 \times 6) + (4 \times 3) = 24$

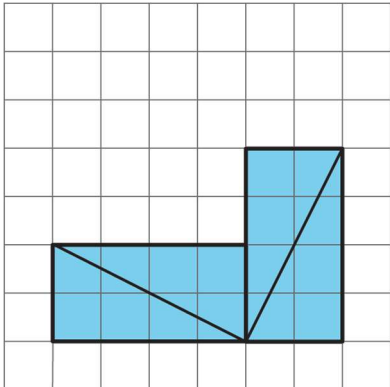


B: 27 square units. Sample strategies:

- Decompose the shape into four rectangles.
- Subtract the area of the inner square from the larger square.  $(6 \times 6) - (3 \times 3) = 27$

C: 16 square units. Sample strategies:

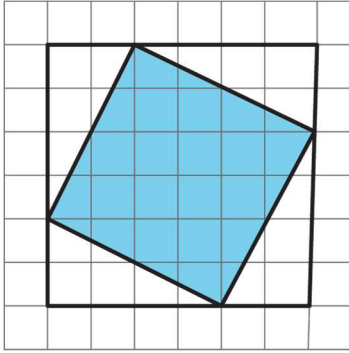
- Decompose into right-angled triangles and rearrange into rectangles.  $(2 \times 4) + (4 \times 2) = 16$



- Find the area of shape D first, and then subtract it from the 6-by-6 square.

D: 20 square units. Sample strategies:

- Decompose the shaded square into four right-angled triangles and a 2-by-2 square. Rearrange the right-angled triangles into two rectangles that are each 2 units by 4 units, with a combined area of 16 square units. Adding the area of the small square (4 square units) gives a total of 20 square units.
- Notice that the shaded square is the inner square of shape C, enclose it in a square as in shape C, and subtract the areas of the four right-angled triangles (i.e., the area of shape C) from the area of the enclosing square.  $(6 \times 6) - 16 = 20$

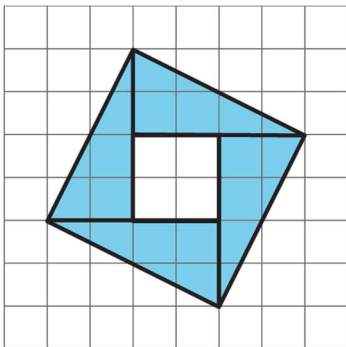


### Are You Ready for More?

Rearrange the triangles from shape C so they fit inside shape D. Draw and colour a diagram of your work.

### Student Response

The triangles fit inside the square, with a smaller 2-by-2 square in the centre.



This arrangement shows that the area of shape D is 4 square units more than the area of shape C.

### Activity Synthesis

Discussion should centre around how different strategies—decomposing, rearranging, subtracting, and enclosing—are used to find area. For each shape, select two students with different strategies to share their work, if possible. Sequence students' presentations so that, for each shape, a subtracting strategy comes last, as that is typically the most challenging.

Before sharing begins, explain to students that they should notice similarities and differences in the strategies shared and be ready to explain them. As students share their strategies, consider recording the moves on each shape for all to see. After each person shares, name the strategy, and poll the class to see if anyone else reasoned the same way. If one of these strategies does not appear in students' work, illustrate it for the class.

- Decomposing (A and B)



- Decomposing and rearranging (C and D)
- Subtracting (B)
- Enclosing, then subtracting (D)

If time permits, give partners a minute to talk to their partner about the similarities and differences they saw in the strategies used to find the areas of the four shapes. Consider displaying sentence starters such as:

- The strategies used to find the areas of shapes \_\_\_ and \_\_\_ are alike in that. . .
- The strategies used to find the areas of shapes \_\_\_ and \_\_\_ are different in that. . .

*Representation: Internalise Comprehension.* Use colour and annotations to highlight distinctions between strategies. Create a display that includes multiple copies of each shape. As students describe their strategies, use colour and annotation to describe their thinking so that it is visible for all students. Be sure to label each shape with the strategy described (decomposing, rearranging, subtracting, or enclosing).

*Supports accessibility for: Visual-spatial processing; Conceptual processing*

*Speaking: Clarify, Critique, Correct.* In *Clarify, Critique, Correct*. Students are given an incorrect or incomplete piece of mathematical work. This may be in the form of a written statement, drawing, problem-solving steps, or another mathematical representation. Pairs of students analyse, reflect on, and improve the written work by correcting errors and clarifying meaning. Typical prompts are “Is anything unclear?” or “Are there any reasoning errors?” The purpose of this routine is to engage students in analysing mathematical thinking that is not their own, and to solidify their knowledge through communicating about conceptual errors and ambiguities in language.

*Design Principle(s): Maximise meta-awareness*

### **How It Happens:**

1. In the class discussion for shape C of this activity, provide the following “draft” explanation:

“I cut a square in the middle, then I saw that it was a bunch of triangles, so then I figured those out to get my answer.”

Prompt students to identify the ambiguity of this response. Ask students, “What do you think this person is trying to say? What is unclear? Did the author use any of the strategies we’ve been using to find area?”

2. Give students 1 minute of individual time to respond to the questions in writing, and then 3 minutes to discuss with a partner.

As pairs discuss, provide these sentence frames for scaffolding: “I think the author is trying to use the strategy \_ because....”, “I think what the author meant by ‘figured it out’ was....”, and “The part that is most unclear to me is ... because....”. Encourage the

listener to press for detail by asking follow-up questions to clarify the intended meaning of the statement. Allow each partner to take a turn as the speaker and listener.

3. Invite students to improve the “draft” response using the target vocabulary and structures.

The targeted vocabulary includes the names of the strategies (decomposing, rearranging, subtracting, enclosing, or any combination of them), and other terms from this lesson such as “alike,” “different,” “area,” and “polygon.”

The targeted structures of the response should include an explanation of each step, order/time transition words (first, next, then, etc.), and/or reasons for decisions made during steps.

Here is one example of an improved response:

“First, I cut out the square in the middle of the shaded regions. Next, I noticed that all the shaded regions were a bunch of right-angled triangles. Then, I rearranged the triangles into rectangles. Lastly, I figured out the area of each rectangle and added them together to get my answer.”

4. Ask each pair of students to contribute their improved response to a poster, the whiteboard, or digital projection. Call on 2–3 pairs of students to present their response to the whole class, and invite the class to make comparisons among the responses shared and their own responses.

Listen for responses that include the strategy of decomposing and then rearranging the triangles. Emphasise that this is one way of figuring out the area of the shaded region.

5. Share one more improved response, discuss to reach a general understanding, and then move on to shape D.

### 3.3 Off the Grid

#### 15 minutes

In this activity, students apply the strategies they learned to find the areas of shapes that are not on a grid.

- Shape A can easily be decomposed and rearranged.
  - Shape B can be decomposed and rearranged into rectangles as for shape C of the previous task. Students cannot use the strategy of subtracting the area of the inner square from that of the outer square because the side lengths of the inner square are unknown.
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- For shape C, students must subtract the area of the inner square from that of the outer square because there is not enough information to decompose and rearrange the shaded regions into rectangles.

As students discuss their approaches in groups, support them in naming the strategies and by asking clarifying questions. Notice any groups that may be stuck in a disagreement on the area of a particular shape. Identify students who observed that the same area-reasoning strategies can be applied both on and off the grid.

Students may not remember from earlier years that if the measurements of side lengths of a rectangle are given in a particular unit, then the area is given in *square units*. Look for students who have trouble giving the appropriate area units (square centimetres) for these shapes.

### Instructional Routines

- Compare and Connect

### Launch

Tell students that they will now find areas of shapes that are not on a grid. Give students access to their geometry toolkits. Allow for 6–7 minutes of quiet time to find all three areas.

Then, arrange students into groups of 4, and ask each group to discuss their answers and strategies, using these guiding questions:

1. What units did you use for each area?
2. Compare your answers and strategies for finding the area of each shape.
  - How are your strategies the same? How are they different?
  - Which strategies are similar to the ones you used in the previous activity?

*Action and Expression: Internalise Executive Functions.* Chunk this task into more manageable parts. After students have found the area of shape A, check in with select groups of students or the whole class. Invite students to share and name the strategy they used, and to predict a strategy they may find useful for shape B.

*Supports accessibility for: Organisation; Attention*

### Anticipated Misconceptions

In shape B, students may estimate the side lengths of the inner square so that its area could be subtracted from that of the outer square. They may struggle to see how the triangles could be rearranged. Suggest that they use tracing paper to help them in their thinking.

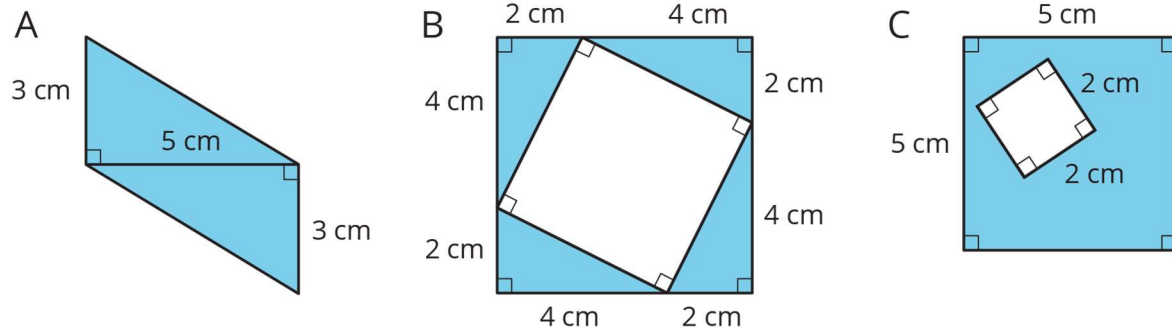
In shapes B and C, students may confuse finding area with finding perimeter. Remind them that area refers to the number of square units it takes to cover a region without gaps or overlaps.

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Students might not be familiar with the symbols that indicate right angles and might think these symbols indicate square units. Remind them that those symbols indicate 90 degree angles.

### Student Task Statement

Find the area of the shaded region(s) of each figure. Explain or show your reasoning.



### Student Response

Reasoning varies.

Shape A: The area is 15 square centimetres. Sample reasoning: Decompose the two triangles and rearrange them to form a rectangle with side lengths of 5 centimetres and 3 centimetres.

Shape B: The area is 16 square centimetres. Sample reasoning: Decompose the triangles and rearrange them to form two rectangles with side lengths of 4 centimetres and 2 centimetres.

Shape C: The area is 21 square centimetres. Sample reasoning: Subtract the area of the inner square from the area of the outer square.  $25 - 4 = 21$

### Activity Synthesis

Reconvene briefly as a class to discuss the question, “Which strategies are similar to the ones you used in the previous activity?”

Select 1–2 previously identified students to share (those who noticed that they decomposed, rearranged, enclosed, and subtracted in both activities). Emphasise that the same strategies for finding area can be used whether we use the measurements indicated by a grid, or whether the measurements are given directly (without a grid).

*Speaking: Compare and Connect.* This is the first time *Compare and Connect* is suggested as a support in this course. In this routine, students are given a problem that can be approached using multiple strategies or representations, and are asked to prepare a visual display of their method. Students then engage in investigating the strategies (by means of a teacher-

led gallery walk, partner exchange, group presentation, etc.), compare approaches, and identify correspondences between different representations. A typical discussion prompt is, “What is the same and what is different?” regarding their own strategy and that of the others. The purpose of this routine is to allow students to make sense of mathematical strategies by identifying, comparing, contrasting, and connecting other approaches to their own, and to develop students’ awareness of the language used through constructive conversations.

*Design Principle(s): Maximise meta-awareness*

### **How It Happens:**

1. Identify which shape (A, B, or C) generated the most variety among students’ strategies. Invite students to create a visual display showing how they made sense of this shape.

Students should include these features on their display:

- the drawing of the shape
  - appropriate units
  - a representation of how the area was calculated
  - the name of the strategy used to find the area (decomposing, rearranging, subtracting, enclosing, or any combination of them)
2. Arrange students in groups of 4, and invite them to investigate each other’s work. Allow 1 minute for each display and signal when it is time to switch. Next, give each student the opportunity to add detail to their own display for 1-2 minutes.
  3. Circulate around the room and invite 2–3 students to present their display to the whole class. Be sure to select a variety of strategies.
  4. After the pre-selected students have finished presenting their displays, lead a discussion comparing, contrasting, and connecting the different approaches.

Consider using these prompts to amplify student language while comparing and contrasting the different approaches:

- “Why did different approaches for shape \_ lead to the same outcome?”
- “What worked well in this approach for shape \_? What did not work well?”
- “What would make this strategy for shape \_ more complete or easy to understand?”

Consider using these prompts to amplify student language while connecting the different approaches:

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- “Can you find any connections between the representations?”
  - “Where are units used in each strategy?”
  - “Is it possible to use the strategy of decomposing for this shape?”
  - “What mathematical features do you see present in all of the representations?”
5. Close the discussion by inviting 3 students to revoice the strategies used in the presentations, and then transition back to the Lesson Synthesis and Cool-Down.

### Lesson Synthesis

This lesson was all about identifying strategies for finding area and applying them to various shapes. We reasoned about the area of a shape on and off a grid by:

- decomposing it into familiar shapes;
- decomposing it and rearranging the pieces into familiar shapes; or
- considering it as a shape with missing pieces, then subtracting the areas of the missing pieces from the area of the shape.

Ask students to go back through the activities and find problems in which these strategies were used—one strategy at a time. Tell students we will have lots of opportunity to use these strategies in upcoming lessons.

### 3.4 Maritime Flag

#### Cool Down: 5 minutes

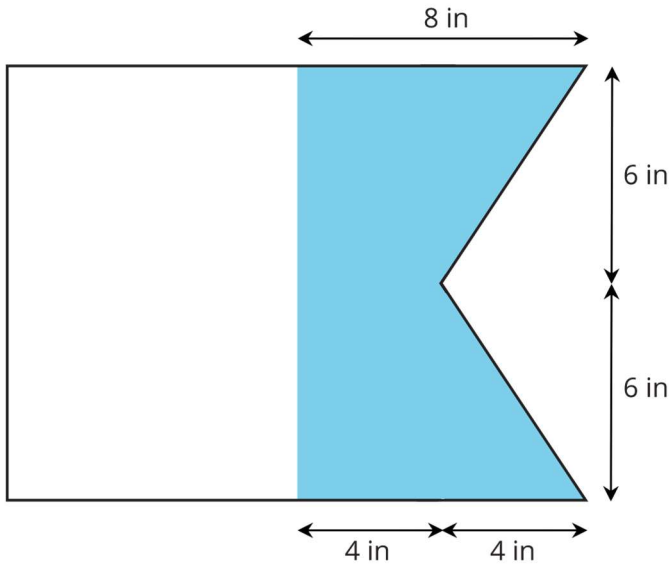
This task does not explicitly ask students to state area units because one purpose of the task is to assess if students understand what units are appropriate given the information presented.

#### Launch

Give students access to their geometry toolkits.

#### Student Task Statement

A maritime flag is shown. What is the area of the shaded part of the flag? Explain or show your reasoning.



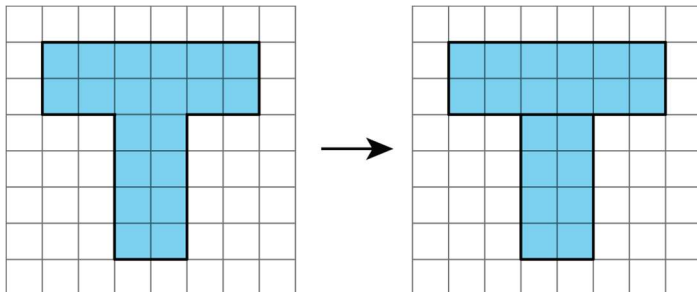
### Student Response

72 square inches. Reasoning varies. Sample reasoning: If we draw a line down the middle of the shaded area, we would have a 4 inch-by-12 inch rectangle on the left and two right-angled triangles. The 4-by-12 rectangle has an area of 48 square inches. The two triangles on the right can be composed into a 4 inch-by-6 inch rectangle, so their combined area is 24 square inches.  $48 + 24 = 72$ .

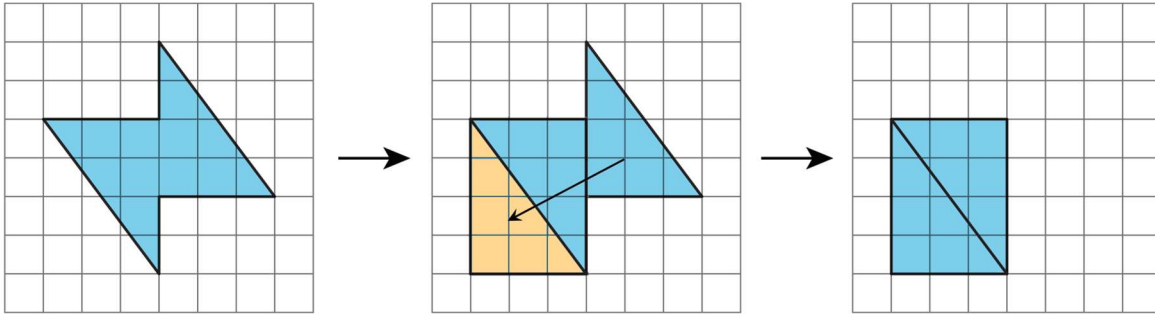
### Student Lesson Summary

There are different strategies we can use to find the area of a region. We can:

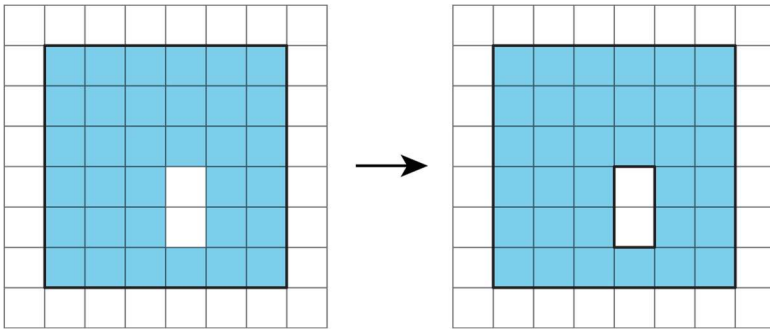
- Decompose it into shapes whose areas you know how to calculate; find the area of each of those shapes, and then add the areas.



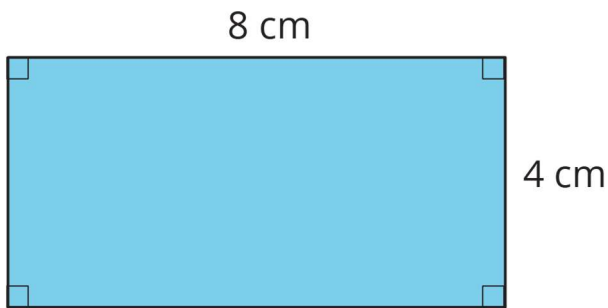
- Decompose it and rearrange the pieces into shapes whose areas you know how to calculate; find the area of each of those shapes, and then add the areas.



- Consider it as a shape with a missing piece; calculate the area of the shape and the missing piece, and then subtract the area of the piece from the area of the shape.



The area of a shape is always measured in square units. When both side lengths of a rectangle are given in centimetres, then the area is given in square centimetres. For example, the area of this rectangle is 32 square centimetres.

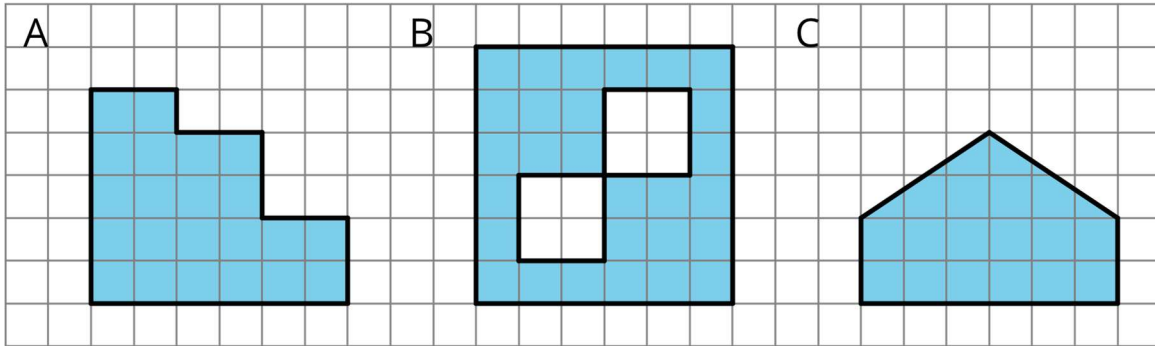




## Lesson 3 Practice Problems

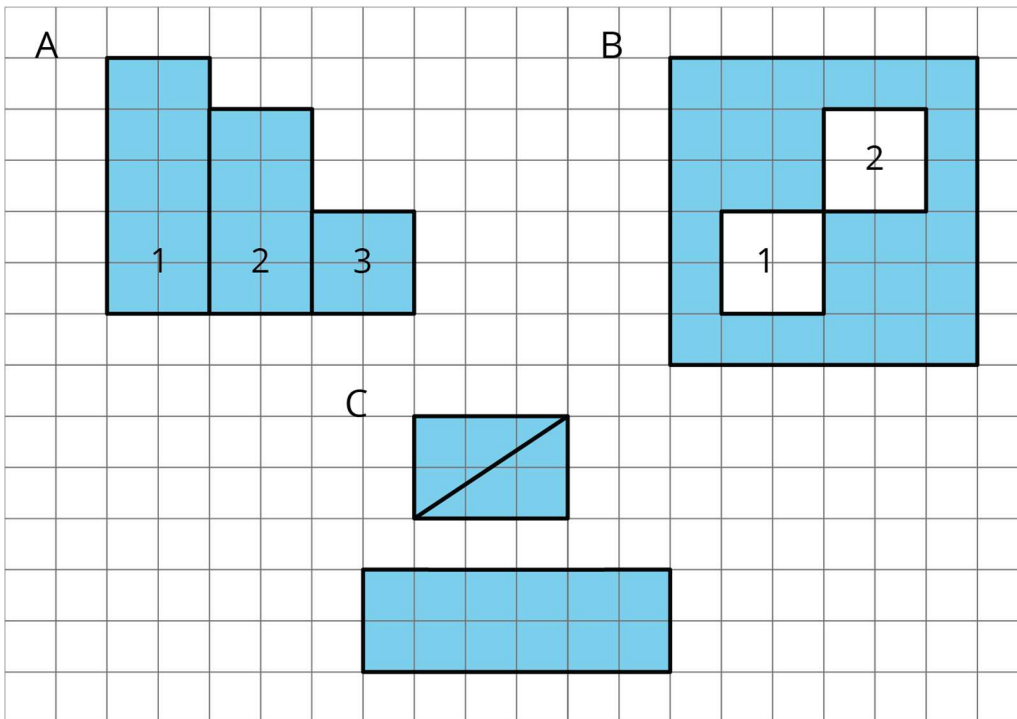
### 1. Problem 1 Statement

Find the area of each shaded region. Show your reasoning.



### Solution

Answers vary. Sample response:



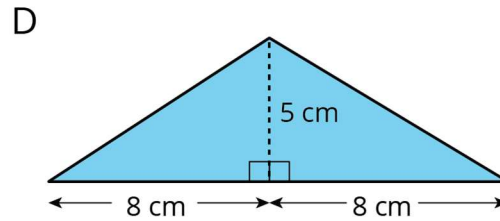
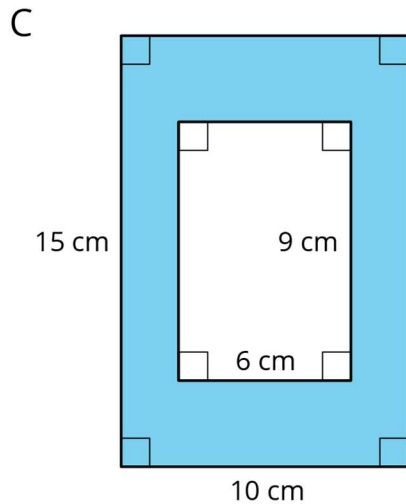
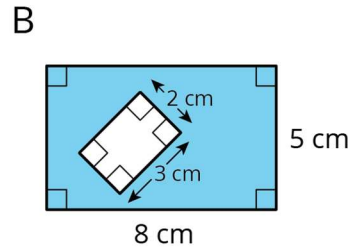
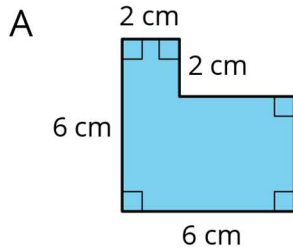
Shape A: 22 square units. The shaded region can be partitioned into rectangles. One way to do this is shown above. Rectangle 1 is 2 units by 5 units, so its area is 10 square units. Rectangle 2 is 2 units by 4 units, so its area is 8 square units. The area of Rectangle 3 is 4 square units. The total shaded area is 22 square units, since  $10 + 8 + 4 = 22$ .

Shape B: 28 square units. The outer square is 6 units by 6 units, so its area is 36 square units. There are two smaller squares inside. Square 1 and Square 2 have been removed. Each small square has an area of 4 square units. To get the shaded area, calculate  $36 - 4 - 4$ , which equals 28.

Shape C: 18 square units. The region can be recomposed to form a 2-by-6 rectangle and a 2-by-3 rectangle.  $(2 \times 6) + (2 \times 3) = 18$ .

## 2. Problem 2 Statement

Find the area of each shaded region. Show or explain your reasoning.



### Solution

Reasoning varies. Sample responses:

A:  $28 \text{ cm}^2$ . A horizontal cut partitions this into a 2 cm-by-2 cm square ( $4 \text{ sq cm}$ ) and a 4 cm-by-6 cm rectangle ( $24 \text{ sq cm}$ ).

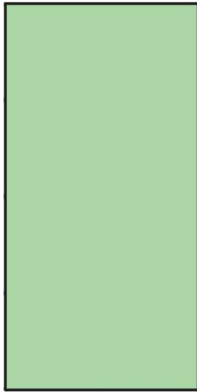
B:  $34 \text{ cm}^2$ . The outer rectangle has an area of  $40 \text{ sq cm}$  while the inner rectangle has an area of  $6 \text{ sq cm}$ .  $40 - 6 = 34$ .

C:  $96 \text{ cm}^2$ . The outer rectangle has an area of  $150 \text{ sq cm}$  while the inner rectangle has an area of  $54 \text{ sq cm}$ .  $150 - 54 = 96$ .

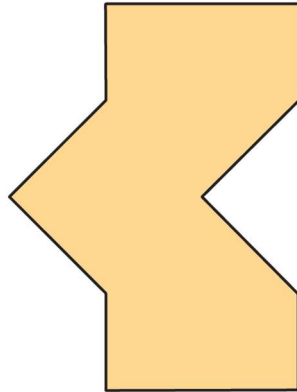
D:  $40 \text{ cm}^2$ . The two right-angled triangles can be put together to make a  $5 \text{ cm}$ -by- $8 \text{ cm}$  rectangle.

### 3. Problem 3 Statement

Two plots of land have very different shapes. Noah said that both plots of land have the same area.



plot A

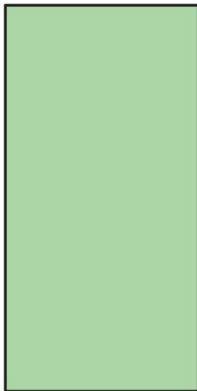


plot B

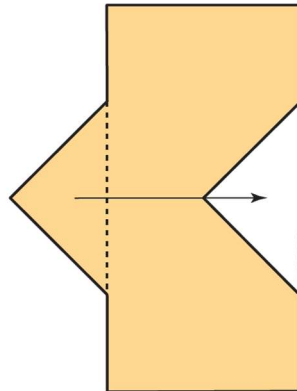
Do you agree with Noah? Explain your reasoning.

#### Solution

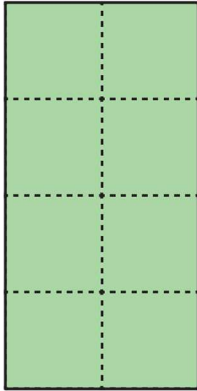
Agree. Answers vary. Sample reasoning: The triangular shape that juts out from the left side of plot B can be cut off and moved to the right side of plot B. The resulting shape is a rectangle that matches exactly with the shape of plot A. We can use tracing paper to verify. Sample diagrams:



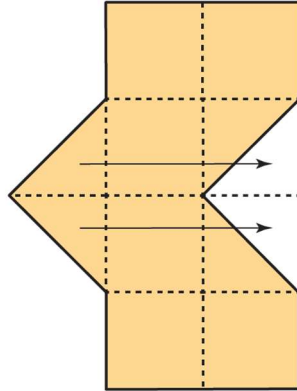
plot A



plot B



plot A



plot B

#### 4. Problem 4 Statement

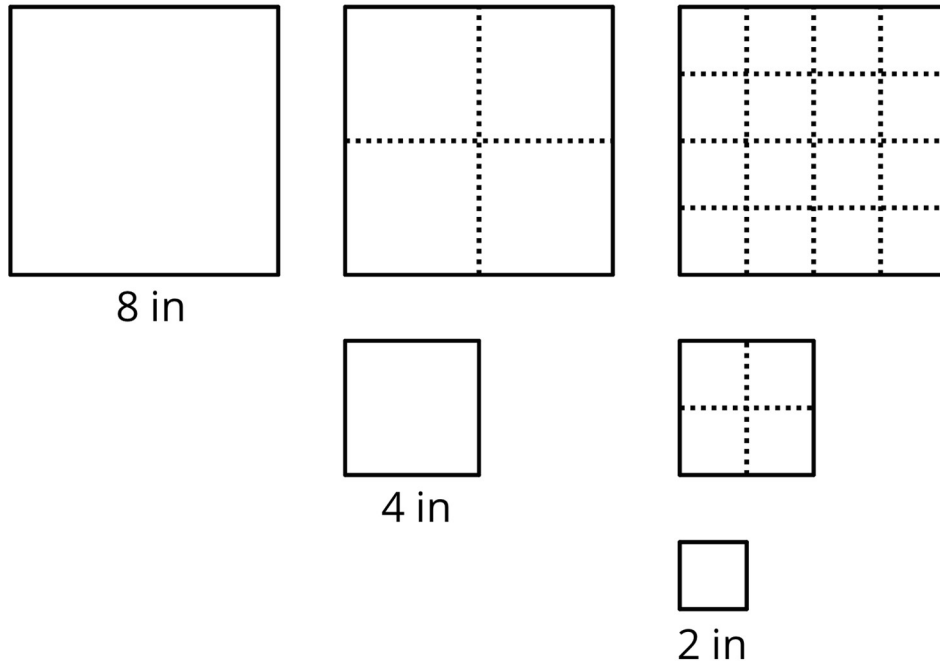
A homeowner is deciding on the size of tiles to use to fully tile a rectangular wall in her bathroom that is 80 inches by 40 inches. The tiles are squares and come in three side lengths: 8 inches, 4 inches, and 2 inches. State if you agree with each statement about the tiles. Explain your reasoning.

- a. Regardless of the size she chooses, she will need the same number of tiles.
- b. Regardless of the size she chooses, the area of the wall that is being tiled is the same.
- c. She will need two 2-inch tiles to cover the same area as one 4-inch tile.
- d. She will need four 4-inch tiles to cover the same area as one 8-inch tile.
- e. If she chooses the 8-inch tiles, she will need a quarter as many tiles as she would with 2-inch tiles.

#### Solution

Explanations vary. Sample explanations and diagram:

- a. Disagree. She will need fewer of the larger tiles and more of the smaller tiles.
- b. Agree. The region being covered does not change regardless of what tiles she chooses.
- c. Disagree. She will need four 2-inch tiles to cover the same area as one 4-inch tile.



- d. Agree. Two rows of two 4-inch tiles cover the same area as one 8-inch tile.
- e. Disagree. Because one 8-inch tile covers the same area as four 4-inch tiles, she will need  $\frac{1}{16}$  as many 8-inch tiles as she would with 2-inch tiles.



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